PRACTICAL CALCULATION

OF

DYNAMO-ELECTRIC MACHINES

A MANUAL
FOR ELECTRICAL AND MECHANICAL ENGINEERS
AND A TEXT-BOOK
FOR STUDENTS OF ELECTRICAL ENGINEERING

CONTINUOUS CURRENT MACHINERY

BY

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PREFACE.

In the following volume an entirely practical treatise on dynamo-calculation is developed, differing from the usual text-book methods, in which the application of the various formulæ given *requires* more or less experience in dynamo-design. The present treatment of the subject is based upon results obtained in practice and therefore, contrary to the theoretical methods, *gives* such practical experience. Information of this kind is presented in the form of more than a hundred original tables and of nearly five hundred formulæ derived from the data and tests of over two hundred of the best modern dynamos of American as well as European make, comprising all the usual types of field magnets and of armatures, and ranging in all existing sizes.

The author's collection of dynamo-data made use of for this purpose contains full particulars of the following types of continuous current machines:

American Machines.

Edison Single Horseshoe Type,	20	sizes.
" Iron-clad Type,	10	"
" Multipolar Central Station Type,	10	"
" Bipolar Arc Light Type,	6	"
" Fourpolar Marine Type,	4	"
" Small Low-Speed Motor Type,	4	"
" Railway Motor Type,	3	"
Thomson-Houston Arc Light Type,	9	"
" " Spherical Incandescent Type,	4	"
" " Multipolar Type,	3	"
" " Railway Motor Type,	2	"
General Electric Radial Outerpole Type,	12	"
Westinghouse Engine Type ("Kodak")	12	"
" Belt Type,	8	"
" Arc Light Type,	3	"
Brush Double Horseshoe ("Victoria") Type,	16	"

Sprague Double Magnet Type, . . .	13	sizes.
Crocker-Wheeler Bipolar Motor Type, . .	6	"
" " Multipolar Generator Type,	2	"
Entz Multipolar Marine Type, . . .	5	"
Weston Double Horseshoe Type, . . .	3	"
Lundell Multipolar Type, ...	3	"
Short Multipolar Railway Motor Type, .	2	"
Walker Multipolar Type, ...	2	"
	162	

English Machines.

Kapp Inverted Horseshoe Type, . . .	4	sizes.
Edison-Hopkinson Single Horseshoe Type, .	3	"
Patterson & Cooper "Phœnix" Type, . .	3	"
Mather & Platt "Manchester" Type, . .	3	"
Paris & Scott Double Horseshoe Type, .	2	"
Crompton Double Horseshoe Type, . .	1	size.
Kennedy Single Magnet Type, . . .	1	"
"Leeds" Single Magnet Type, . . .	1	"
Immisch Double Magnet Type, . . .	1	"
"Silvertown" Single Horseshoe Type, .	1	"
Elwell-Parker Single Horseshoe Type, . .	1	"
Sayers Double Magnet Type, . . .	1	"
	22	

German Machines.

Siemens & Halske Innerpole Type, . .	3	sizes.
" " Single Horseshoe Type, .	2	"
Allgemeine E. G., Innerpole Type, . .	3	"
" " Outerpole Type, . .	3	"
Schuckert Multipolar Flat Ring Type, . .	3	"
Lahmeyer Iron-clad Type, ...	3	"
Naglo Bros. Innerpole Type, . . .	2	"
Fein Innerpole Type, ...	2	"
" Iron-clad Type, ...	2	"
" Inward Pole Horseshoe Type, . .	2	"
Guelcher Multipolar Type, ...	2	"
Schorch Inward Pole Type, ...	1	size.
Kummer & Co. Radial Multipolar Type, .	1	"
Bollmann Multipolar Disc Type, . . .	1	"
	30	

French Machines.

Gramme Bipolar Type, ...	3 sizes.
Marcel Deprez Multipolar Type, . . .	2 "
Desrozier Multipolar Disc Type, . . .	1 size.
Alsacian Electric Construction Co. Innerpole Type, ...	1 "
	7

Swiss Machines.

Oerlikon Multipolar Type, ...	4 sizes.
" Bipolar Iron-clad Type, . . .	2 "
" Bipolar Double Magnet Type, .	2 "
Brown Double Magnet Type (Brown, Boveri & Co.), ...	2 "
Thury Multipolar Type, ...	1 size.
Alioth & Co. Radial Outerpole Type ("Helvetia"), ...	1 "
	12

In this list are contained the generators used in the central stations of New York, Brooklyn, Boston, Chicago, St. Louis, and San Francisco, United States; of Berlin, Hamburg, Hanover, Duesseldorf, and Darmstadt, Germany; of London, England; of Paris, France; and others; also the General Electric Company's large power generator for the Intramural Railway plant at the Chicago World's Fair, and other dynamos of fame.

The author believes that the abundance and variety of his working material entitles him to consider his formulæ and tables as universally applicable to the calculation of any dynamo.

Although being intended as a text-book for students and a manual for practical dynamo-designers, anyone possessing a but fundamental knowledge of arithmetic and algebra will by means of this work be able to successfully calculate and design any kind of a continuous-current dynamo, the matter being so arranged that all the required practical information is given wherever it is needed for a formula.

The treatise as here presented has originated from notes prepared by the author for the purpose of instructing his

classes of practical workers in the electrical field, and upon the success experienced with these it was decided to publish the method for the benefit of others.

Since the book is to be used for actual workshop practice, the formulæ are so prepared that the results are obtained in inches, feet, pounds, etc. But since the time is approaching when the metric system will be universally employed, and as the book is written for the future as well as for the present, the tables are given both for the English and metric systems of measurement.

As far as the principles of dynamo-electric machinery are concerned, the time-honored method of filling one-third to one-half of each and every treatise on dynamo design with chapters on magnetism, electro-magnetic induction, etc., has in the present volume been departed from, the subject of it being the *calculation* and not the *theory* of the dynamo. For the latter the reader is referred to the numerous text-books, notably those of Professor Silvanus P. Thompson, Houston and Kennelly, Professor D. C. Jackson, Carl Hering, and Professor Dr. E. Kittler. Descriptions of executed machines have also been omitted from this volume, a fairly complete list of references being given instead, in Chapter XIV.

The arrangement of the Parts and Chapters has been carefully worked out with regard to the natural sequence of the subject, the process of dynamo-calculation, in general, consisting (1) in the calculation of the length and size of conductor required for a given output at a certain speed; (2) in the arrangement of this conductor upon a suitable armature; (3) in supplying a magnet frame of proper cross-section to carry the magnetic flux required by that armature, and (4) in determining the field winding necessary to excite the magnetizing force required to produce the desired flux.

Numerous complete examples of practical dynamo calculation are given in Part VIII., the single cases being chosen with a view of obtaining the greatest possible variety of different designs and varying conditions. The leakage examples in Chapter XXX. not only demonstrate the practical application of the formulæ given in Chapters XII. and XIII., but also show the accuracy to which the leakage factor of a

dynamo can be estimated from the dimensions of its magnet frame by the author's formulæ.

A small portion of the subject matter of this volume first appeared as a serial entitled "Practical Notes on Dynamo Calculation," in the *Electrical World*, May 19, 1894 (vol. xxiii. p. 675) to June 8, 1895 (vol. xxv. p. 662), and reprinted in the *Electrical Engineer* (London), June 1, 1894 (vol. xiii., new series, p. 640), to July 12, 1895 (vol. xvi. p. 43). This portion has been thoroughly revised, and by considering all the literature that has appeared on the subject since the serial was written has been brought to date.

It has been the aim of the author to make the book thoroughly practical from beginning to end, and he expresses the hope that he may have attained this end.

The author's thanks are extended to all those firms who upon his request have so courteously supplied him with the data of their latest machines, without which it would not have been possible to bring this work up to date.

Due credit, finally, should also be given to the publishers, who have spared neither trouble nor expense in the production of this volume.

ALFRED E. WIENER.

SCHENECTADY, N. Y.,
September 20, 1897.

PREFACE TO THE SECOND EDITION.

In preparing the second edition, it has been the aim to bring this volume up to date in every particular. For this purpose, data of the latest machines of the most prominent manufacturers were procured by the author and compared with the information given in the book. Since the practice in regard to direct-current machinery has changed but little during the past few years, however, only comparatively few changes in the tables have been found necessary.

A number of new tables have been inserted, in order to facilitate the work of the inexperienced designer to a still greater extent. The most import of these additions to the text are those to § 17 and to § 89. The new matter in § 17 gives additional guidance in the selection of the conductor-velocity, it having been found that too much uncertainty was formerly left in the assumption of this most important factor. With the added help, even a novice in dynamo designing is now enabled to obtain a practical value of the conductor-velocity for any kind of machine. Table LXXXIXa, § 89, serves to check the design with respect to the relation between armature and field. By its use, the performance of a machine in operation can be predicted, thereby avoiding the liability of building a dynamo which would give trouble due to excessive sparking. The importance of such a check will be appreciated by designers who have had experience.

Other new matter has been added, referring to double-current generators, multi-circuit arc dynamos, secondary generators, etc.

Besides these additions to the text, three appendices have been added to the book. Appendix I. gives dimensions and armature data of various types of modern dynamos, thus affording to the student a means of comparing his results with existing machines as he proceeds in the design. Appendix II. contains wire tables and winding data necessary in determining

the windings of dynamos; these tables are added in order to make the book complete in itself, the designer now having close at hand all the necessary data referring to standard wires, rods, cables, etc. Appendix III., finally, in which the causes, localization, and remedies of the usual troubles occurring in dynamo-electric machines are compiled, is given for two purposes: first, to enable the designer, by calling his attention to the ordinary short-comings of electrical machinery, to take such preventive measures in designing a machine as will reduce the liability of trouble in operation to a minimum, thus making his dynamo good in performance as well as economical in operation; and second, to assist the attendant of a dynamo plant in going about in a systematic manner in finding the causes of troubles, so that, by their prompt elimination, unnecessary delay or even a shut-down may be obviated.

In conclusion, the author takes this opportunity to express his sincere thanks to his professional confrères in this country as well as abroad, for the encouraging comments on the first edition of his book. A. E. W.

BROOKLYN, *November*, 1901.

CONTENTS.

CHAPTER IV. DIMENSIONS OF ARMATURE CORE.

CHAPTER VII. MECHANICAL EFFECTS OF ARMATURE WINDING.

CHAPTER VIII. ARMATURE WINDING OF DYNAMO-ELECTRIC MACHINES.

CHAPTER IX. DIMENSIONING OF COMMUTATORS, BRUSHES, AND CURRENT-CONVEYING PARTS OF DYNAMO.

CHAPTER X. MECHANICAL CALCULATIONS ABOUT ARMATURE.

Part III. Calculation of Magnetic Flux.

Chapter XI. Useful and Total Magnetic Flux.

CHAPTER XVIII. MAGNETIZING FORCES.

Part VI. Calculation of Magnet Winding.

CHAPTER XIX. COIL WINDING CALCULATIONS.

CHAPTER XX. SERIES WINDING.

CHAPTER XXVI. CALCULATION OF UNIPOLAR DYNAMOS.

CHAPTER XXVII. CALCULATION OF DYNAMOTORS; GENERATORS FOR SPECIAL PURPOSES, ETC.

APPENDIX I. TABLES OF DIMENSIONS OF MODERN DYNAMOS.

APPENDIX II. WIRE TABLES AND WINDING DATA.

APPENDIX III. LOCALIZATION AND REMEDY OF TROUBLES IN DYNAMOS AND MOTORS IN OPERATION.

LIST OF SYMBOLS.

Throughout the book a uniform system of notation, based upon the standard Congress-notation, is adhered to, the same quantity always being denoted by the same symbol. The following is a complete list of these symbols, here compiled for convenient reference:

AT, at = ampere-turns.

AT = total number of ampere-turns on magnets, at normal load, or magnetizing force.

AT' = total magnetizing force required for maximum output of machine.

AT'' = total magnetizing force required for minimum output of machine.

AT_1 = total magnetizing force required for maximum speed of machine.

AT_2 = total magnetizing force required for minimum speed of machine.

AT_0 = total magnetizing force required at open circuit.

at_a = magnetizing force required for armature core, normal load.

at_{a_0} = magnetizing force required for armature core, open circuit.

$at_{c.i.}$ = magnetizing force required for cast iron portion of magnetic circuit, normal load.

$at_{c.i._0}$ = magnetizing force required for cast iron portion of magnetic circuit, no load.

$at_{c.s.}$ = magnetizing force required for cast steel portion of magnetic circuit, normal load.

$at_{c.s._0}$ = magnetizing force required for cast steel portion of magnetic circuit, no load.

at_g = magnetizing force required for air gaps, normal load.

at_{g_0} = magnetizing force required for air gaps, open circuit.

$at_{g.a.r.}$ = combined magnetizing force required for air gaps, armature core, and reactions.

at_m = magnetizing force required for magnet frame, normal output.

at_{m_o} = magnetizing force required for magnet frame, open circuit.

at_p, at_{p_o} = magnetizing forces required for polepieces.

at_r = magnetizing force required for compensation of armature reactions.

at_s = magnetizing force required to produce a reversing field of sufficient strength for sparkless collection.

$at_{w.i.}$ = magnetizing force required for wrought iron portion of magnetic circuit, normal load.

$at_{w.i._o}$ = magnetizing force required for wrought iron portion of magnetic circuit, no load.

at_y, at_{y_o} = magnetizing forces required for yoke, or yokes.

α = half pole-space angle (also angle of brush-displacement).

$\mathfrak{B}$ = magnetic flux density in magnetic material, in lines per square centimetre.

$\mathfrak{B}''$ = magnetic flux density in magnetic material, in lines per square inch.

$\mathfrak{B}_a$, $\mathfrak{B}''_a$ = average density of magnetic lines in armature core.

$\mathfrak{B}_{a_1}$, $\mathfrak{B}''_{a_1}$ = maximum density of magnetic lines in armature core.

$\mathfrak{B}_{a_2}$, $\mathfrak{B}''_{a_2}$ = minimum density of magnetic lines in armature core.

$\mathfrak{B}_{c.i.}$, $\mathfrak{B}''_{c.i.}$ = mean density of magnetic lines in cast iron portion of frame.

$\mathfrak{B}_{c.s.}$, $\mathfrak{B}''_{c.s.}$ = mean density of magnetic lines in cast steel portion of frame.

$\mathfrak{B}_p$, $\mathfrak{B}''_p$ = mean density of magnetic lines in polepieces.

$\mathfrak{B}_{p_1}$, $\mathfrak{B}''_{p_1}$ = maximum density of magnetic lines in polepieces.

$\mathfrak{B}_{p_2}$, $\mathfrak{B}''_{p_2}$ = minimum magnetic density in polepieces.

$\mathfrak{B}_t$, $\mathfrak{B}''_t$ = magnetic density in armature teeth.

$\mathfrak{B}_{w.i.}$, $\mathfrak{B}''_{w.i.}$ = magnetic density in wrought iron portion of magnetic circuit.

b = breadth, width.

b_a = breadth of armature cross-section, or radial depth of armature core.

b'_a = maximum depth of armature core.

b_b = width of commutator brush.
b_B = breadth of belt.
b_k = circumferential breadth of brush contact.
b_s = width of armature slot.
b'_s = available width of armature slot.
b''_s = width of armature slot for minimum tooth-density.
b_S = smallest breadth of armature spoke (parallel to shaft).
b_t = width, at top, of armature tooth.
b'_t = radial depth to which armature tooth is exposed to magnetic field.
b''_t = width, at root, of armature tooth.
b_y = breadth of yoke.
β = angle embraced by each pole.
β_1 = percentage of polar arc.
β'_1 = percentage of effective arc, or effective field circumference.
γ = electrical conductivity, in mhos.
D, d, δ = diameter.
D_m = external diameter of magnet coil.
D_p = diameter of armature pulley.
d_a = diameter of armature core.
d'_a = mean diameter of armature winding.
d''_a = external diameter of armature (over winding).
d'''_a = mean diameter of armature core.
d_b = diameter of armature bearings.
d_c = diameter of core-portion of armature shaft.
d_f = mean diameter of magnetic field.
d_h = diameter of front head of (drum) armature.
d'_h = diameter of back head of (drum) armature.
d_k = diameter of commutator.
d_m = diameter of magnet core.
d_p = diameter of bore of polepieces.
d_w = diameter of car wheel, in inches.
δ_a = diameter of armature wire, in mils.
δ'_a = width of insulated armature conductor, in inches.
δ''_a = height of insulated armature conductor, in inches.
δ'''_a = pitch of conductors on armature circumference.
δ_i = thickness of iron laminæ in armature core, in inches.
δ_m = diameter of magnet wire, bare, in mils.
δ'_m = diameter of magnet wire, insulated, in mils.

δ_{se} = diameter of series field wire.
δ_{sh} = diameter of shunt field wire.
$(\delta_a)^2$ = sectional area of armature conductor, in circular mils.
$(\delta_a)^2_{mm}$ = sectional area of armature conductor in square millimetres.
$(\delta_{a_1})^2$ = sectional area of single armature wire, in circular mils.
E, e = electromotive force, or pressure, in volts.
E = normal E. M. F. output, or voltage, of generator; terminal E. M. F., or supply voltage of motor.
E' = total E. M. F. induced in armature of generator; counter E. M. F. of motor.
E_0 = total E. M. F. active in armature, on open circuit.
E_1 = total E. M. F. active in armature, at minimum load.
E_2 = total E. M. F. active in armature, at maximum load.
E_m = E. M. F. between terminals of magnet winding.
e = unit armature induction per pair of poles, volts per foot.
e_1 = unit armature induction per pair of poles, volts per metre.
e' = specific induction of active armature conductor, volts per foot.
e'_1 = specific induction of active armature conductor, volts per metre.
e'' = specific generating power of motor, *i. e.*, volts of counter E. M. F., produced at a speed of 1 revolution per minute.
e_2 = volts generated per 100 conductors, per 100 revolutions per minute, and 1 megaline of flux per pole.
e_s = average volts between commutator segments per megaline and per 100 revolutions per minute.
e_a = drop of voltage due to armature resistance.
ε = factor of eddy current loss in armature, English measure (watts per cubic foot).
ε' = factor of eddy current loss in armature, metric measure (watts per cubic metre).
ε_1 = eddy current constant.
$\mathfrak{F}$ = magneto-motive force, in gilberts.
F, f = force, or pull, in pounds.
F_a = total peripheral force of armature, in pounds.
F'_a = peripheral force corresponding to safe working strength of armature spokes, in pounds.

F_B = tension on tight side of belt, in pounds.
F_p = pull at pulley circumference, in pounds.
f_a = peripheral force per armature conductor, in pounds.
f_B = tension on slack side of belt, in pounds.
f_h = horizontal effort, or draw-bar pull of railway motor.
f_k = specific tangential pull due to brush-friction, at 1000 feet per second, in pounds per square inch of contact area.
f'_k = specific tangential pull due to brush-friction, at any velocity, in pounds per square inch of contact area.
f_t = armature thrust, *i. e.*, displacing force acting on armature due to unsymmetrical field.
Φ = useful flux, *i. e.*, number of lines of force cutting armature conductors, at normal output.
Φ_o = useful flux, *i. e.*, number of lines of force cutting armature conductors, at open circuit.
Φ' = total flux, or total number of lines generated, at normal output (maxwells).
Φ'' = total flux per magnetic circuit.
Φ'_P = relative efficiency of magnetic field (maxwells per watt of output at unit conductor velocity).
g = grade of railway track, in per cent.
$\mathcal{H}$ = magnetic flux density in air, or field density, in gausses (lines of force per square centimetre).
$\mathcal{H}''$ = field density, in lines of force per square inch.
$\mathcal{H}_1$, $\mathcal{H}''_1$ = density on stronger side of an unsymmetrical field.
$\mathcal{H}_2$, $\mathcal{H}''_2$ = density on weaker side of an unsymmetrical field.
h = height, thickness.
h_a = total height of winding space in armature (depth of slots).
h'_a = available height of armature winding space.
h_B = thickness of belt, in inches.
h_c = radial height of clearance between external diameter of finished armature and polepieces.
h_i = thickness of commutator side insulation, in inches.
h'_i = thickness of commutator bottom insulation, in inches.
h''_i = thickness of commutator end insulation, in inches.
h_m = height of winding space on field magnets.
h'_m = net height of field winding.
h_p = height of polepieces.

h_s = smallest thickness of armature spoke (perpendicular to shaft).
h_y = height of yoke.
h_z = height of zinc block.
HP, hp = horse power.
η = factor of hysteresis loss in armature, English measure (watts per cubic foot).
η' = factor of hysteresis loss in armature, metric measure (watts per cubic metre).
η_1 = hysteretic resistance.
η_c = commercial efficiency.
η_e = electrical efficiency.
η_g = gross efficiency, or efficiency of conversion.
I, i = intensity of current, amperes.
I = current output, or amperage, of generator; current supplied to motor terminals.
I' = total current active in armature, in amperes.
I_{I}, I_{II}, . . . = currents flowing in coils I, II, . . . of series field regulator.
I_m = current in magnet winding, in amperes.
I_{se} = total series current, in amperes.
I_{sh} = total shunt current, in amperes.
i_a = current density in armature conductor, circular mils per ampere.
i_c = circumferential current density of armature (amperes per unit length of core circumference).
i_m = current density in magnet wire, circular mils per ampere.
i_{se} = current density in series wire, circular mils per ampere.
i_{sh} = current density in shunt wire, circular mils per ampere.
K, k = constants.
k_1, k_2, k_3, . . . = various constants depending upon material, manner of manufacture, and similar conditions.
L, l = length, distance.
L_a = active length of armature conductor, in feet.
L_e = effective length of armature conductor, in feet.
L_m = total length of magnet wire, in feet.
L_{se} = total length of series wire, in feet.
L_{sh} = total length of shunt wire, in feet.
L_t = total length of armature conductor.
l_a = length of armature core, in inches.

l''_a = length of magnetic circuit in armature core, in inches.
l_b = length of armature bearings, in inches.
l'_b = length of gap between adjacent commutator brushes.
l_c = total length of commutator brush contact surface.
$l''_{c.i.}$ = length of magnetic circuit in cast iron portion of field frame.
$l''_{c.s.}$ = length of magnetic circuit in cast steel portion of field frame.
l_f = mean length of magnetic field.
l''_g = length of magnetic circuit in air gaps, in inches.
l_h = length of drum armature heads.
l_k = effective axial length of commutator brush contact surface.
l_m = length of magnet core, in inches.
l'_m = total length of magnet cores, in inches.
l''_m = total length of magnetic circuit in entire field magnet frame.
l_p = length of polepieces, parallel to armature inductors.
l'_p = mean distance between pole-corners, in inches.
l''_p = length of magnetic circuit in polepieces, in inches.
l_s = distance of smallest armature spoke section from active conductors, or leverage at smallest section of armature spokes.
l_t = mean length of turn of field magnet winding, in feet.
l_T = mean length of turn of field magnet winding, in inches.
l'_T = length of mean series turn, in inches.
l''_T = length of mean shunt turn, in inches.
$l''_{w.i.}$ = length of magnetic circuit in wrought iron portion of field frame, in inches.
l'_y = length of yoke, in inches.
l''_y = length of magnetic circuit in yoke, in inches.
λ = factor of magnetic leakage.
λ' = factor of core leakage in machines with toothed or perforated armature.
$\lambda_m = \frac{1}{\rho_m}$ = specific length of magnet wire, in feet per ohm
λ'_m = specific length of magnet wire, in feet per pound.
λ_{se} = specific length of series wire, in feet per ohm.
λ_{sh} = specific length of shunt wire, in feet per ohm.
$M, M_1, \ldots$ = mass, volume.

M = mass of iron in armature core, in cubic feet.
M_1 = mass of iron in armature core, in cubic metres.
M'_1 = mass of iron in armature core, in cubic centimetres.
M_m = volume of coil space on field magnets, in cubic inches.
m = magnetizing force per centimetre length.
m'' = magnetizing force per inch length.
m_a, m''_a = specific magnetizing force of armature core.
$m_{c.i.}$, $m''_{c.i.}$ = specific magnetizing force of cast iron portion of magnetic circuit.
$m_{c.s.}$, $m''_{c.s.}$ = specific magnetizing force of cast steel portion of magnetic circuit.
m_m, m''_m = specific magnetizing force of magnet frame.
m_p, m''_p = specific magnetizing force of polepieces.
m_t, m''_t = specific magnetizing force of armature teeth.
$m_{w.i.}$, $m''_{w.i.}$ = specific magnetizing force of wrought iron portion of magnetic circuit.
m_y, m''_y = specific magnetizing force of yoke.
μ = magnetic permeability.
N, n = number.
N = number of revolutions of armature per minute.
N' = number of revolutions of armature per second.
N_1 = frequency of magnetic reversals, or number of cycles per second.
N_2 = speed of dynamo, when run as motor.
N_a = total number of turns on armature.
N_c = number of conductors around pole-facing circumference of armature.
N_m = number of turns on magnets.
N_{se} = number of series turns.
N_{sh} = number of shunt turns.
n = speed ratio, *i. e.*, abnormal divided by normal speed of machine.
n_1 = speed ratio for maximum speed.
n_2 = speed ratio for minimum speed.
n_a = number of turns per armature coil.
n_b = number of commutator brushes, at one point of commutation.
n_c = number of armature coils, or number of commutator divisions.
n'_c = number of armature slots.

n_δ = number of wires stranded in parallel to make up one armature conductor.

n_f = number of separate field coils in each magnetic circuit.

n_k = number of commutator bars covered by one set of brushes.

n_l = number of layers of wire on armature.

n_m = number of independent armature windings in multiple.

n_p = number of pairs of magnet poles.

n'_p = number of pairs of parallel branches in armature, or number of bifurcations of current in armature.

n''_p = number of pairs of brush sets.

n_r = number of steps, or divisions, in shunt field regulator.

n_s = number of armature circuits connected in series in each of the parallel branches.

n_S = total number of spokes in armature spiders.

n_{se} = number of wires constituting one series field conductor.

n_w = number of armature wires per layer.

n_z = number of magnetic circuits in dynamo.

$\mathfrak{P}$, $\mathfrak{P}_1$, $\mathfrak{P}_2$, . . . = permeances.

$\mathfrak{P}_1$ = relative permeance of gap-spaces.

$\mathfrak{P}_2$ = relative average permeance across magnet cores.

$\mathfrak{P}_3$ = relative permeance across polepieces.

$\mathfrak{P}_4$ = relative permeance between polepieces and yoke.

$\mathfrak{P}'$ = relative permeance of clearance space between poles and external surface of armature.

$\mathfrak{P}''$ = relative permeance of teeth.

$\mathfrak{P}'''$ = relative permeance of slots.

P = electrical energy at terminals of machine; *i. e.*, output of generator, intake of motor.

P' = total electrical energy, active in armature, or electrical activity of machine.

P'' = mechanical energy at dynamo shaft; *i. e.*, driving power of generator, output of motor.

P_A = total energy absorbed in armature.

P_M = total energy absorbed in field circuits.

P_a = energy absorbed in armature winding (C^2R-loss).

P'_a = running value of armature; *i. e.*, energy developed per unit weight of copper at unit speed and unit field density.

P_e = energy absorbed by eddy currents, in watts.

P'_e = energy absorbed by eddy currents, in ergs.
P_f = energy absorbed by brush-friction.
P_h = energy absorbed by hysteresis, in entire armature core.
P'_h = energy absorbed by hysteresis, in solid portion of slotted armature core.
P''_h = energy absorbed by hysteresis, in iron projections of toothed and perforated armatures.
P_k = energy absorbed by contact resistance of brushes.
P_m = energy absorbed in magnet windings.
P_o = energy loss due to air-resistance, brush friction, journal friction, etc.
P'_o = energy required to run dynamo at normal speed on open circuit.
P_{se} = energy absorbed in series winding.
P_{sh} = energy absorbed in shunt winding.
P'_{sh} = energy absorbed in entire shunt-circuit, at normal load.
P_r = energy absorbed in shunt regulating resistance.
P''_x = any load of a motor, in watts.
p_s = safe pressure, or working load, of materials, in pounds per square inch.
π = ratio of circumference to diameter of circle, = 3.1416.
$\mathcal{R}$ = reluctance of magnetic circuit, in oersteds.
R, r = electrical resistance, in ohms.
R = resistance of external circuit.
R_a = total resistance of armature wire, all in series.
r_a = armature resistance, cold, at 15.5° Centigrade.
r'_a = armature resistance, hot, at $(15.5 + \theta_a)$ degrees Cent.
r_m = magnet-resistance, cold, at 15.5° Centigrade.
r'_m = magnet-resistance, warm, at $(15.5 + \theta_m)$ degrees Cent.
r_r = resistance of shunt field regulator.
r_{se} = resistance of series winding, cold, at 15.5° Centigrade.
r'_{se} = resistance of series winding, warm, at $(15.5 + \theta_m)$ degrees Centigrade.
r_{sh} = resistance of shunt winding, cold, at 15.5° Centigrade.
r'_{sh} = resistance of shunt winding, warm, at $(15.5 + \theta_m)$ degrees Centigrade.
r_x = extra-resistance, or shunt regulating resistance in circuit at normal load, in per cent. of magnet resistance.
r_I, r_{II}, ... = resistances of coils I, II, ... of series field regulator.

ρ_k = resistivity of brush-contact, in ohms per square inch of surface.

ρ_m = resistivity of magnet-wire, in ohms per foot.

S = surface, sectional area.

S_A = radiating surface of armature.

S_a = sectional area (corresponding to average specific magnetizing force) of magnetic circuit in armature core.

S_{a_1} = minimum cross-section of armature core.

S_{a_2} = maximum cross-section of armature core.

$S_{c.i.}$ = sectional area of magnetic circuit in cast iron portion of field frame.

$S_{c.s.}$ = sectional area of magnetic circuit in cast steel portion of field frame.

S_f = actual field area ; *i. e.*, area occupied by effective inductors.

S_g = sectional area of magnetic circuit in air gaps.

S'_g = area of clearance spaces in toothed and perforated armature.

S_M = radiating surface of magnets.

S'_M = surface of magnet-cores.

S_m = sectional area of magnet-frame, consisting of but one material.

S_p = area of magnet circuit in polepieces of uniform cross-section.

S_{p_1} = minimum cross-section of polepieces.

S_{p_2} = maximum area of magnetic circuit in polepieces.

S_s = sectional area of armature slot, in metric units.

S''_s = sectional area of armature slot, in square inches.

$S_{w.i.}$ = sectional area of magnetic circuit in wrought iron portion of field frame.

S_y = area of magnetic circuit in yoke.

σ = factor of magnetic saturation.

T, t = time.

τ = torque, or torsional moment.

θ_a = rise of temperature in armature, in degrees Centigrade.

θ'_a = specific temperature increase in armature, in degrees Centigrade.

θ_m = rise of temperature in magnets, in degrees Centigrade.

θ'_m = specific temperature increase in magnets, in degrees Centigrade.

v = velocity, linear speed.
v_B = belt velocity, in feet per minute.
v'_B = belt velocity, in feet per second.
v_c = conductor velocity, or cutting speed, in feet (or metres) per second.
v_k = commutator velocity, in feet per second.
v_m = velocity of railway car, in miles per hour.
W_t, wt = weight.
W_t = total weight to be propelled by railway motor, in tons.
wt_a = weight of armature winding, bare wire, in pounds.
wt'_a = weight of armature winding, covered wire.
wt_m = weight of magnet winding, bare wire.
wt'_m = weight of magnet winding, covered wire.
wt_{se} = weight of series winding, bare wire.
wt'_{se} = weight of series winding, covered wire.
wt_{sh} = weight of shunt winding, bare wire.
wt'_{sh} = weight of shunt winding, covered wire.
X_α, x_α = value of an ordinate corresponding to position, or angle, α.
x = any integer, in formula for number of armature conductors. Exponent of size ratio to give output ratio of two dynamos.
Y = relative hysteresis-heat per unit volume of teeth.
y = connecting-pitch, or spacing, of armature winding; average pitch.
y_b = back-pitch; *i. e.*, connecting-distance on back of armature.
y_f = front-pitch; *i. e.*, connecting distance on front of armature (commutator-side).
z = ratio of speed reduction of railway motor; *i. e.*, ratio of armature revolutions to those of car-axle.
⊕ = conductor carrying current toward observer.
⊙ = conductor carrying current from observer.
○ = singly re-entrant simplex armature winding.
◎ = doubly re-entrant simplex armature winding.
⊚ = triply re-entrant simplex armature winding.
○○ = singly re-entrant duplex armature winding.
◎◎ = doubly re-entrant duplex armature winding.
○○○ = singly re-entrant triplex armature winding.
⊚⊚⊚ = triply re-entrant triplex armature winding.

PART I.

PHYSICAL PRINCIPLES

OF

DYNAMO-ELECTRIC MACHINES.

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PHYSICAL PRINCIPLES OF DYNAMO-ELECTRIC MACHINES.

CHAPTER I.

PRINCIPLES OF CURRENT GENERATION IN ARMATURE.

1. Definition of Dynamo-Electric Machinery.

A *dynamo electric machine*, or a *dynamo*, is a machine in which mechanical energy is converted into electrical energy, or *vice-versa*, by means of electromagnetic induction.

According to this definition, every dynamo-electric machine is capable of serving either as a *generator* or as a *motor*, according to whether it is supplied with mechanical or electrical energy, and whether it is, therefore, giving out electrical or mechanical energy, respectively.

In an electric *generator*, mechanical energy is converted into electrical by means of continuous relative motion between electrical conductors and a magnetic field, or fields, such motion causing the conductors to cut, or traverse, the lines of force of the field.

In an electric *motor*, electrical energy is transformed into mechanical by means of continuously supplying a system of electrical conductors with an electric current which causes a magnetic force to act between the conductors carrying it and the magnetic field, or fields, thereby producing continuous relative motion between the conductors and the magnetic fields.

2. Classification of Armatures.

A system of electrical conductors, arranged for the purpose of converting continuous motion into electrical energy, or of electrical energy into rotation, and attached to a suitable frame, or structure, is called the *armature* of a dynamo-electric machine. According to the manner of the arrangement of the rotating conductors the following kinds of armatures may be distinguished:

(1) *Cylinder* or *Drum Armatures*, in which the conductors are wound longitudinally upon the surface of a cylinder, or drum;

(2) *Ring Armatures*, in which the conductors are wound spirally around a ring-shaped core;

(3) *Pole* or *Star Armatures*, in which the coils are arranged in the form of a star with their axes pointing radially;

(4) *Disc Armatures*, in which the conductors are placed radially upon the surface of a disc-shaped frame, thus forming flat coils having their axes parallel to the shaft;

(5) *Smooth-Core Armatures*, in which the conductors are exterior to the iron core;

(6) *Toothed* or *Slotted Armatures*, in which the conductors are imbedded in slots, or channels, provided upon the surface of the core;

(7) *Perforated* or *Tunnel Armatures*, in which the conductors are drawn through holes, or ducts, extending near the surface of, but entirely within, the iron body of the armature core.

3. Production of Electromotive Force.

When relative motion takes place between a conductor and a magnetic field, two kinds of such movement may be distinguished, according to whether the conductor does or does not *cut* across the lines of magnetic force of the field.

If a conductor $A\ B$, of which $A'\ B'$, Fig. 1, is the plan, $A''\ B''$ the elevation, and $A'''\ B'''$ an end-view, is moved either in the direction of the arrows 1 or 1′, coinciding with its longitudinal axis, or in the direction of the arrows 2 or 2′, coinciding with that of the magnetic lines, or if its motion is composed of both the former and the latter direction, then it either merely passes end-wise through the field between two rows of magnetic lines parallel to its axis, or it slides along

a row of lines, or its motion is compounded of both these movements, passing longitudinally between two rows of lines and in the same time sliding along the lines laterally, respectively, but in none of these cases the conductor intersects, or traverses, the lines of force by cutting them with its length. If, however, the direction of the motion does not fall wholly within the plane containing both the axis of the conductor and the direction of the lines of force, for instance if the motion is perpendicular to the direction of the magnetic lines and also to the axis of the conductor, as shown by arrows 3, 3′,

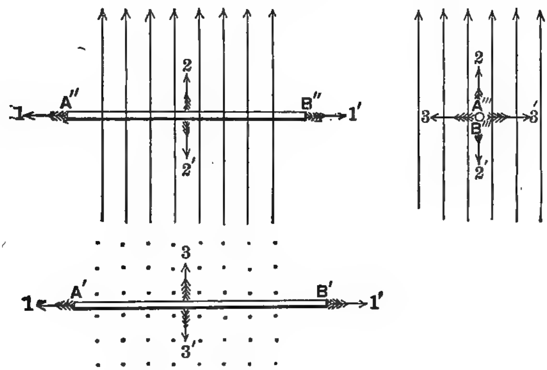

Fig. 1.—Motion of Conductor in Uniform Magnetic Field.

Fig. 1, then the moving conductor *cuts* the lines of induction along its length as it passes through the field.

In case the motion is of the first kind, *i. e.*, if the conductor *does not cut* across any lines of force, no difference of state can be detected in it due to such motion. But if, in case of a motion of the second kind, the conductor *does cut* the magnetic lines, then a difference of electric potential is observed between the ends *A* and *B* during the motion, that is to say, an electromotive force is set up or *induced* in the conductor, which therefore more appropriately may be termed an *inductor*, while it cuts the lines of the magnetic field. Due to this induced E. M. F. the one end of the inductor is raised to a higher potential than the other, in consequence of which there is a tendency for electricity to flow along the inductor, and if the two ends are electrically connected exterior to the

field so as to complete a closed circuit of conducting material, as in Fig. 2, this tendency will be called into action, and a current will flow.

4. Magnitude of Electromotive Force.

The *magnitude* of the E. M. F. produced in the conductor depends upon the *rate* at which the lines of force are cut, that

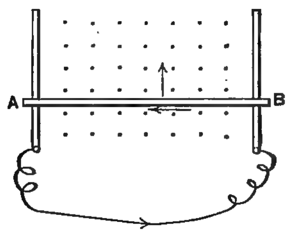

Fig. 2.—Moving Conductor, forming Part of Complete Electric Circuit.

is, upon the total number of magnetic lines cut by the inductor in a unit of time. The number of lines cut in any period of time is given by the actual field area swept through by the moving inductor and by the number of lines per unit of field

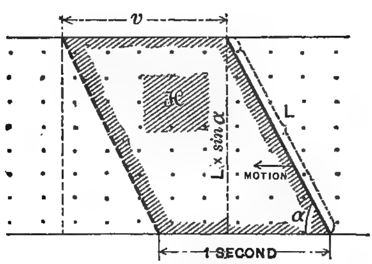

Fig. 3.—Moving Conductor in Uniform Magnetic Field.

area, *i. e.*, by the density of the magnetic lines within that area; the rate at which the lines are cut, therefore, depends upon the *length* of the inductor, the *speed* of its motion, the *strength* of the magnetic field, and upon the *angle* between the moving conductor and the direction of its motion.

If L, Fig. 3. is the length of the inductor, α its angle with

the direction of motion, v its linear velocity per second, and $\mathcal{H}$ the uniform density of the lines, then the total number of lines cut per second, Φ, is the product of the area swept and of the density, thus:

$$\Phi = L \times \sin \alpha \times v \times \mathcal{H} \quad \ldots\ldots\ldots\ldots (1)$$

In practical dynamos the inductors are usually so arranged upon the armature that their axes are perpendicular to the direction of the motion, *i. e.*, so that $\alpha = 90^\circ$, and for this practical case we have:

$$\begin{aligned}\Phi &= L \times \sin 90^\circ \times v \times \mathcal{H} \\ &= L \times v \times \mathcal{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)\end{aligned}$$

The E. M. F. induced in the moving inductor is proportional to this number, hence:

$$E = k \times \Phi = k \times L \times v \times \mathcal{H}, \quad \ldots\ldots (3)$$

where E = E. M. F. induced in moving inductor;
Φ = total number of lines cut per second;
L = length of moving inductor;
v = linear velocity of inductor per second;
$\mathcal{H}$ = average density of magnetic field;
k = constant, whose value depends upon units chosen.

Now, the absolute electric and magnetic systems of units are so related with each other that, if the number of magnetic lines cut per second is expressed in C. G. S. units, the result of formula (3) gives directly the E. M. F. induced, expressed in absolute units, or in other words, if an inductor cuts 1 C. G. S. line per second, the difference of potential induced in its length by the motion causing such cutting, is 1 absolute unit of E. M. F. In the C. G. S. system, consequently, the constant $k = 1$. The practical unit of E. M. F., *1 volt*, is one hundred million times greater than the absolute unit, which is inconveniently small, and, in consequence, 100,000,000 C. G. S. lines of force cut per second produce one volt of E. M. F. If, therefore, Φ is reckoned in C. G. S. lines, and E is to be measured, as usual, in volts, the value of the constant is

$$k = \frac{1}{100,000,000} = 10^{-8};$$

and the formula for the E. M. F., in practical units, becomes:

$$E = L \times v \times \mathcal{H} \times 10^{-8} \text{ volts}, \quad \ldots\ldots\ldots(4)$$

and now: L = length of inductor, in centimetres;
v = cutting-velocity, in centimetres per second;
$\mathcal{H}$ = density of field, in C. G. S. lines per square centimetre.

5. Average Electromotive Force.

If the *rate* of cutting lines of force is constant, the E. M. F. induced at any instant is the same throughout the motion of

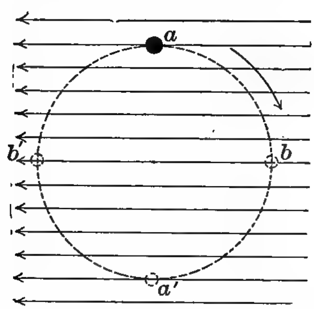

Fig. 4.—Inductor Describing Circle in Magnetic Field.

the conductor, but if either the cutting-speed or the density of the field varies, the instantaneous values of the E. M. F. vary accordingly, and the *average E. M. F.* generated in the inductor is the geometrical mean of all the instantaneous values.

In a dynamo each inductor is carried in a circle through a more or less homogeneous field; in two diametrically opposite positions therefore, at a and a', Fig. 4, its motion is parallel to the lines of force, while at two positions, b and b', at right angles to a and a', the inductor moves perpendicular to the lines. In positions a and a', consequently, no lines are cut, and the induced E. M. F. is $E = 0$, while at b and b' the maximum number of lines is cut in unit time, and E has its maximum value. Between these two extremes any possible value of E exists, according to the angular position of the inductor.

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armature *coils* is formed. According to the number of inductors in each loop there are two kinds of armature coils. In *ring* armatures, Fig. 7, each coil contains but *one inductor* per turn, while in *drum* armatures, Fig. 8, every convolution of the coil is formed of *two inductors* and two connecting conductors. A

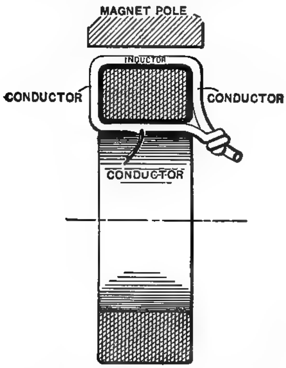


Fig. 7.—Ring Armature Coil.

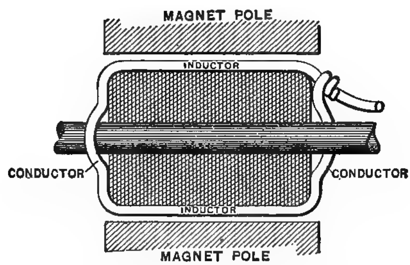


Fig. 8.—Drum Armature Coil.

ring armature coil, therefore, when moved so as to cut the lines of a magnetic field, has only one E. M. F. induced in it; in a drum armature coil, however, E. M. Fs. are induced in both the inductors, and these two E. M. Fs. may be of the same or of opposite directions, according to the manner in which the coil

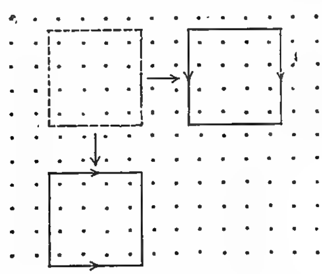
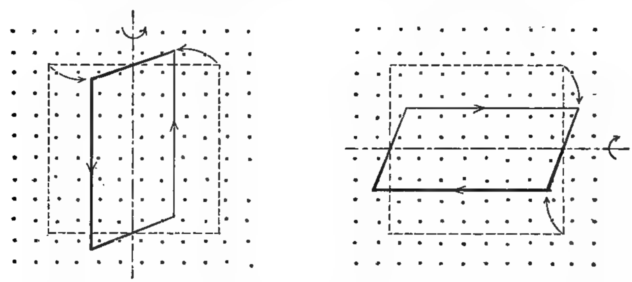


Fig. 9.—Closed Coil moving Horizontally in Magnetic Field.

is moved with respect to the lines of force. If the relative position between the magnetic axis of the coil and the direction of the lines does not change, that is, if the angle enclosed by them remains the same during the entire motion of the coil, as in Fig. 9, the E. M. Fs. induced in the two halves counter-

act each other, while when the coil is revolved about an axis perpendicular to the direction of the lines of force, as in Figs. 10 and 11, the E. M. Fs. in the two inductors have opposite directions, and therefore add each other when flowing around the coil.

Since in the former case, Fig. 9, the number of lines through the coil does not *change*, while in the latter case, Figs. 10 and 11, it does, it follows that *E. M. F. is induced in a closed circuit, if this circuit moves in a magnetic field so that the number of lines of force passing through it is altered during the motion.* By applying the finger-rule to the single elements of the coil it is found that



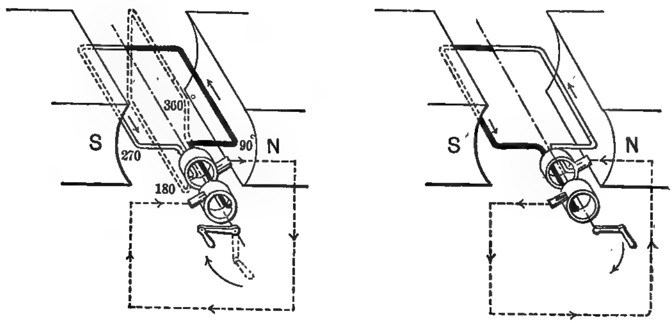
Figs. 10 and 11.—Closed Coil Revolving in Magnetic Field.

the direction of the induced current is *clockwise*, viewed in the direction with the lines, if the motion is such as to cause a *decrease* in the number of lines; and is *counter-clockwise*, if the motion effects an *increase* in the number of lines.

7. Collection of Current from Armature Coil.

If a coil is revolved in a uniform magnetic field, the number of lines threading through it will twice in each revolution be zero, once a maximum in one direction, and once in the other. If, therefore, the current of that coil is collected by means of *collector-rings* and *brushes*, Figs. 12 and 13, it will traverse the external circuit, from brush to brush, in one direction for one-half of a revolution and in the opposite direction in the other half, or an *alternating current* is produced by the coil. In plotting the positions of the coil in the magnetic field as ordinates and the corresponding instantaneous values of the

induced E. M. F. as abscissæ, the *curve of induced E. M. Fs.*, or, since the electrical resistance of the circuit is constant during the motion of the coil, the *curve of induced currents* is



Figs. 12 and 13.—Collection of Armature Current.

obtained, Fig. 14. Since the instantaneous value e_φ at any moment is expressed by the product of the maximum value and the sine of the angle through which the coil has moved,

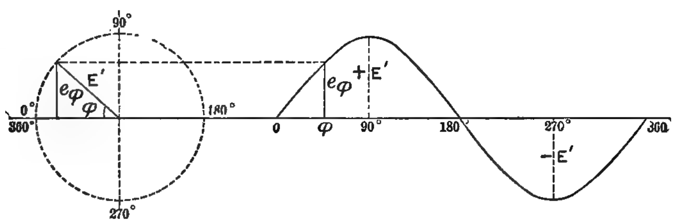


Fig. 14.—Curve of Induced E. M. Fs.

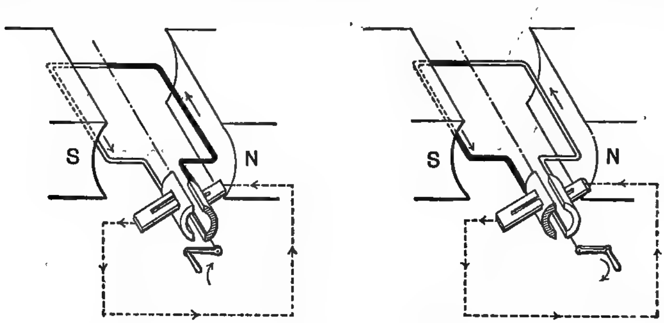
viz., $e_\varphi = E' \times \sin \varphi$, the curve of the induced E. M. Fs., in a uniform magnetic field, is a *sine-wave*, or a *sinusoid.*

8. Rectification of Alternating Currents.

By means of a device called a *commutator*, the alternating current delivered by the coil to the external circuit can be *rectified* so as to flow always in the same direction, the negative inductions being *commutated* into positive ones, and the alternating current transformed into a *uni-directed* or *continuous current.*

A commutator employed for this purpose in continuous current dynamos consists of as many conducting cylinder segments

or circle-sectors as there are coils, in case of a ring armature, and has twice as many commutator-bars or -divisions as there are coils in the case of a drum armature, each commutator-bar being insulated from its neighbors, but in electrical connection with the armature coils and rotating with them. The process



Figs. 15 and 16.—Commutation of Armature Current.

of rectification of the currents generated in the drum armature coil of Figs. 12 and 13 by means of a two-division commutator is shown in Figs. 15 and 16, of which the former refers to the first and the latter to the second half-revolution of the coil. The corresponding curve of the induced E. M. Fs. is represented in Fig. 17, which shows that the current issuing from a

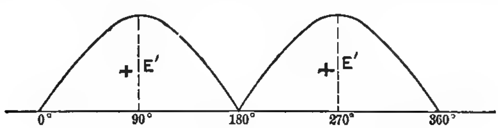


Fig. 17.—Rectified Curve of E. M. Fs.

single coil is of a pulsating character, its value periodically increasing from zero to a maximum, and decreasing again to zero.

9. Fluctuations of Commutated Currents.

The instantaneous E. M. Fs. induced in a single coil varying between the values $e_{\min} = 0$ and $e_{\max} = E'$, the mean E. M. F. is

$$\frac{1}{2}(0 + E') = \frac{E'}{2},$$

and the amount of fluctuation, with a two-division commutator, is

$$\left.\begin{aligned}\frac{e_{\text{max}} - e_{\text{mean}}}{e_{\text{max}}} &= \frac{E' - \frac{E'}{2}}{E'} \\ \frac{e_{\text{min}} - e_{\text{mean}}}{e_{\text{max}}} &= \frac{0 - \frac{E'}{2}}{E'}\end{aligned}\right\} = \pm .5, \text{ or } \pm 50\%.$$

In order to obtain a less fluctuating current, it is necessary to employ more than one armature coil, the current growing

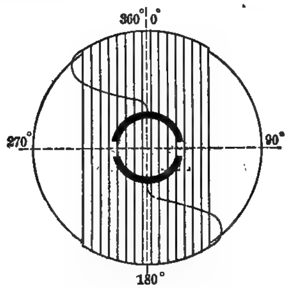


Fig. 18.—One-Coil Armature.

the steadier, the greater the number of the coils. If a coil of, say, 16 turns, Fig. 18, generating a maximum E. M. F. of $e_{\text{max}} = E'$ volts, is split up into two coils of half the number of turns each, which are set at right angles to each other, Fig. 19, each will only generate half the maximum E. M. F.

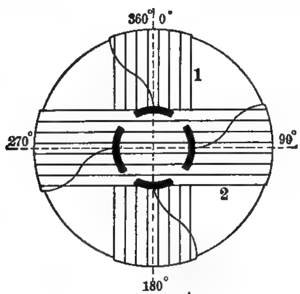


Fig. 19.—Two-Coil Armature.

of the original coil, viz.:

$$(e_1)_{\text{max}} = (e_2)_{\text{max}} = \frac{E'}{2},$$

but each of them will have this maximum value while the other one passes through the position of zero induction, as is shown in Fig. 20. Hence, if the E. M. Fs. of the two coils are

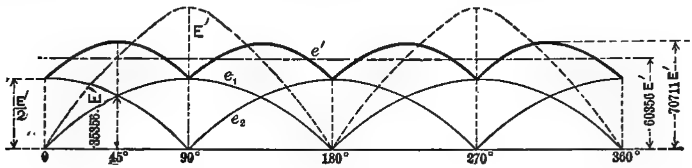


Fig. 20.—Fluctuation of E. M. F. in Two-Coil Armature.

added by means of a four-division commutator, the minimum joint E. M. F. in this case is

$$e'_{\min} = \frac{E'}{2},$$

while the total maximum E. M. F., the maximum inductions in the two coils not occurring at the same time, does not reach the maximum value E' of the undivided coil, but, being the

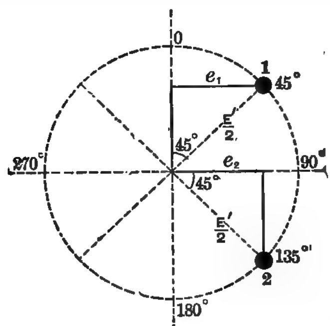


Fig. 21.—E. M. Fs. in Two-Coil Armature at one-eighth Revolution.

sum of the E. M. Fs. induced at one-eighth revolution, when both partial E. M. Fs. are equal, is, with reference to Fig. 21,

$$e'_{\max} = (e_1)_{45^\circ} + (e_2)_{45^\circ} = \frac{E'}{2}(\sin 45^\circ + \cos 45^\circ)$$

$$= \frac{E'}{2}\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}\right) = \frac{E'}{\sqrt{2}} = .70711\ E'.$$

The mean E. M. F., therefore, is

$$e'_{\text{mean}} = \frac{1}{2}(e'_{\min} + e'_{\max}) = \frac{1}{2}(.5 + .70711)\ E' = .60356\ E',$$

and the fluctuation of the E. M. F., with a four-division commutator, amounts to

$$\left.\begin{aligned}\frac{e'_{\max}-e'_{\text{mean}}}{e'_{\max}} &= \frac{(.70711-.60356)\,E'}{.70711\,E'}\\ \frac{e'_{\min}-e'_{\text{mean}}}{e'_{\max}} &= \frac{(.5-.60356)\,E'}{.70711\,E'}\end{aligned}\right\} = \pm\frac{.10356}{.70711} = \pm .1465,$$

or 14.65%.

If each of the two coils 1 and 2, Fig. 19, is again subdivided into two coils of half the number of turns, four coils, 1′, 2′, 3′,

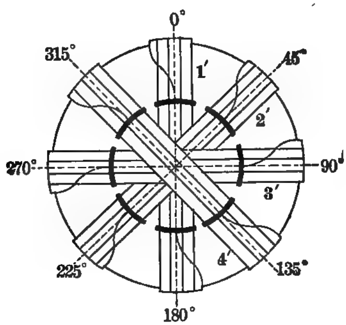


Fig. 22.—Four-Coil Armature.

and 4′, are obtained which make angles of 45° with each other, Fig. 22. Plotting the curves of E. M. Fs., therefore, we get

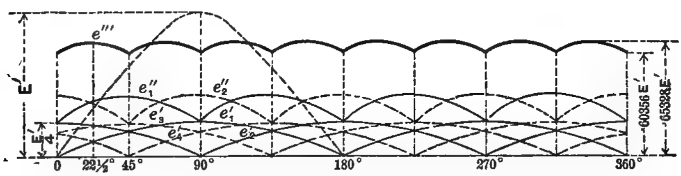


Fig. 23.—Fluctuations of E. M. F. in Four-Coil Armature.

four waves, e_1', e_2', e_3' and e_4', Fig. 23, each varying between the values

$$(e_1')_{\min} = (e_2')_{\min} = (e_3')_{\min} = (e_4')_{\min} = 0$$

and

$$(e_1')_{\max} = (e_2')_{\max} = (e_3')_{\max} = (e_4')_{\max} = \frac{E'}{4},$$

and each starting 45° from its neighbor. In combining each two waves 90° apart, by adding their respective ordinates, the

four waves are reduced to two, viz., e_1'' and e_2'', the addition of which, finally, renders the resultant curve of reduced E. M. F., e''', which fluctuates between the values

$$e'''_{\text{min}} = (e_1'')_{\text{max}} + (e_2'')_{\text{min}} = 2 \times \frac{E'}{4} \sin 45^\circ + \frac{E'}{4}$$

$$= \frac{E'}{2} \times \frac{1}{2} \sqrt{2} + \frac{E'}{4} = (.35356 + .25)\, E' = .60356\, E',$$

and

$$e'''_{\text{max}} = (e_1'')_{22\frac{1}{2}^\circ} + (e_2'')_{22\frac{1}{2}^\circ} = 2 \times \frac{E'}{4} (\sin 22\tfrac{1}{2}^\circ + \cos 22\tfrac{1}{2}^\circ)$$

$$= \frac{.38268 + .92388}{2} E' = .65328\, E'.$$

From this follows the mean E. M. F. obtained with an eight-division commutator:

$$e'''_{\text{mean}} = \frac{1}{2} (.60356 + .65328)\, E' = .62842\, E',$$

giving a fluctuation of the maximum E. M. F. in the amount of

$$\left.\begin{array}{l} \dfrac{e'''_{\text{max}} - e'''_{\text{mean}}}{e'''_{\text{max}}} = \dfrac{(.65328 - .62842)\, E'}{.65328\, E'} \\[2ex] \dfrac{e'''_{\text{min}} - e'''_{\text{mean}}}{e'''_{\text{max}}} = \dfrac{(.60356 - .62842)\, E'}{.65328\, E'} \end{array}\right\} = \pm \frac{.02486}{.65328}$$

$$= \pm .0386, \text{ or } 3.86\%.$$

The above calculations show that the percentage of fluctuation rapidly diminishes as the number of armature coils increases, and in continuing the process of subdividing the coils into sections symmetrically spaced at equal angles, we will get for resultants curves which more and more resemble a straight line, and thus indicate the approaching entire disappearance of fluctuations and, therefore, continuity of the E. M. F. In the following Table I. the numerical results of such continued subdivision of the armature coils are given, the original maximum E. M. F. E' being for convenience taken as unity:

TABLE I.—FLUCTUATION OF E. M. F. OF COMMUTATED CURRENTS.

NUMBER OF COMMUTATOR DIVISIONS.	ANGLE EMBRACED BY EACH COIL.	MAXIMUM E. M. F.	MINIMUM E. M. F.	MEAN E. M. F.	AMOUNT OF FLUCTUATION.	FLUCTUATION IN P. CENT. OF MAX. E. M. F.
2	180°	1.	0.	.5	±.5	±50%
4	90	.70711	.5	.60356	.10356	14.65
8	45	.65328	.60356	.62842	.02456	3.86
12	30	.64395	.62201	.63298	.01097	1.70
18	20	.63987	.63014	.63500	.00487	.76
24	15	.63844	.63298	.63571	.00273	.43
36	10	.63743	.63501	.63622	.00121	.19
48	$7\frac{1}{2}$	.63708	.63571	.636395	.000685	.107
60	6	.63691	.63604	.636475	.000435	.068
90	4	.63675	.63637	.63656	.000190	.030
180	2	.63665	.63656	.636605	.000045	.007
360	1	.63664	.63660	.63662	.000020	.003

The *average* E. M. F., that is, the geometrical mean of all the sums of instantaneous E. M. Fs. induced in the various

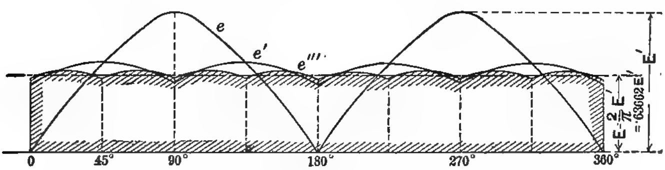


Fig. 24.—Average E. M. F. Induced in Rotating Armature.

subdivisions of the coil, must be the same in every case, for, the total number of turns, the speed, and the field-strength remain the same for any number of commutator divisions.

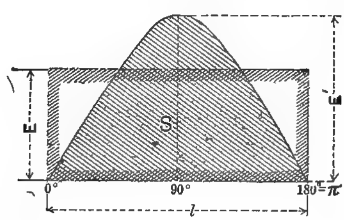


Fig. 25.—Average E. M. F. of One-Coil Armature.

Numerically, the average E. M. F. is the height of a rectangle having an area equal to the surface extending between the axis of abscissæ, the two end-ordinates, and the curve of

E. M. F., as shown in Fig. 24. In case of the one-coil armature the average E. M. F., in considering one-half of a revolution, is the height of a rectangle equal to the area of a single wave having E' as its amplitude. The area S inclosed by a sinusoid of amplitude E' and length l, Fig. 25, is:

$$S = \frac{l}{\pi} E' \int_0^\pi \sin x \, dx = \frac{E' l}{\pi} (-\cos 180^\circ - (-\cos 0^\circ))$$
$$= \frac{E' l}{\pi} (1 + 1) = \frac{2}{\pi} (E' l),$$

therefore the average E. M. F.

$$E = \frac{S}{l} = \frac{2}{\pi} \times E' = .63662\, E'. \quad \ldots\ldots.(7)$$

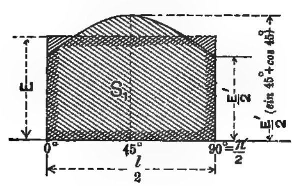


Fig. 26.—Average E. M. F. of Two-Coil Armature.

For the two-coil armature the area S_1, Fig. 26, of one-quarter of a revolution is the sum of a rectangle of length

$$\frac{l}{2}$$

and height

$$\frac{E'}{2},$$

and of a wave of amplitude

$$\frac{E'}{2} \left\{ (\sin 45^\circ + \cos 45^\circ) - 1 \right\}$$

and length

$$\frac{l}{2},$$

or:

$$S_1 = \frac{E'}{2} \times \frac{l}{2} + \frac{l}{\pi} E' \int_0^{\frac{\pi}{2}} \left[\frac{1}{2} (\sin x + \cos x) - .5 \right] dx$$

$$= E'l \left\{ \frac{1}{4} + \frac{1}{\pi} \left(-\frac{1}{2} \cos x + \frac{1}{2} \sin x + \frac{1}{2} x \right)_0^{\frac{\pi}{2}} \right\}$$

$$= E'l \left\{ \frac{1}{4} + \frac{1}{\pi} \left(0 + \frac{1}{2} - \frac{\pi}{4} + \frac{1}{2} - 0 + 0 \right) \right\}$$

$$= E'l \left\{ \frac{1}{4} + \frac{1}{\pi} \left(1 - \frac{\pi}{4} \right) \right\} = E'l \left(\frac{1}{4} + \frac{1}{\pi} - \frac{1}{4} \right) = \frac{E'l}{\pi}$$

The average E. M. F. in this case is:

$$E = \frac{S_1}{\frac{l}{2}} = \frac{2\,E'l}{\pi\,l} = \frac{2}{\pi} E' = .63662\,E',$$

which is the same as obtained above for the case of a one-coil armature. In the same manner the average E. M. F. is obtained for any number of coils and is invariably found to be .63662 of the maximum E. M. F. produced if all of the inductive wire is wound in but one coil and connected to the external circuit by a two-division commutator.

As might be expected from the definition of the average E. M. F., it will be noted that the values of the *mean* E. M. F., column 5, Table I., for increasing number of commutator divisions, approach the figure .63662 for the *average* E. M. F. as a limit.

CHAPTER II.

THE MAGNETIC FIELD OF DYNAMO-ELECTRIC MACHINES.

10. Unipolar, Bipolar, and Multipolar Induction.

From the previous chapter it is evident that an E. M. F. will be induced in a conductor:

(1) When the conductor is moved across the lines of force of the field in a direction perpendicular to its own axis and perpendicular to the direction of the lines, Fig. 27; and

(2) When the conductor is revolved in the field about an axis perpendicular to the direction of the lines, Fig. 28.

In the first case, the inductor *aa*, Fig. 27, as it cuts the lines

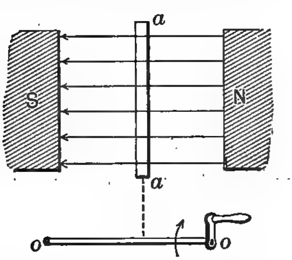


Fig. 27.—Unipolar Induction.

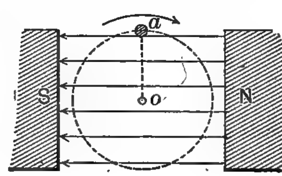


Fig. 28.—Bipolar Induction.

of the magnetic field but *once* in each revolution around the axis *oo*, and in the *same* direction each time, is the seat of a *uni-directed* or *continuous E. M. F.* In the second case, however, the inductor *a*, Fig. 28, in revolving about the axis *o*, cuts the lines of the field *twice* in each revolution, and cuts them in the *opposite* direction alternately; the inductor *a*, therefore, is the seat of an *alternating E. M. F.* whose direction undergoes reversal twice every revolution. If the conductor *a* is made to rotate in a multiple field formed of more than one pair of magnet poles, Fig. 29, it cuts the lines of all the individual fields, between each two poles, in alternate directions, and an alternating E. M. F. is induced in it, whose direction reverses

as many times in every revolution as there are poles to form the multiple field. Since the induced E. M. F. in the first case always has the same direction along the length of the conductor, in the second case has two reversals in every revolution, and in the third case reverses its direction as many times as there are poles, three different kinds of inductions are dis-

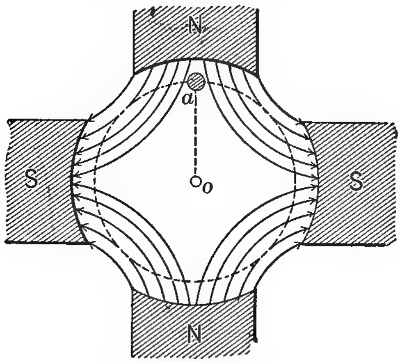


Fig. 29.—Multipolar Induction.

tinguished accordingly, viz.: *Unipolar*, *Bipolar*, and *Multipolar* induction, respectively.

As induction due to but one pole cannot exist, the term "unipolar induction," if strictly interpreted, is both incorrect and misleading, and Professor Silvanus P. Thompson, in the latest (fifth) edition of his "Dynamo-Electric Machinery," therefore uses the word *homopolar* (homo=alike) for unipolar, and *heteropolar* (hetero=different) for bi- and multipolar induction.

11. Unipolar Dynamos.

In carrying out practically the principle of unipolar induction, as illustrated in Fig. 27, the poles of the magnet are made tubular and the conductor extended into the form of a disc or of a cylinder-ring, Figs. 30 and 31, respectively, in order to cause the unidirected E. M. F. to be maintained continuously at a constant value. The solid disc or solid cylinder-ring inductor is to be considered as a number of contiguous strips, in electrical contact with each other, thus forming a number of conductors in parallel which carry a correspondingly larger

current, but which do not increase the amount of E. M. F. induced.

In order to increase the E. M. F. it would be necessary to connect two or more conductors in series, thereby multiplying the inducing length. But heretofore all methods which have been experimented with to achieve the end of grouping in series the conductors on a unipolar dynamo armature have failed, for the reason that the conductor which would have to

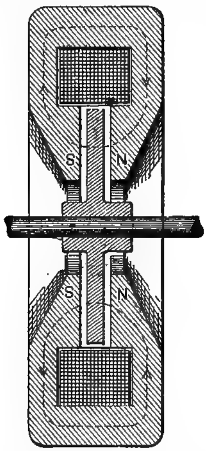


Fig. 30.—Unipolar Disc Dynamo.

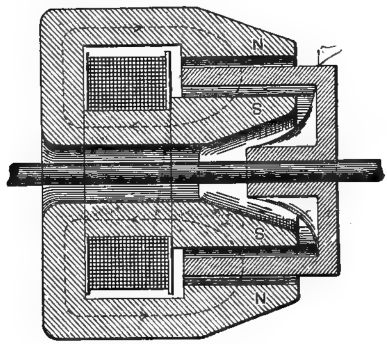


Fig. 31.—Unipolar Cylinder Dynamo.

be used to connect the two inductors with each other will itself become an inductor, and, being joined to oppositely situated ends of the two adjoining inductors, will neutralize the E. M. F. produced in a length of inductor equal to its own length. No matter, therefore, how many inductors are placed "in series" on the armature, the resulting E. M. F. will correspond to the length of but one of them. By adapting the ring armature to this class of machines, winding the conductor alternately backward and forward across the field which is made discontinuous by dividing up the polefaces into separate projections, loops of several inductors in series can be formed, round which the E. M. F. and current alternate, the characteristic feature of the unipolar continuous current dynamo being thereby lost, and unipolar *alternators* being obtained.

Unipolar dynamos being the only natural continuous current

machines not requiring commutating devices, it is but a matter of course that attempts are continually being made to render these machines useful for technical purposes; but unless the points brought out in the following are kept in mind, such attempts will be of no avail.[1]

From the fact that unipolar dynamos have practically but one conductor, it is evident that its length must be made rather great, and the whole machine rather cumbersome in consequence, in order to obtain sufficient voltage for commercial uses. But since a very large amount of current may be drawn from a solid disc or cylinder-ring, it follows that unipolar dynamos can be practical machines only if built for very large current outputs, such as will be required for metallurgical purposes and for central station incandescent lighting.

Professor F. B. Crocker and C. H. Parmly[2] have recently taken up this subject in a paper presented to the American Institute of Electrical Engineers, and have shown that the only practical manner in which the unipolar dynamo problem can be solved, is by the use of large solid discs or cylinder-rings of wrought iron or steel run at very high speed between the poles of strong tubular magnets. The greatest advantage of such unipolar machines is their extreme simplicity, the armature having no winding and no commutator. The almost infinitesimal armature resistance not only effects increased efficiency and decreased heating, but also causes the machine to regulate more closely either as a generator or as a motor. Furthermore, there is no hysteresis, because the armature and field are always magnetized in exactly the same direction and

[1] See "Unipolar Dynamos which will Generate No Current," by Carl Hering, *Electrical World*, vol. xxiii. p. 53 (January 13, 1894); A. Randolph, *Electrical World*, vol. xxiii. p. 145 (February 3, 1894); Bruce Ford, *Electrical World*, vol. xxiii. p. 238 (February 24, 1894); G. M. Warner, *Electrical World*, vol. xxiii. p. 431 (March 31, 1894); A. G. Webster, *Electrical World*, vol. xxiii. p. 491 (April 14, 1894); Professor Lecher, *Elektrotechn. Zeitschr.*, January 1, 1895, *Electrical World*, vol. xxv. p. 147 (February 2, 1895); Professor Arnold, *Elektrotechn. Zeitschr.*, March 7, 1895, *Electrical World*, vol. xxv. p. 427 (April 6, 1895).

[2] "Unipolar Dynamos for Electric Light and Power," by F. B. Crocker and C. H. Parmly, *Trans. A. I. E. E.*, vol. xi. p. 406 (May 16, 1894); *Electrical World*, vol. xxiii. p. 738 (June 2, 1894); *Electrical Engineer*, vol. xvii. p. 468 (May 30, 1894).

to precisely the same intensity. For similar reasons there are no eddy currents, since the E. M. F. generated in any element of the armature is exactly equal to that induced in any other element, the magnetic field being perfectly uniform, owing to the exactly symmetrical construction of the magnet frame. The armature conductor consists of only one single length, consequently the maximum magnetizing effect of the armature in ampere turns is numerically equal to its current capacity, and since the field excitation is considerably greater than this, the armature reaction cannot be great. The armature reaction has the effect of distorting and slightly lengthening the lines of force, so that they do not pass perpendicularly from one pole surface to the other in the air gap and have a spiral path in the iron. For, the field current tends to produce lines in planes passing through the axis, while the armature current acts at right angles to the field current and produces an inclined resultant. There can, of course, be no change of distribution of magnetism as a result of armature reaction, which is the really objectionable effect that it produces in bipolar and multipolar machines. Unipolar machines having no back ampere turns, an extremely small air gap, and but very little magnetic leakage, their exciting power needs to be but very small, comparatively, and they have, therefore, a very economical magnetic field. Machines of the type recommended by Professor Crocker, finally, are practically indestructible, since they are so simple and can be made so strong that they are not likely to be damaged mechanically, while it is almost impossible to conceive of an armature being burnt out or otherwise injured electrically, as the engine would be stalled by the current before it reached the enormous strength necessary to fuse the armature.

Machines possessing all these important advantages certainly deserve a prominent place in electrical engineering, whereas they now have practically no existence whatever.

12. Bipolar Dynamos.

While the homopolar (unipolar) dynamo is naturally a continuous current dynamo, the heteropolar (bipolar and multipolar) dynamo is naturally an alternating current machine, and has to be artificially made to render continuous currents by

means of a commutator. But in heteropolar machines any number of inductors may be connected in series, and consequently high E. M. Fs. may be produced with comparatively small-sized armatures. In Fig. 32 a ring armature placed in a bipolar field is shown. The magnetic lines emanating from the *N*-pole, in passing over to the *S*-pole of the field magnet, first cross the adjacent gap-space, then traverse the armature core, and finally pass across the gap-space at the opposite side. The inductors of the armature as they revolve will cut these magnetic lines twice in every revolution, once each as

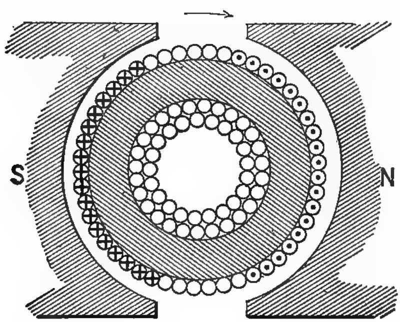


Fig. 32.—Ring Armature in Bipolar Field.

they pass through either gap. If the rule for the direction of the induced E. M. F., as given in § 6, is now applied, it is found that in all the inductors that *descend* through the right-hand gap-space the direction of the induced current is *from* the observer, while in all inductors that *ascend* through the left-hand gap-space it is *toward* the observer.

If an armature is wound as a *ring*, the currents which are produced in the inductors in the gap-space are added up by conductors carrying the currents through the inside of the ring; when, however, the armature is wound as a *drum*, the currents simply cross at the ends of the core through connecting conductors provided to complete a closed electric circuit. In this manner armature *coils* are formed, in ring as well as in drum armatures, which are grouped symmetrically around the armature core. In order to yield a continuous current these coils must be connected at regular intervals to the respective bars of a commutator, as illustrated by Fig. 33. The currents

induced in the two gap-spaces will then unite at the top-bar b, and will flow together in the upper brush, which, therefore, is the positive brush in this case, and thence will return, through the external circuit, to the lower or negative brush and will there re-enter the armature at the lowest bar b_1 of the commutator, dividing again into two parts and flowing through the two halves of the winding in parallel circuits. The preceding equally applies to a drum winding, but owing to the overlapping

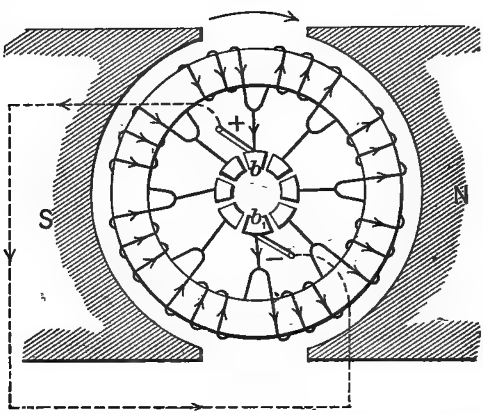


Fig. 33.—Commutator Connections of Bipolar Ring Armature.

of the two halves of the windings, the paths of the currents cannot be followed up as easily as in a ring winding.

By inspection of the diagram, Fig. 33, it is seen that the current after having divided in its two paths goes from coil to coil without flowing down in any of the commutator bars, until both streams unite at the other side and pass down into the bar of the commutator which is at the time passing under the brush. At the instant when one of the commutator segments is just leaving contact with the brush and another one is coming into contact with it, the brush will rest upon two adjacent bars and will momentarily short-circuit one of the coils. While this lasts the two streams will unite by both flowing into the same brush from the two adjacent commutator segments. A moment later the short-circuited coil when it has passed the brush will belong to the other half of the armature, that is to say, in the act of passing the brush

every coil will be *transferred* from one half of the armature to the other, and will have its current *reversed*. This is, in fact, the *act of commutation*, and the conditions under which it takes place govern the proper functioning of the machine when running, as they directly control the presence and amount of *sparking* at the brushes.

The production of sparks is a consequence of the property of *self-induction* in virtue of which, owing to the current in a conductor setting up a magnetic field of its own, it is impossible to instantaneously start, stop, or reverse a current. If the act of commutation occurs exactly at the point when the short-circuited coils under the two brushes are not cutting any magnetic lines at all, no E. M. F. is induced in them at the time and they are perfectly idle when entering the other half of the armature winding. On account of the self-induction the current cannot instantly rise to its full strength in these idle coils, and it will spark across the commutator bars as the brushes leave them. From this can be concluded that the ideal arrangement is attained if the brushes are shifted just so far beyond the point of maximum E. M. F. that, while each successive coil passes under the brush and is short-circuited, it should actually have a reverse E. M. F. of such an amount induced in it as to cause a current of the opposite direction to circulate in it, exactly equal in strength to that which is flowing in the other half of the armature which it is then ready to join without sparking. A magnetic field of the proper intensity to cause the current in the short-circuited coil to be stopped, reversed, and started at equal strength in the opposite direction can usually be found just outside the tip of the polepiece, for here the fringe of magnetic lines presents a density which increases very rapidly toward the polepiece. Since a more intense field is needed to reverse a large current than is required for a small one, it follows that for sparkless commutation the brushes must be shifted through the greater an angle the greater the current output of the armature. Since it takes a certain length of time to reverse a current, the brushes must be of sufficient thickness to short-circuit the coils for that length of time, while on the other hand they must not be so wide as to short-circuit a number of coils at the time, as this again would

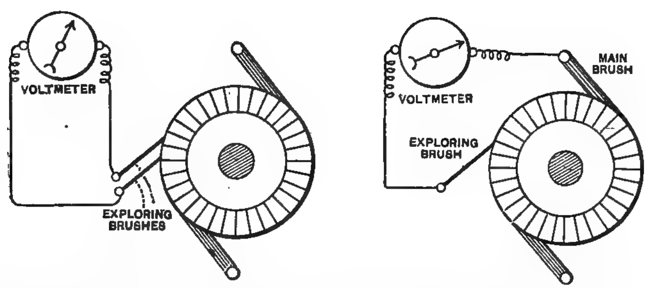
increase the tendency to sparking on account of increased self-induction. From the preceding, then, it is evident that sparkless commutation will be promoted (1) by dividing up the armature into many sections so as to do the reversing of the current in detail; (2) by making the field magnet relatively powerful, thereby securing between the pole tips a fringe of field of sufficient strength to reverse the currents in the short-circuited coils; (3) by so shaping the pole surfaces as to give a fringe of magnetic field of suitable extent; (4) by choosing brushes of proper thickness and keeping their contact surfaces well trimmed.

Since the direction of a current causing a certain motion is opposite to the direction of the current caused by that motion, it follows that in a *generator* the current induced in the short-circuited coil at a certain position has just the opposite direction with relation to the current flowing in the armature from that induced in the short-circuited coil of a *motor* in the same position, when rotating in the same direction. That is to say, if in a generator the brushes are shifted so that the current induced in the short-circuited coil has the same direction as the current flowing in the half of the armature it is about to join, in a motor revolving in the same direction and having its brushes set in exactly the same position, the current in the commuted coil, which absolutely of course has the same direction as in case of the generator, would relatively have a direction opposite to that flowing in the half of the armature to which it is transferred by the act of commutation. While the brushes, in order to attain sparkless commutation, must therefore be shifted with the direction of rotation, or must be given an *angle of lead* in a generator, in a motor they have to be shifted backward, or have to be given an *angle of lag*.

In a generator the effect of commutation is a tendency to increase the aggregate magnetomotive force and therefore to strengthen the field; in a motor, however, the effect of commutation is to decrease the magnetomotive force and to weaken the field. Iron is very sensitive to slight increases of magnetomotive force, while on the other hand it is comparatively insensible to considerable decrease of magnetomotive force; in generators, therefore, the danger of

sparking due to improper setting of the brushes is much greater than in motors.

If the magnetic field is perfectly uniform in strength all around the armature, the E. M. Fs. generated in the separate coils will be all of equal amount; but in actual dynamos the distribution of the magnetic lines in the gaps is always more or less uneven, and the E. M. Fs. in the different coils, therefore, have more or less varying strengths. In well-designed machines, however, the magnetic lines, although unevenly distributed around the armature, are symmetrically



Figs. 34 and 35.—Methods of Exploring Distribution of Potential around Armature.

situated in the two air gaps, and the total E. M. F. of either half of the winding, being the sum of the individual E. M. Fs. of the separate coils, will be equal to the total E. M. F. of the other half, from brush to brush. As the distribution of the magnetic flux around the armature directly affects the distribution of the potential, an examination of the latter will allow conclusions to be drawn as to the former.

There are two ways of studying the distribution of the potential around the armature: (1) by observing the voltmeter-deflections caused by the individual coils, a set of exploring brushes being placed, in turn, against every two adjacent commutator bars, Fig. 34, and (2) by taking a voltmeter-reading for every bar, the voltmeter being connected between one of the main brushes and an exploring brush sliding upon the commutator, Fig. 35. By plotting the voltmeter readings, in the first case a curve is obtained which shows the relative

amount of E. M. F. induced in each armature coil when brought in the various parts of the magnetic field, while the curve received in the second case gives the totalized or "integrated" potential around the armature, such as is found for any point in one of the armature halves by adding up the E. M. Fs. of all the coils from the brush to that point.

The investigation of the distribution of the potential around the commutator is very useful in practice, as it may disclose unsymmetrical distribution of the magnetic field due to faulty design of the magnet frame, or to incorrect shape of the pole-pieces, or to other causes. Fig. 36 shows the curves of

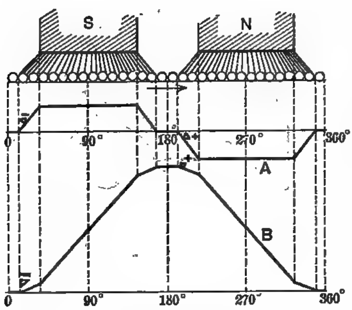


Fig. 36.—Curves of Potentials around Armature at No Load.

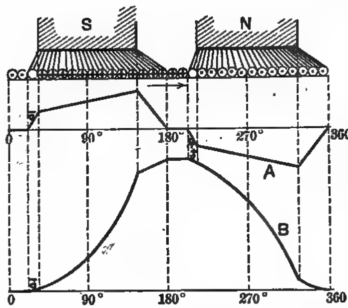


Fig. 37.—Curves of Potentials around Armature at Full Load.

potentials around an armature rotating in an evenly distributed field, such as will exist in a well-proportioned dynamo when there is no current flowing in the armature, that is to say, when the machine is running on open circuit. In Fig. 37 similar curves are given for a correctly designed dynamo with unevenly but symmetrically distributed field, as distorted by the action of the armature current when running on closed circuit. In both diagrams *A* is the curve of potentials in each coil, obtained by the first method, and *B* the curve of integrated potential, obtained by the second method of exploring the distribution of potential around the commutator.

If either one of the curves *A* or *B* is given by experiment, the ordinates of the other may be directly obtained by one of the following formulæ given by George P. Huhn:[1]

[1] "On Distribution of Potential," by George P. Huhn, *Electrical Engineer*, vol. xv. p. 186 (February 15, 1893).

$$X_a = x_a \times \frac{1 - \cos \alpha}{\sin \alpha} \times \frac{2n_c}{\pi}$$

and

$$x_a = X_a \times \frac{\sin \alpha}{1 - \cos \alpha} \times \frac{\pi}{2\, n_c},$$

in which X_a = ordinate, at angle α from starting position of curve of integral potential;
x_a = ordinate, at angle α from starting position of curve of potential in each coil;
n_c = number of commutator divisions.

The potentials may also with advantage be plotted out round a circle corresponding to the circumference of the commutator, the reading for each coil being projected radially from the

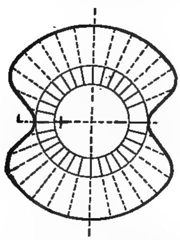


Fig. 38.—Distribution of Potential around Commutator at No Load.

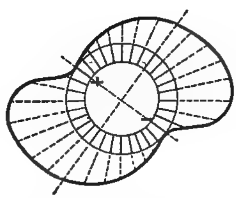


Fig. 39.—Distribution of Potential around Commutator at Full Load.

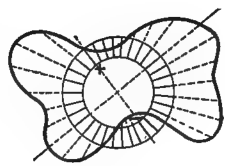


Fig. 40.—Distribution of Potential around Commutator of Faulty Dynamo.

respective commutator division. Fig. 38 shows, thus plotted, the curve of potentials at no load, and Fig. 39 that at full load of a well-arranged dynamo, while Fig. 40 depicts the distribution of potential around the commutator of a badly designed machine.

13. Multipolar Dynamos.

While bipolar dynamos offer advantages when small capacities are required, their output per unit of weight does not materially increase with increasing size, and a more economical form of machine is therefore desired for large outputs. In order that the *weight-efficiency* (output per pound of weight) of a dynamo may be increased without increasing the periphery velocity of the armature, or dangerously increasing the tem-

perature limit, it is necessary to decrease the reluctance of the magnetic circuit, that is, to reduce the ratio of the length of the air gap to the area of its cross section. Since the length of the armature cannot be increased beyond certain limits governed by mechanical as well as magnetical conditions, the only means of increasing the gap area remains to increase the armature diameter. Increasing the diameter of an armature allows a greater circumference on which to wind conductors, and therefore the depth of the winding may be proportionally decreased. Thus the increase of the armature diameter not only increases the gap area, but also decreases its length, and consequently very effectively reduces the reluctance of the magnetic circuit. With armatures of such large diameters, in order to more evenly distribute the magnetic flux, and to more economically make use of space and weight of the magnet frame, it is advantageous to divide the magnetic circuit, resulting in dynamos with more than one pair of poles, or *multipolar dynamos.*

For small multipolar dynamos drum armatures are often used; large machines for continuous current work, however, have always ring armatures. In a multipolar armature there are as many neutral and commutating planes as there are pairs of poles, and, therefore, as many sets of brushes as there are poles. Often, however, all commutator segments that are symmetrically situated with respect to the separate magnetic circuits are cross-connected among each other, so that the separate portions of the armature winding corresponding to the separate magnetic circuits are actually connected in parallel within the machine, and then only two brushes, in any two subsequent planes of commutation, are necessary. But unless the armature is in excellent electric and magnetic balance, and all the magnetic circuits of the machine have an equal effect on the armature, excessive heating and sparking are bound to result from this arrangement. This trouble may be avoided by winding the armature so that the current is divided between only two paths, exactly as in a bipolar machine. When such a two-path, or *series*, winding is used, the wire of each coil must cross the face of the core as many times as there are field-poles, the turns being spaced at a distance equal to nearly the pitch of the poles. Series-wound multipolar armatures will

operate satisfactorily regardless of inequalities in the strength of the magnetic circuits. Unless specially arranged, these armatures require only two brushes which are 180° apart in machines having an odd number of pairs of poles, and at an angular distance apart equal to the pitch of the poles in machines having an even number of pairs of poles.

Sometimes the commutators of series armatures are arranged with twice as many bars as there are coils in the armature, in which case the extra bars are properly cross-connected to the active bars, so that four brushes may be used in order to give a greater current-carrying capacity. To economize wire in multipolar armatures, it is of advantage to arrange the winding so that no wires have to pass through the inside of the ring, the inductors being connected by conductors on either face of the core. An armature so wound is termed a *drum-wound ring armature.*

If the dynamos are to be directly coupled to the steam engines, particularly low rotative speeds of the armatures are required, and their diameters are then made extra large in order to give them low speed without too great a reduction of periphery velocity. To fully utilize the large armature circumference of such *low speed multipolar machines*, the number of poles is usually made very high, their actual number depending upon the capacity of the machine and the service required of it. Great reductions of rotative speed can, however, only be obtained either by considerable sacrifice of weight-efficiency, or by sacrificing sparkless operation. The former, when carried to an extreme, makes too expensive a machine, and the latter causes increased repairs and depreciation; a mean between the two must therefore be followed in practice.

14. Methods of Exciting Field Magnetism.

In modern dynamos the field magnetism is excited by current from the armature of the machine itself. According to the manner in which current is taken from the armature and sent through the field winding, we distinguish, as far as continuous current machines are concerned, the following classes of dynamos: (*a*) Series-wound, or *Series* dynamo; (*b*) Shunt-wound, or *Shunt* dynamo, and (*c*) Compound-wound, or *Compound* dynamo.

a. Series Dynamo.

In the series-wound dynamo the whole current from the armature is carried through the field-magnet coils, the latter being wound with comparatively few turns of heavy copper

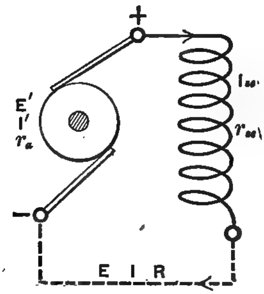


Fig. 41.—Diagram of Series-Wound Dynamo.

wire, cable, or ribbon, and connected in series with the main circuit, Fig. 41.

Denoting by

E' = total E. M. F. generated in armature;
I' = total current generated in armature;
r_a = armature resistance;
E = terminal voltage, or potential of dynamo;
I = useful current flowing in external circuit;
R = resistance of external or working circuit;
I_{se} = current in series field;
r_{se} = resistance of series-field coil;
η_e = electrical efficiency;

the following equations exist, by virtue of Ohm's law of the electric circuit, for the series dynamo:

$$\left.\begin{aligned} I' &= \frac{E'}{R + r_a + r_{se}} \\ I &= \frac{E}{R} = I' \\ I_{se} &= \frac{E' - E}{r_{se}} = I = I' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (8)$$

$$\left.\begin{aligned} E' &= E + I\,(r_a + r_{se}) \\ &= E\left(1 + \frac{r_a + r_{se}}{R}\right) \end{aligned}\right\} \cdots\cdots\cdots\cdots(9)$$

$$\left.\begin{aligned} \eta_e &= \frac{\text{useful energy}}{\text{total energy}} = \frac{E\,I}{E'\,I'} = \frac{E}{E'} \\ &= \frac{I^2 R}{I'^2 (R + r_a + r_{se})} = \frac{R}{R + r_a + r_{se}} \end{aligned}\right\} \cdots(10)$$

From equations (8) it is evident that an increase in the working resistance directly diminishes the current in the field coils, therefore reducing the amount of the effective magnetic flux, and that on the other hand a decrease of the external resistance tends to increase the excitation and, in consequence, the flux. The constancy of the flux thus depending upon the constancy of the current strength in series-wound dynamos, these machines are best adapted for service requiring a constant current, such as series arc lighting.

Equation (9) shows that the current generated in the armature of a series dynamo, in order to overcome the resistances of armature and series field, loses a portion of its E. M. F.; the E. M. F. to be generated in the armature of a series-wound machine, therefore, is equal to the required useful potential, increased by the drops in the armature and in the series-field winding. Series machines having but one circuit the current intensity is the same throughout, and consequently the current to be generated in the armature is equal to the current required in the external circuit.

The end result of equation (10) shows that the electrical efficiency of a series dynamo is obviously a maximum when the armature resistance and field resistance are both as small as possible. In practice they are usually about equal.

The series-wound dynamo has the disadvantage of not starting action until a certain speed has been attained, or unless the resistance of the circuit is below a certain limit, the machine refusing to excite when there is too much resistance or too little speed.

b. Shunt Dynamo.

In the shunt-wound dynamo the field-magnet coils are wound with many turns of fine wire, and are connected to the brushes of the machine, constituting a by-pass circuit of high

resistance through which only a small portion of the armature current passes, Fig. 42.

Using similar symbols as in the case of the series dynamo,

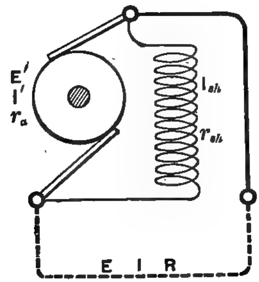


Fig. 42.—Diagram of Shunt-Wound Dynamo.

the following fundamental equations for the shunt dynamo can be derived:

$$\left.\begin{aligned} I' &= I + I_{\text{sh}} = I + \frac{E}{r_{\text{sh}}} \\ &= \frac{E'}{r_a + \dfrac{R \times r_{\text{sh}}}{R + r_{\text{sh}}}} = \frac{E}{R}\left(1 + \frac{R}{r_{\text{sh}}}\right) \\ I &= \frac{E}{R}; \quad I_{\text{sh}} = \frac{E}{r_{\text{sh}}} \end{aligned}\right\} \ldots (11)$$

$$\left.\begin{aligned} E' &= E + I' r_a \\ &= E\left(1 + \frac{r_a}{R} + \frac{r_a}{r_{\text{sh}}}\right) \\ &= E \times r_a \left(\frac{1}{R} + \frac{1}{r_a} + \frac{1}{r_{\text{sh}}}\right) \end{aligned}\right\} \ldots\ldots (12)$$

$$\left.\begin{aligned} \eta_e &= \frac{EI}{E'I'} = \frac{I^2 R}{I^2 R + I'^2 r_a + I_{\text{sh}}^2 r_{\text{sh}}} \\ &= \frac{I^2 R}{I^2 R + \left(I \times \dfrac{R + r_{\text{sh}}}{r_{\text{sh}}}\right)^2 r_a + \left(I \dfrac{R}{r_{\text{sh}}}\right)^2 r_{\text{sh}}} \\ &= \frac{1}{1 + \dfrac{r_a}{R} + 2\dfrac{r_a}{r_{\text{sh}}} + \dfrac{(r_a + r_{\text{sh}})R}{r_{\text{sh}}^2}} \end{aligned}\right\} (13)$$

Equations (11) show that in a shunt dynamo an increase of the external resistance, by diminishing the current in the working circuit, increases the shunt current, and with it the magnetic flux, while a decrease of the working resistance increases the useful current, the sum of which and the shunt current is a constant as long as the total current generated in the armature remains the same, thereby reducing the exciting current and ultimately decreasing the magnetic flux. The flux remains constant only when the potential of the machine is kept the same, as then the shunt current, which is the quotient of the terminal pressure and the constant shunt resistance, is also constant; shunt-wound machines, therefore, are best adapted for service demanding a constant supply of pressure, such as parallel incandescent lighting.

Since the stronger a current flows through the shunt circuit the less is the current intensity of the main circuit, a shunt machine will refuse to excite itself if the resistance of the main circuit is too low.

From (11) and (12) it is seen that the armature current of a shunt dynamo suffers a loss both in E. M. F. and in intensity within the machine; E. M. F. being lost in overcoming the armature resistance, and current intensity in supplying the shunt circuit. In consequence, the E. M. F. to be generated in a shunt dynamo must be equal to the potential required in the working circuit, plus the drop in the armature; and the total current is equal to the useful amperage required, plus the current strength used for field excitation.

The efficiency of a shunt dynamo, by equation (13), becomes maximum under the condition [1] that

$$R = r_{sh} \sqrt{\frac{r_a}{r_a + r_{sh}}}. \quad \ldots\ldots\ldots\ldots (14)$$

Inserting this value in (13) we obtain the equation for the maximum electrical efficiency of a shunt dynamo:

$$\eta_e = \frac{1}{1 + 2\,\dfrac{\sqrt{r_a\,(r_a + r_{sh})}}{r_{sh}} + 2\,\dfrac{r_a}{r_{sh}}}. \quad \ldots\ldots (15)$$

[1] Sir W. Thomson (Lord Kelvin), *La Lumière Électr.*, iv., p. 385 (1881).

Now, since the armature-resistance is usually very small compared with the shunt-field resistance, the sum $r_a + r_{sh}$ may be replaced by r_{sh}, and the quotient

$$\frac{r_a}{r_{sh}}$$

may be neglected, when the following very simple approximate value of the efficiency is obtained:

$$\eta_e = \frac{1}{1 + 2\sqrt{\frac{r_a}{r_{sh}}}}, \qquad \ldots\ldots\ldots\ldots(16)$$

and this, by transformation, furnishes

$$\frac{r_{sh}}{r_a} = \left(\frac{2\,\eta_e}{1 - \eta_e}\right)^2. \qquad \ldots\ldots\ldots\ldots(17)$$

By means of equation (16) the approximate electrical efficiency of any shunt dynamo can be computed if armature and magnet resistance are known; and from formula (17) the ratio of shunt resistance to armature resistance for any given percentage of efficiency can directly be calculated. In the following Table II. these ratios are given for electrical efficiencies from $\eta_e = .8$, to $\eta_e = .995$, or from 80 to 99.5 per cent.:

TABLE II.—RATIO OF SHUNT TO ARMATURE RESISTANCE FOR DIFFERENT EFFICIENCIES.

PERCENTAGE OF ELECTRICAL EFFICIENCY. $100\,\eta_e$	RATIO OF SHUNT TO ARMATURE RESISTANCE. $\frac{r_{sh}}{r_a}$	PERCENTAGE OF ELECTRICAL EFFICIENCY. $100\,\eta_e$	RATIO OF SHUNT TO ARMATURE RESISTANCE. $\frac{r_{sh}}{r_a}$
80%	64	95.5%	1,802
85	128	96	2,304
87.5	196	96.5	3,041
90	324	97	4,182
91	409	97.5	6,084
92	529	98	9,604
93	706	98.5	17,248
94	982	99	39,204
95	1,444	99.5	158,404

c. *Compound Dynamo.*

Compound winding is a combination of shunt and series excitation. The field coils of a compound dynamo are partly wound with fine wire and partly with heavy conductors, the fine winding being traversed by a shunt current and the heavy winding by the main current. The shunt circuit may be derived from the brushes of the machine or from the terminals of the external circuit; in the former case the combination is termed a *short shunt compound winding*, or an *ordinary compound winding*, Fig. 43, in the latter case a *long shunt compound winding*, Fig. 44.

Employing the same symbols as before, the application of

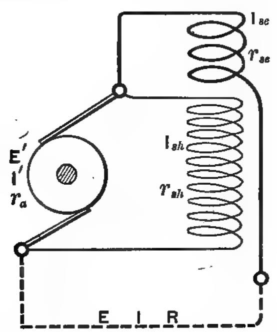


Fig. 43.—Diagram of Ordinary Compound-Wound Dynamo.

Ohm's law furnishes the following equations for the compound dynamo:

(1) *Ordinary Compound Dynamo* (Fig. 43).

$$\left.\begin{aligned} I' &= I + I_{sh} = I_{se} + I_{sh} \\ &= I\left(1 + \frac{r_{se} + R}{r_{sh}}\right) \\ &= I \times \frac{r_{se} + r_{sh} + R}{r_{sh}} \\ I_{sh} &= \frac{E_{sh}}{r_{sh}} = \frac{E + I\,r_{se}}{r_{sh}} = I \times \frac{r_{se} + R}{r_{sh}} \end{aligned}\right\} \ldots\ldots(18)$$

$$\left.\begin{aligned} E' &= E + I' r_a + I r_{se} \\ &= E + \left(I + \frac{E + I r_{se}}{r_{sh}}\right) r_a + I r_{se} \\ &= E\left(1 + \frac{r_a + r_{se}}{R} + \frac{r_a (r_{se} + R)}{r_{sh} \times R}\right) \end{aligned}\right\} \ldots (19)$$

$$\left.\begin{aligned} \eta_e &= \frac{E\,I}{E'\,I'} = \frac{I^2 R}{I'^2 r_a + I_{sh}^2 r_{sh} + I^2 (R + r_{se})} \\ &= \frac{1}{1 + \frac{r_a + r_{se}}{R} + \frac{R}{r_a}\left(1 + \frac{r_{se}}{R}\right)^2 \times \left(1 + \frac{r_a}{r_{sh}} + 2\frac{r_a}{R + r_{se}}\right)} \end{aligned}\right\} \ldots (20)$$

(2) *Long Shunt Compound Dynamo* (Fig. 44).

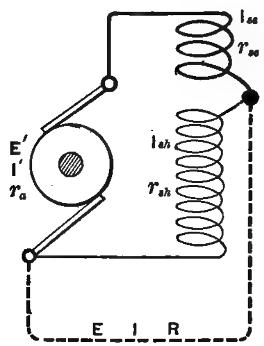


Fig. 44.—Diagram of Long Shunt Compound-Wound Dynamo.

$$\left.\begin{aligned} I' &= I_{se} = I + I_{sh} \\ &= I + \frac{E}{r_{sh}} = I \times \frac{R + r_{sh}}{r_{sh}} \\ I_{sh} &= \frac{E}{r_{sh}} = I \times \frac{R}{r_{sh}} \end{aligned}\right\} \ldots\ldots\ldots (21)$$

$$\left.\begin{aligned} E' &= E + I'(r_a + r_{se}) = E + \left(I + \frac{E}{r_{sh}}\right)(r_a + r_{se}) \\ &= E\left(1 + \frac{R + r_{sh}}{R \times r_{sh}}(r_a + r_{se})\right) \end{aligned}\right\} (22)$$

$$\left.\begin{aligned}\eta_e &= \frac{I^2 R}{I'^2(r_a + r_{se}) + I_{sh}^2 r_{sh} + I^2 R} \\ &= \frac{1}{\left(1+\frac{R}{r_{sh}}\right)^2 \times \frac{r_a + r_{se}}{R} + \frac{R}{r_{sh}} + 1} \\ &= \frac{1}{1 + \frac{r_a + r_{se}}{R} + 2\,\frac{r_a + r_{se}}{r_{sh}} + \frac{R\,(r_a + r_{se} + r_{sh})}{r_{sh}^2}}\end{aligned}\right\} \quad (23)$$

By combining the shunt and series windings, the excitation of the dynamo can be held constant, as the main current diminishes and the shunt current increases with increasing working resistance, and the main current rises and the shunt current decreases with decreasing external resistance. A compound-wound dynamo, therefore, if properly proportioned, will maintain a constant potential for varying load. In the case of the ordinary compound dynamo, the potential between the brushes is thus kept constant, in case of the long shunt compound dynamo the potential between the terminals of the working circuit. Although, therefore, the latter arrangement is the more desirable in practice, in a well-designed dynamo it makes very little difference whether the shunt is connected across the brushes or across the terminals of the external circuit.

In the ordinary compound dynamo the series winding supplies the excitation necessary to produce a potential equal in amount to the voltage lost by armature resistance and by armature reaction; in the long shunt compound dynamo the series winding compensates for armature reaction, and for the drop in the series field as well as for that in the armature. The series winding may even be so proportioned that the increase of pressure due to it exceeds the lost voltage, and then the dynamo is said to be *over-compounded*, and gives higher voltage at full load than on open circuit. Compound dynamos used for incandescent lighting are usually about 5 per cent. over-compounded in order to compensate for drop in the line from the machine to the lamps.

The armature current of a compound dynamo suffering a drop both in potential and in intensity within the machine, in calcu-

lating a compound-wound machine the total E. M. F. to be generated must be taken equal to the required potential plus the voltage necessary to overcome armature and series-field resistances; and the total current strength of the armature equal to the intensity of the external circuit increased by the current used in exciting the shunt field.

PART II.

CALCULATION OF ARMATURE.

CHAPTER III.

FUNDAMENTAL CALCULATIONS FOR ARMATURE WINDING.

15. Unit Armature Induction.

It is evident that a certain length of wire moving with the same speed in magnetic fields of equal strengths will invariably generate the same electromotive force, no matter whether the said length of wire be placed on the circumference of a drum or of a ring armature, and no matter whatever may be the shape of the field magnet frame, or the number of poles of the different magnetic fields.

In order to obtain such a constant, suitable for practical purposes, we start from the definition: "*One volt E. M. F. is generated by a conductor when cutting a magnetic field at the rate of 100,000,000 C. G. S. lines of force per second.*"

Since the English system of measurement is still the standard in this country, we will take one foot as the unit length of wire, and one foot per second as its unit linear velocity, and for the unit of field strength we take an intensity of one line of force per square inch. At the same time, however, for calculation in the metric system, one metre is taken as the unit for the length of the conductor, one metre per second as the unit velocity, and one line per square centimetre as the unit of field density.

Based upon the law: "The E. M. F. generated in a conductor is directly proportional to the length and the cutting speed of the conductor, and to the number of lines of force cut per unit of time," we can then derive the unit amounts of E. M. F. generated in the respective systems of measurement, with the following results:

"*Every foot of inductor moving with the velocity of one foot per second in a magnetic field of the density of one line of force per square inch generates an electromotive force of* 144×10^{-8} *volt,*" and "*Every metre of inductor cutting at a speed of one metre per second through a field having a density of one line per square centimetre generates* 10^{-4} *volt.*"

The derivation of these two laws from the fundamental definition is given in the following Table III.:

TABLE III.—UNIT INDUCTIONS.

LENGTH OF INDUCTOR.	CUTTING VELOCITY.	DENSITY OF FIELD.	E. M. F. GENERATED.
1 foot	1 ft. per second	100,000,000 lines per sq. ft.	1 Volt
1 foot	1 ft. per second	100,000,000 lines per sq. in.	144 Volts.
1 foot	1 ft. per second	1 line per sq. in.	144×10^{-8} Volt
1 cm.	1 cm. per second	100,000,000 lines per sq. cm.	1 Volt
1 metre	1 m. per second	100,000,000 lines per sq. m.	1 Volt
1 metre	1 m. per second	100,000,000 lines per sq. cm.	10,000 Volts.
1 metre	1 m. per second	1 line per sq. cm.	10^{-4} Volt

If two or more equal lengths are connected in parallel, in each of these wires every unit of length will produce the respective unit of induction, but these parallel E. M. Fs. will not add, but the total E. M. F. generated in one length will also be the total E. M. F. output of the combination.

In an ordinary bipolar armature, now, there are two such parallel branches, each branch generating the total E. M. F. This necessitates one foot of generating wire in each of these two parallel circuits, or altogether two feet of wire, under our unit conditions, in order to obtain an E. M. F. output of 144×10^{-8} volt; or, in other words: *Every foot of the total generating wire on a bipolar armature, at a cutting speed of one foot per second, in a field of one line per square inch, generates* 72×10^{-8} *volt of the output E. M. F.* And by a similar consideration we find for the metric system: *Every metre of the actual inductive wire on a bipolar armature revolving with a cutting velocity of one metre per second in a field of one line per square centimetre, generates* 5×10^{-5} *volt of the output E. M. F.*

In multipolar armatures the number of the electrically parallel portions of the winding generally is $2n'_p$, the number of *pairs* of parallel armature circuits, or the number of *bifurcations* of the current in the armature being denoted by n'_p, and usually $2n'_p$ is equal to the number of poles, $2n_p$, the number of pairs of poles being denoted by n_p. In such armatures it therefore takes $2n'_p$ feet of generating conductor to produce 144×10^{-8} volt of output, or the share of E. M. F. contrib-

uted to the total output by every foot of the generating wire on the entire pole-facing circumference is

$$\frac{144 \times 10^{-8}}{2n'_p} = \frac{72 \times 10^{-8}}{n'_p}$$

volt; that is, *72 × 10⁻⁸ volt per pair of armature circuits*, or per pair of poles, respectively. In metric units the share of the E. M. F. contributed to the output of a multipolar armature by every metre of the inductive length of the armature conductor is

$$\frac{5 \times 10^{-5}}{n'_p}$$

volt, or *5×10^{-5} volt per bifurcation.*

These theoretical values of the "*unit armature induction*," however, have to undergo a slight modification for practical use, owing to the fact that generally only a portion of the total generating or *active* wire of an armature is *effective.* "Active" is all the wire that is placed upon the pole-facing surface of the armature, "effective" only that portion of it which is actually generating E. M. F. at any time; that is, the portion immediately opposite the poles and within the reach of the lines of force, at that time.

The percentage of effective polar arc, in modern dynamos, according to the number and arrangement of the poles, varies from 50 to 100 per cent. and, usually, lies between 70 and 80 per cent., corresponding to a pole angle of 120° to 144°, respectively. The lowest values of the effective arc, 50 to 60 per cent. of the total circumference, are found in the multipolar machines made by *Schuckert*, with poles parallel to the armature shaft, and having no separate pole shoes; in these the space taken up by the magnet winding prevents the poles from being as close together as in machines of other types. The highest figure, 100 per cent., is met in some of the *Allgemeine Elektricitaets Gesellschaft* dynamos, in which the poles are united by a common cast-iron ring (*Dobrowolsky's* pole bushing. See § 76, Chap. XV.).

In fixing a preliminary value of this precentage, β_1, in case of a new design, take 67 to 80 per cent., or $\beta_1 = .67$ to $.80$, for *smooth drum* armatures; $\beta_1 = .75$ to $.85$ for *smooth rings*, and

$\beta_1 = .70$ to $.90$ for *toothed* and *perforated* armatures. The lower of the given limits refers to small, and the upper to large sizes, for the final value of β_1 is determined with reference to the length of the air gaps, and the latter are comparatively much smaller in large than in small dynamos. Also the number of the magnet poles somewhat affects the selection of β_1, the smaller a percentage usually being preferable the larger the number of field poles.

For these various percentages the author has found the average values of the unit armature induction given in the following Table IV.:

TABLE IV.—PRACTICAL VALUES OF UNIT ARMATURE INDUCTION.

PERCENTAGE OF POLAR ARC.	E. M. F. PER PAIR OF ARMATURE CIRCUITS.			
	ENGLISH UNITS. Volt per Foot.		METRIC UNITS. Volt per Metre.	
	BIPOLAR DYNAMOS.	MULTIPOLAR DYNAMOS.	BIPOLAR DYNAMOS.	MULTIPOLAR DYNAMOS.
β_1	e	e	e_1	e_1
1.00	72×10^{-8}	72×10^{-8}	5×10^{-5}	5×10^{-5}
.95	71	68	4.9	4.8
.90	70	65	4.8	4.6
.85	67.5	62.5	4.7	4.4
.80	65	60	4.6	4.2
.75	62.5	57.5	4.4	4
.70	60	55	4.2	3.8
.65	57.5	52.5	4	3.6
.60	55	50	3.8	3.4
.55	52.5	47.5	3.6	3.2
.50	50	45	3.4	3

It will be noticed that the values for multipolar machines run somewhat below those for bipolar ones. This means that, at the same rate of polar embrace, a greater percentage of the total active wire is effective in the case of a bipolar machine, which is undoubtedly due to a greater circumferential spread of the lines of force of bipolar fields.

16. Specific Armature Induction.

Knowing the values of the induction per unit length of active armature wire under unit conditions, a general expression can now easily be derived for the "specific armature induction" at any given conductor speed and field density. The induction per unit length of active conductor, in any armature, is

$$e' = \frac{e}{n'_p} \times v_c \times \mathcal{H}'', \qquad \ldots\ldots\ldots\ldots (24)$$

where e' = specific induction of active armature conductor, in volts per foot;

e = unit armature induction per pair of armature circuits, in volts per foot, from Table IV.;

n'_p = number of bifurcations of current in armature, or number of pairs of parallel armature circuits; n'_p has the following values, to be multiplied by the number of independent windings in case of multiplex grouping (§ 44):

$n'_p = 1$ for bipolar dynamos and for multipolar machines having ordinary series grouping,

$n'_p = n_p$ for multipolar dynamos with parallel grouping, n_p being the number of pairs of magnet poles,

$n'_p = \frac{n_p}{n_s}$ for multipolar dynamo with series-parallel grouping, n_s being the number of armature circuits connected in series in each of the $2n'_p$ parallel circuits;

v_c = conductor-velocity, or cutting speed, in feet per second, from Table V.;

$\mathcal{H}''$ = field density, in lines of force per square inch, from Table VI.

In order to obtain the specific armature induction in the metric system, e is to be replaced by the corresponding value of e_1, Table IV.; the conductor velocity is to be expressed in metres per second, Table V., and the field density, $\mathcal{H}$, in lines per square centimetre, from Table VII.; then (24) gives the specific armature induction in volts per metre of active conductor.

17. Conductor Velocity.

The E.M.F. of a dynamo, according to formula (4), is proportional to the velocity v_c of the moving conductor; since, therefore, the output of a given dynamo can be raised by simply increasing its speed, it will be *best economy* to run a dynamo-electric machine at as high a conductor speed as practically possible.

The velocity, however, is limited mechanically as well as electrically; *mechanically*, because the friction in the bearings and the strain in the revolving parts due to centrifugal force, must not exceed certain limits; and *electrically*, because the heating of the armature caused by the resistance of the winding and by hysteresis and eddy currents in the iron, must be kept reasonably low by limiting the power loss, which increases with the output, and therefore is the greater, the higher the conductor speed is chosen. Furthermore, if the number of revolutions of the armature is given, either by the speed of the engine in case of a direct-driven machine, or otherwise, the above mechanical and electrical limitations alone are not sufficient for choosing the conductor velocity, for, when the number of revolutions is fixed, the diameter of the armature is proportional to the peripheral velocity, and abnormal sizes may be obtained by assuming a value of v_c, which is permissible from all the other considerations.

The limits of v_c established by practice are from 25 to 100 feet per second, according to the kind, size, and revolving speed of the machine. A common value is 50 feet per second, or 3000 feet per minute, which in the metric system corresponds to about 15 metres per second, or 900 metres per minute. For *drum armatures*, the average practical values of the conductor velocity range between 25 and 50 feet per second, and for *ring armatures*, which offer a better ventilation and are lighter than drum armatures of the same diameter, conductor velocities up to 100 feet per second are employed. Values near the upper limits are chosen for *high-speed* machines, in which the selection of a low peripheral velocity would result in too small an armature diameter; the radiating surface, or more properly called the *cooling surface*, of the armature would consequently be inadequate, and excessive heating would be inevitable. Values near the lower limit, on the

other hand, are taken for *low-speed* machines, because too large a conductor velocity would in their case excessively increase the diameter of the armature, and in consequence would bring the size of the entire machine out of proportion to its output.

The following Table V will serve the unexperienced designer as a guide in selecting the proper value of v_c for various sizes of drum and ring armature machines. This table is compiled from the data of a great many practical machines, the scope of which can best be seen from the list of machines given in the Preface. The averages given for drum armatures are intended for the usual case of high-speed drum machines, but they hold also good for medium and low-speeds, if it is considered that the figures given in the table are in each case averaged from widely differing actual values of the conductor velocity, so that good practical values of v_c for each size may be taken from about 20 to 25 per cent. below to about as much above the

TABLE V.—AVERAGE CONDUCTOR VELOCITIES.

CAPACITY IN KILOWATTS.	CONDUCTOR VELOCITY, IN FEET PER SECOND.				CONDUCTOR VELOCITY, IN METRES PER SECOND.			
	Drum Armature	Ring Armature.			Drum Armature	Ring Armature.		
		High Speed.	Medium Speed.	Low Speed.		High Speed.	Medium Speed.	Low Speed.
.1	25	50	25	..	7.5	15	7.5	..
.25	27	55	27	..	8	16.5	8	..
.5	30	60	30	25	9	18	9	7.5
1	35	65	30	25	10.5	19.5	9	7.5
2.5	40	70	35	25	12	21	10.5	7.5
5	45	70	40	26	13.5	21	12	8
10	45	75	40	28	13.5	22.5	12	8.5
25	50	75	45	30	15	22.5	13.5	9
50	50	80	50	32	15	24	15	10
100	50	80	50	35	15	24	15	11
200	50	85	50	40	15	25.5	15	12
300	50	85	55	40	15	25.5	16.5	12
500	..	90	55	45	..	27	16.5	13.5
1000	..	90	60	45	..	27	18	13.5
2000	..	95	60	45	..	28.5	18	13.5
5000	..	100	65	50	..	30	19.5	15

given average. Thus, for instance, the conductor velocity of a 2.5-KW drum armature may be chosen between $.75 \times 40 = 30$ and $1.25 \times 40 = 50$ feet per second; and the velocity of drum

machines above 25 KW output may be taken within the limits of $.75 \times 50 = 37.5$ and $1.25 \times 50 = 62.5$ feet per second. In the case of ring armatures, in which the peripheral velocities vary in much wider limits than in drum armatures, separate averages are given for high, medium, and low speeds; and in each case a deviation of about 15 per cent. above or below the given average is within good practical limits. For example, the value of v_c for a 5-KW high-speed ring armature may be selected between $.85 \times 70 = 60$ and $1.15 \times 70 = 80$ feet per second.

It will be noted that the value of v_c for a ring armature of given output varies considerably with the speed at which the machine is run, for the reasons given above. Since the size of the armature, and therefore the general proportion of the entire machine, depends directly upon the value chosen for v_c, it is evident that the proper selection of the conductor velocity is one of the most important assumptions to be made by the designer.

TABLE V*a*.—High, Medium, and Low Dynamo Speeds.

Capacity in Kilowatts.	Drum Armatures.			Ring Armatures.		
	High Speeds.	Medium Speeds.	Low Speeds.	High Speeds.	Medium Speeds.	Low Speeds.
.1	3000 to 2400	2400 to 1800	1800 to 1200	2600 to 2200	2200 to 1600	......
.25	2800 " 2200	2200 " 1600	1600 " 1000	2400 " 2000	2000 " 1400	......
.5	2600 " 2000	2000 " 1500	1500 " 800	2200 " 1800	1800 " 1200	1200 to 600
1	2400 " 1800	1800 " 1400	1400 " 700	2000 " 1600	1600 " 1000	1000 " 500
2.5	2200 " 1600	1600 " 1200	1300 " 600	1800 " 1400	1400 " 800	800 " 400
5	2000 " 1400	1400 " 1000	1000 " 500	1600 " 1200	1200 " 700	700 " 300
10	1800 " 1200	1200 " 800	800 " 400	1400 " 1000	1000 " 600	600 " 250
25	1500 " 1000	1000 " 600	600 " 300	1200 " 850	850 " 500	500 " 200
50	1200 " 800	800 " 500	500 " 250	1000 " 700	700 " 400	400 " 150
100	1000 " 600	600 " 400	400 " 200	800 " 550	550 " 300	300 " 125
200	800 " 450	450 " 300	300 " 150	600 " 400	400 " 200	200 " 100
500	......	......	200 " 100	500 " 300	300 " 150	150 " 80
1000	......	......	......	400 " 250	250 " 125	125 " 70
2000	......	......	......	300 " 200	200 " 100	100 " 60
5000	......	......	......	250 " 150	150 " 80	80 " 50

The diameter of the armature must be of such magnitude that the required length of armature conductor can be placed upon the core, if made of the proper length for that diameter, without causing the winding depth to be too great, or without causing an abnormal length of the armature when

wound to a certain depth proper for the diameter in question. And furthermore, the dimensions of the armature must be such that the size of its superficial area is adequate to liberate the heat generated in the winding and in the core. For these reasons it is an advantage to calculate the armature for several values of the conductor velocity, and to select the best size obtained, all things considered.

In Table V*a*, p. 52*b*, the usual speeds of various sizes of dynamos and their classification into *high*, *medium*, and *low* speeds are given.

The values of v_c for ring armatures, in Table V., refer to the average of the respective speeds in Table V*a*. If the dynamo in the given problem is a high-speed machine running near the lower limit, or a medium-speed machine running at a speed near the upper limit given in Table V*a*, a value of v_c about halfway between the high-speed and the medium-speed average is to be taken. If the speed specified for the dynamo to be designed is near the lower medium, or the upper low-speed limit, a value near the mean of the medium and the low-speed values of v_c should be selected. For speeds near the upper high-speed or the lower low-speed limits, a value of v_c somewhat higher than the high-speed average, or lower than the low-speed average, respectively, should be chosen.

For instance, if a ring armature for a 25-KW dynamo is to be designed to run at 800 revolutions per minute, the average conductor velocity is obtained as follows: From Table V*a* it is seen that the given speed, though found under the head of "medium speed," approaches the lower limit given for high speeds; the average value of v_c for medium-speed machines of the size in question is 45; the average for 25-KW high-speed ring armatures is 75; the mean of the two averages is

$$\frac{45+75}{2} = 60 \text{ feet per second.}$$

Therefore, in the present case, an average value of v_c of about 55 feet per second should be chosen.

In order to check the value of the conductor velocity so obtained, the tables of dimensions of modern machines which have been added for the guidance of the student may be used.

These tables, which will be found in Appendix I., include drum as well as ring armature machines, and give the principal dimensions of armatures and field frames for all ordinary sizes of high-, medium-, and low-speed dynamos. Tables CXIII. and CXIV., which contain the armature diameters and lengths of all the machines given in the Dynamo Tables CVII. to CXII., have been prepared especially for the purpose of checking the conductor velocity and the consequent armature dimensions.

The conductor velocity having been ascertained by the method given above, the armature diameter is computed by means of formula (30), p. 58, and is compared with the diameter of the machine of nearest size and speed given in Table CXIII. or CXIV., respectively. Thus, for the above example of a 25-KW 800-revolution dynamo, formula (30) gives a diameter of

$$d'_a = 230 \times \frac{55}{800} = 15\tfrac{3}{4} \text{ inches.}$$

The nearest machine in Table CXIII. is a 20-KW dynamo running at 700 revolutions, which would furnish 25 KW at 875 revolutions; its diameter is 16 inches. The close agreement of the two diameters shows that the conductor velocity chosen is a good value for the case on hand.

If Table CXIII. or CXIV. contains no machine of sufficiently near output and speed to allow of a comparison, the required diameter may be obtained by interpolation as shown in Appendix I, p. 663.

18. Field Density.

The specific strength of the magnetic field is chosen according to the size of the machine, the number of poles, the form of the armature, and the material of the polepieces. In general, higher field densities are taken for large than for small dynamos, and in multipolar machines higher values of $\mathcal{H}$ are admitted than in bipolar ones. In dynamos with smooth-core armatures, the field densities are usually taken somewhat greater than in those with toothed armature bodies, for the reason that in the latter a portion of the lines enters the teeth

and passes from tooth to tooth without cutting the conductors, and that in such armatures it therefore takes more lines per square inch of pole area to produce the same field density (per square inch of area occupied by armature conductors) than for smooth cores; consequently, smaller field densities must be employed with toothed armatures in order to prevent over-saturation of the polepieces, and, eventually, of the frame. This leakage through the armature teeth takes place in the higher a degree, the greater the width of the teeth compared to that of the slots, and therefore still smaller field densities are to be chosen in case of armature cores with tangentially projecting teeth, and of those with closed slots. Finally, in machines having wrought iron or cast steel polepieces, the densities can be taken about fifty per cent. higher than those with cast iron pole-shoes.

Practical average values of $\mathcal{H}''$ for ordinary dynamos and motors are tabulated in Table VI., which gives the average densities in lines of force per square inch, while Table VII. contains the corresponding values of $\mathcal{H}$ in lines per square centimetre.

The values of $\mathcal{H}$ will also depend on the method to be employed for obviating armature reaction. Modern designers often rely upon a strong magnetic field to assist in preventing the distorting effect of the armature reaction (see §§ 93 and 124), and, therefore, higher gap inductions are generally used now than were a few years ago. If a strong field is desired for the above purpose, a value of the field density about 20 or 30 per cent. in excess of the respective value given in Table VI. or VII., respectively, is advisable.

For machines designed for a very low voltage, such as electro-plating dynamos, battery motors, etc., or for dynamos in which the amperage is very high, comparatively, as in incandescent generators of large outputs, the field density is usually made about two-thirds or three-quarters of the corresponding density employed under similar conditions for ordinary machines.

For machines generating very high voltages, the field density should, on the other hand, be chosen considerably higher than the averages given, values from 25 to 50 per cent. in excess of those given being quite common for such machines.

For considerations governing the design of the above and other special kinds of machines, the student is referred to § 123, pp. 455 to 463.

TABLE VI.—PRACTICAL FIELD DENSITIES, IN ENGLISH MEASURE.

Capacity in Kilo Watts	Field Densities, in Lines of Force per square inch, $\mathcal{H}''$.												Capacity in Kilo Watts
	Bipolar Dynamos						Multipolar Dynamos						
	Smooth Armature Core		Toothed Armature Core				Smooth Armature Core		Toothed Armature Core				
			Straight Teeth		Projecting Teeth				Straight Teeth		Projecting Teeth		
	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	
.1	10000	15000	8000	12000	—	—	14000	20000	12000	18000	—	—	.1
.25	12000	18000	10000	15000	—	—	16000	24000	14000	21000	—	—	.25
.5	14000	20000	12000	18000	—	—	18000	27000	16000	24000	—	—	.5
1	15000	22000	13000	19000	8000	12000	19000	28000	17000	25000	10000	15000	1
2.5	16000	24000	14000	20000	9000	14000	20000	29000	18000	26000	11000	16000	2.5
5	17000	25000	15000	22000	10000	15000	21000	30000	19000	28000	12000	18000	5
7.5	18000	26000	16000	24000	11000	16000	22000	32000	20000	30000	13000	20000	7.5
10	19000	28000	17000	25000	12000	18000	24000	35000	21000	32000	14000	21000	10
25	20000	30000	18000	27000	13000	20000	26000	38000	22000	35000	15000	23000	25
50	22000	33000	20000	30000	14000	22000	28000	41000	23000	38000	16000	24000	50
100	24000	36000	22000	33000	16000	24000	30000	44000	25000	40000	17000	25000	100
200	27000	40000	24000	36000	18000	27000	32000	47000	27000	42000	18000	26000	200
300	30000	45000	27000	40000	20000	30000	35000	50000	29000	44000	19000	28000	300
500	—	—	—	—	—	—	38000	53000	31000	46000	20000	30000	500
1000	—	—	—	—	—	—	41000	56000	33000	48000	22000	32000	1000
2000	—	—	—	—	—	—	45000	60000	35000	50000	24000	35000	2000

TABLE VII.—PRACTICAL FIELD DENSITIES, IN METRIC MEASURE.

Capacity in Kilo Watts	Field Densities, in Lines of Force per square Centimetre, $\mathcal{H}$.												Capacity in Kilo Watts
	Bipolar Dynamos						Multipolar Dynamos						
	Smooth Armature Core		Toothed Armature Core				Smooth Armature Core		Toothed Armature Core				
			Straight Teeth		Projecting Teeth				Straight Teeth		Projecting Teeth		
	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	Cast Iron Polepieces	Wr't Iron or Steel Polepieces	
.1	1550	2300	1250	1850	—	—	2150	3100	1850	2800	—	—	.1
.25	1850	2800	1550	2300	—	—	2500	3700	2150	3250	—	—	.25
.5	2150	3100	1850	2800	—	—	2800	4200	2500	3700	—	—	.5
1	2300	3400	2000	2950	1250	1850	2950	4350	2650	3850	1550	2300	1
2.5	2500	3700	2150	3100	1400	2150	3100	4500	2800	4000	1700	2500	2.5
5	2650	3850	2300	3400	1550	2300	3250	4700	2950	4350	1850	2800	5
7.5	2800	4000	2500	3700	1700	2500	3400	5000	3100	4700	2000	3100	7.5
10	2950	4350	2650	3850	1850	2800	3700	5400	3250	5000	2150	3300	10
25	3100	4700	2800	4200	2000	3100	4000	5900	3400	5400	2300	3500	25
50	3400	5100	3100	4700	2150	3400	4350	6400	3550	5900	2500	3700	50
100	3700	5600	3400	5100	2500	3700	4700	6800	3850	6200	2650	3850	100
200	4200	6200	3700	5600	2800	4200	5000	7300	4200	6500	2800	4000	200
300	4700	7000	4200	6200	3100	4700	5400	7750	4500	6800	2950	4350	300
500	—	—	—	—	—	—	5900	8200	4800	7200	3100	4700	500
1000	—	—	—	—	—	—	6400	8700	5100	7500	3400	5000	1000
2000	—	—	—	—	—	—	7000	9300	5400	7800	3700	5400	2000

19. Length of Armature Conductor.

By means of the specific armature induction obtained from formula (24), the total length of active wire to be wound upon the pole-facing surface of any armature can be readily determined. If E' denotes the total E. M. F. generated in an armature, and L_a the total length of active wire wound on it, then E' divided by L_a will give the specific armature induction, e'. The length of active conductor for any armature can therefore be obtained from the formula

$$L_a = \frac{E'}{e'}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which L_a = total length of active conductor (on whole circumference opposite polepieces), in feet, or in metres;

E' = total E. M. F. to be generated in armature, *i. e.*, volt output plus additional volts to be allowed for internal resistances (see Table VIII.); and

e' = specific induction of active armature wire, calculated by formula (24), in volts per foot, or in volts per metre, respectively.

Introducing the value of e' from (24) into (25), the formula for the length of active armature conductor becomes:

$$L_a = \frac{E'}{\frac{e}{n'_p} \times v_c \times \mathcal{H}''} = \frac{n'_p \times E'}{e \times v_c \times \mathcal{H}''}. \quad \ldots (26)$$

The length L_a is obtained in feet, if e is given in volts per foot, v_c in feet per second, and $\mathcal{H}''$ in lines per square inch; and is obtained in metres, if e is replaced by e_1 in volt per metre, v_c expressed in metres per second, and if $\mathcal{H}''$ is replaced by $\mathcal{H}$ in lines per square centimetre.

To find the total electromotive force, E', to be generated by the armature, increase the electromotive force E wanted in the external circuit, by the percentages given in Table VIII. The figures in the second column of this table refer to shunt-wound dynamos, and, therefore, take into account the armature resistance only. The percentages in the third and fourth

columns are to be used for series- and for compound-wound dynamos respectively, and, consequently, include allowances for armature resistance as well as for series field resistance:

TABLE VIII.—E. M. F. ALLOWED FOR INTERNAL RESISTANCES.

CAPACITY IN KILOWATTS.	ADDITIONAL E. M. F. IN PER CENT. OF OUTPUT E. M. F.		
	Shunt Dynamos.	Series Dynamos.	Compound Dynamos.
Up to .5	20 % to 12 %	40 % to 25 %	30 % to 20 %
" 1	12 " 10	25 " 20	20 " 15
" 2.5	10 " 8	20 " 16	15 " 12
" 5	8 " 7	16 " 14	12 " 10
" 10	7 " 6	14 " 12	10 " 8
" 25	6 " 5	12 " 10	8 " 7
" 50	5 " 4	10 " 8	7 " 6
" 100	4 " 3½	8 " 6	6 " 5
" 200	3½ " 3	6 " 5	5 " 4
" 500	3 " 2½	5 " 4	4 " 3
" 1,000	2½ " 2	4 " 3	3 " 2½
" 2,000	2 " 1½	3 " 2½	2½ " 2

20. Size of Armature Conductor.

The sectional area of the armature conductor is determined by the strength of the current it has to carry. For general work the current densities usually taken vary between 400 and 800 circular mils (.25 to .5 square millimetre) per ampere; in special cases, however, a conductor area may be provided at the rate of as low as 200 to 400 circular mils (.125 to .5 square millimetre) per ampere, or as high as 800 to 1,200 circular mils (.5 to .75 square millimetre) per ampere. The low rate refers to machines which only are to run for a short while at the time, as, for instance, motors to drive special machinery (private elevators, pumps, sewing machines, dental drilis, etc.), while the high rate is to be employed for dynamos which have a fifteen or twenty hours' daily duty, as is the case for central-station, power-house, and marine generators, etc.

Taking 600 circular mils per ampere as the average current density (= 475 square mils, or .000475 square inch per ampere, or about 2,100 amperes per square inch), the sectional area of the armature conductor, in circular mils, is to be

$$\delta_a^2 = 600 \times \frac{I'}{2n'_p} = \frac{300 \times I'}{n'_p}, \quad \ldots\ldots (27)$$

where δ_a^2 = sectional area of armature conductor, in circular mils;

δ_a = diameter of armature wire, in mils;

I' = total current generated in armature, in amperes; and

n'_p = number of pairs of parallel armature circuits.

In the metric system, taking .4 square millimetre per ampere (= 2.5 amperes per square millimetre) as the average current density in the armature conductor, the sectional area of the inductor, in square millimetres, is obtained:

$$(\delta_a)^2_{mm} = \frac{.4 \times I'}{n'_p} = .2 \times \frac{I'}{n'_p}, \quad \ldots\ldots (28)$$

from which, in case of a circular conductor, the diameter can be derived:

$$(\delta_a)_{mm} = \sqrt{\frac{4}{\pi} \times .2 \frac{I'}{n'_p}} = .5 \sqrt{\frac{I'}{n'_p}}. \quad \ldots (29)$$

The size of conductor may be taken from the wire gauge tables by selecting a wire, the sectional area of one or more of which makes up, as nearly as possible, the cross-section obtained by formula (27).

The total armature current, I', in shunt and compound dynamos is the sum of the current output, I, and the exciting current of the shunt circuit. The latter quantity, however, generally is very small compared with the former, and in all practical cases, consequently, it will be sufficient to use the given I instead of the unknown I' for the calculation of the conductor area. A supplementary allowance may, then, be made by correspondingly rounding off the figures obtained by (27), or by selecting the wires of such a gauge that the actual conductor area is somewhat in excess of the calculated amount.

CHAPTER IV.

DIMENSIONS OF ARMATURE CORE.

21. Diameter of Armature Core.

If the speed of the dynamo is given, the proper conductor velocity taken from Table V. will at once determine the diameter of the armature. Let N denote this known speed, in revolutions per minute, and d'_a the mean diameter of the

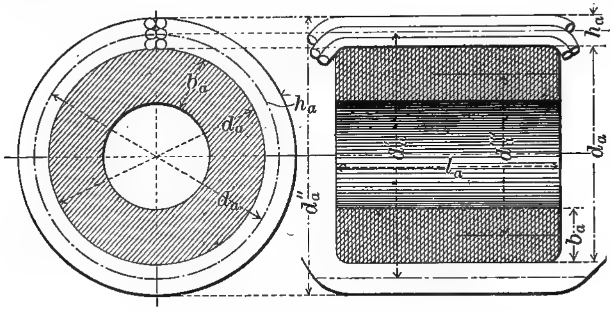


Fig. 45.—Principal Dimensions of Armature.

armature winding, in inches, then the cutting speed, in feet per second, is

$$v_c = \frac{d'_a \times \pi}{12} \times \frac{N}{60},$$

from which follows:

$$d'_a = \frac{12 \times 60}{\pi} \times \frac{v_c}{N} = 230 \times \frac{v_c}{N}. \quad \ldots\ldots \mathbf{(30)}$$

In the metric system the mean diameter of the armature winding, in centimetres, is given by

$$d'_a = \frac{100 \times 60}{\pi} \times \frac{v_c}{N} = 1{,}900 \times \frac{v_c}{N}, \quad \ldots\mathbf{(31)}$$

in which v_c is to be expressed in metres per second.

From this mean winding diameter, d'_a, then, the diameter of the armature core, d_a, Fig. 45, is found by making allowance for the height of the armature winding. For small armatures—under two feet in diameter—the coefficients given in the following Table IX. may be used for this purpose; for larger ones it is sufficient to simply round off the result of formula (30), or (31), respectively, to the next lower round figure:

TABLE IX.—RATIO BETWEEN CORE DIAMETER AND MEAN WINDING DIAMETER FOR SMALL ARMATURES.

SIZE OF ARMATURE.		RATIO $\frac{d_a}{d'_a}$,	
English Measure.	Metric Measure.	Drum Armatures.	Ring Armatures.
Up to 2 ins. dia.	Up to 5 cm. dia.	.88	—
" 4 "	" 10 "	.92	.95
" 8 "	" 20 "	.94	.97
" 12 "	" 30 "	.95	.975
" 16 "	" 40 "	.96	.98
" 20 "	" 50 "	.965	.9825
" 24 "	" 60 "	.97	.985

For dynamos with internal poles, the reciprocals of these coefficients are to be taken, or the result is to be rounded off to the next higher round figure, respectively; and the dimension thus obtained is the internal diameter of the armature core. In the case of machines with internal as well as external poles, the mean winding diameter, d'_a, is identical with the mean diameter of the armature body.

If the speed of the dynamo is not prescribed by the conditions for its service, the following Tables X., XI., and XII. will be found useful. Table X. gives practical data for speeds, conductor velocities, and corresponding diameters of drum armatures. Table XI. contains similar information relating to high-speed ring armatures, and in Table XII. data for the speeds of low-speed ring armatures are compiled and the corresponding armature diameters computed:

TABLE X.—SPEEDS AND DIAMETERS OF DRUM ARMATURES.

CAPACITY IN KILOWATTS.	SPEED, IN REVOLUTIONS PER MINUTE. N	ENGLISH MEASURE. Conductor Velocity, in ft. per sec. v_c	ENGLISH MEASURE. Armature Diameter, in inches. d_a	METRIC MEASURE. Conductor Velocity, in m. per sec. v_c	METRIC MEASURE. Armature Diameter, in cm. d_a
.1	3,000	25	$1\frac{3}{4}$	8	4.5
.25	2,700	30	$2\frac{1}{4}$	9	5.5
.5	2,400	32	$2\frac{3}{4}$	10	7
1	2,200	34	$3\frac{1}{4}$	11	8.5
2	2,000	36	$3\frac{3}{4}$	12	10
3	1,900	40	$4\frac{1}{2}$	13	12
5	1,800	45	$5\frac{1}{4}$	14	14
10	1,700	50	6	15	16
15	1,600	50	$6\frac{3}{4}$	15	17
20	1,500	50	$7\frac{1}{2}$	15	18
25	1,350	50	$8\frac{1}{4}$	15	20
30	1,200	50	9	15	23
50	1,050	50	$10\frac{1}{2}$	15	26
75	900	50	$12\frac{1}{2}$	15	30
100	750	50	15	15	37
150	600	50	$18\frac{1}{2}$	15	46.5
200	500	50	$22\frac{1}{2}$	15	56
300	400	50	28	15	70

TABLE XI.—SPEEDS AND DIAMETERS OF HIGH-SPEED RING ARMATURES.

CAPACITY IN KILOWATTS.	SPEED, IN REVOLUTIONS PER MINUTE. N	ENGLISH MEASURE. Conductor Velocity, in ft. per sec. v_c	ENGLISH MEASURE. Armature Diameter, in inches. d_a	METRIC MEASURE. Conductor Velocity, in m. per sec. v_c	METRIC MEASURE. Armature Diameter, in cm. d_a
.1	2,600	50	4	15	10
.25	2,400	55	5	17	12.5
.5	2,200	60	6	18.5	15
1	2,000	65	7	20	18
2.5	1,700	70	9	21.5	23
5	1,500	75	11	23	28
10	1,250	80	14	24	35
25	1,000	80	18	25	46
50	800	85	24	26	60
100	600	85	32	26	80
200	500	88	40	27	100
300	450	90	46	28	115
400	400	92	52	28	130
600	350	95	62	28	150
800	300	95	72	29	180
1,000	250	95	87	30	225
1,500	225	100	102	30	250
2,000	200	100	115	32	300

TABLE XII.—SPEEDS AND DIAMETERS OF LOW-SPEED RING ARMATURES.

CAPACITY IN KILOWATTS.	SPEED, IN REVOLUTIONS PER MINUTE. N	ENGLISH MEASURE.		METRIC MEASURE.	
		Conductor Velocity, in ft. per sec. v_c	Armature Diameter, in inches. d_a	Conductor Velocity, in m. per sec. v_c	Armature Diameter, in cm. d_a
2.5	400	25	14	7.5	35
5	350	26	17	8	42
10	300	28	21	8.5	53
25	250	30	27	9	70
50	200	32	36	9.5	90
100	175	35	46	11	115
200	150	40	60	12	150
300	125	42	78	13.25	200
400	100	44	100	13.25	250
600	90	45	115	13.75	290
800	80	45	129	13.75	325
1,000	75	45	138	13.75	350
1,500	70	45	148	13.75	375
2,000	65	45	158	13.75	400

22. Dimensioning of Toothed and Perforated Armatures.

Armatures with toothed and with perforated core discs, which have been much used in recent years, offer the following *advantages* over smooth armatures: (1) Excellent means for driving the conductors; (2) mechanical protection of the winding, especially in cores with tangentially projecting teeth, and in perforated bodies; (3) lessening of the resistance of the magnetic circuit, and, therefore, saving in exciting power; (4) prevention of eddy currents in the conductors; (5) lessening of the difference between the amounts of field-distortion at open circuit and at maximum output, and therefore possibility of sparkless commutation for varying load without shifting brushes; and (6) taking up of the magnetic drag by the core instead of by the conductors. Their *disadvantages* are: (1) Increased cost of manufacture; (2) necessity for special devices to insulate the winding from the core; (3) eddy currents set up by the teeth in the polar faces; (4) additional heat generated in the iron projections by

hysteresis; (5) increase of self-induction in short-circuited armature coils due to imbedding them in iron, especially in high amperage machines; (6) increased length of the gap-space and consequent greater expenditure in exciting power when saturation of the teeth takes place; and (7) leakage of lines of force through the armature core, exterior to the winding, particularly in case of projecting teeth and of perforated cores.

Comparing these advantages and disadvantages with each other we find that the conditions that have to be fulfilled in order to bring to prominence certain advantages will also favor the conspicuousness of certain of the disadvantages, and moreover we see that what is an advantage in one case may be a decided disadvantage in another. All considered, therefore, there are no such striking advantages in either the toothed or the smooth core as to make any one of them superior in all cases over the other, and a general decision whether a toothed or a smooth-core armature is preferable, cannot be arrived at. As a matter of fact, in practice it chiefly depends upon the purpose of the machine to be designed whether a smooth or a toothed core is preferably used in its armature. In machines with toothed and with perforated armatures an increase of the load has the effect of increasing the saturation of the iron projections and therewith the reluctance of the air gap; the counter-magnetomotive force of the armature, which also increases with the load, has therefore to overcome a greater reluctance as it increases itself, in consequence of which the demagnetizing effect of the armature is kept very nearly constant at all loads. Hence the distribution of the field in the gap remains nearly the same and the angle inclosed between the planes of commutation at no load and at maximum output is reduced to a minimum. For cases where sparkless commutation is required without shifting the brushes for varying loads, as for instance in railway generators, in which due to the continual and sudden fluctuations of the load a shifting of the brushes is impracticable, the employment of toothed armatures is preferable, for the attainment of the desired end in this case outweighs all their disadvantages. On the other hand, the self-induction in smooth-core armatures, owing to the absence of iron between

the conductors, is much less, and consequently they are chosen in cases of machines in which large currents are commutated at low voltages, such as in central station lighting generators and in electro-metallurgical machines. In the latter case the disadvantage of increased self-induction in the toothed armature is the main consideration and drives it out of competition with the smooth armature, in spite of all advantages which it may have otherwise. Again, in the case of motors, where a large torque is the desideratum, especially in low-speed motors, such as single reduction and gearless railway motors, the toothed armature answers best, as in this instance its advantage of increased drag upon the teeth is considered the prominent one. Toothed armatures must further be employed if, in a series motor, a constant speed under all loads is to be attained, for at light loads the teeth, being worked at a low point of magnetization, offer but little reluctance to the flux through the armature, while at heavy loads the teeth become saturated and considerably increase the reluctance of the magnetic circuit, thereby preventing the induction from increasing with increased field excitation, the result being a motor that comes much nearer being self-regulating than one with a smooth-core armature.

In order to more definitely determine the mechanical advantage of the iron projections, W. B. Sayers[1] compared the pull on the conductors in toothed and smooth-core armatures. He found that in toothed armatures the driving force is borne directly by the iron instead of by the conductors as in case of smooth-core machines. Taking the case of an armature in which the thickness of the tooth is equal to the width of the slot, he shows that, when the density in the teeth is 100,000 lines per square inch (= 15,500 lines per square centimetre), that in the slots is about 300 lines per square inch (= 47 per square centimetre), while in a smooth-core armature the field density would be about 50,000 per square inch (= 7,750 per square centimetre), from which follows that the force acting upon the conductors is about 167 times greater in the latter case than in the former. In another example he takes a higher magnetic density and finds that the pull in case of the toothed arma-

[1] London *Electrician*, April 19, 1895; *Electrical World*, vol. xxv. p. 562 (May 11, 1895).

ture is only 16 times as great as in a corresponding smooth armature. If the density is so high that the teeth become saturated, the field density in the slots will approach that in the gap of an equally sized smooth armature, and the forces will be about equal in both cases.[1]

When a toothed or perforated armature is placed in a magnetic field, the lines of force concentrate toward the teeth in form of bunches, Fig. 46, and thereby destroy the uniformity

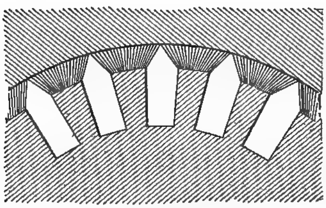


Fig. 46.—Distribution of Magnetic Lines around Toothed Armature at Rest.

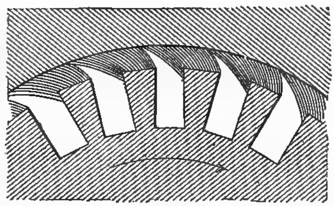


Fig. 47.—Distribution of Magnetic Lines around Toothed Armature in Motion.

of the field. If the armature is now revolved these bunches are taken along by the teeth until a position, Fig. 47, is reached in which the lines have been distorted to such a degree that the reluctance of their path has reached the maximum value the magnetomotive force of the field is able to overcome. At this moment each bunch of lines will commence to change over to the next following tooth of the armature, and thus every line of each bunch will in succession cross the slot immediately behind the tooth through which it had passed previously. In this manner every line of force passing through the armature core cuts all the inductors on the armature during each revolution. By the action of changing over from one tooth to the next, an oscillation or quivering of the magnetic lines is caused, which tends to set up eddy currents in the teeth and in the polar faces. In order to obviate excessive heating from this cause it is necessary that

[1] See also "On the Seat of the Electrodynamic Force in Ironclad Armatures," by E. J. Houston and A. E. Kennelly, *Electrical World*, vol. xxviii. p. 3 (July 4, 1896). Comment, by Townsend Wolcott, *Electrical World*, vol. xxviii. p. 271 (September 5, 1896), and by William Baxter, Jr., *Electrical World*, vol. xxviii. p. 299 (September 12, 1896).

the teeth must be made numerous and narrow, and that the length of the air gap between the pole face and the top of the iron projection must bear a definite relation to the width of the slot. In practice it has been found that—according to the field density employed—air gaps having a radial length of from one-fourth to one-half the width of the slot, in large and medium size machines respectively, and a ratio of gap to slot up to 1 in very narrow slotted small armatures, give the best results.

a. Toothed Armatures.

The mean winding diameter, d'_a, of a toothed armature being determined by means of formula (30), its *core diameter* or diameter at bottom of slots, d_a, Fig. 48, is obtained by deducting

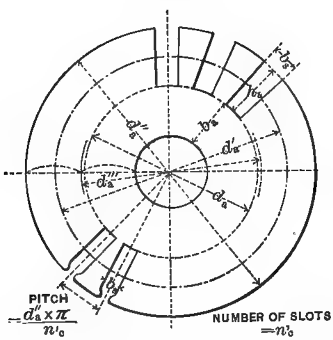


Fig. 48.—Dimensions of Toothed Core Disc.

ing from d'_a the height of the winding space, or, in this case, the depth of the slots, h_a, averages for which are given in the fourth column of Table XVIII., § 23. The outside diameter of the armature, d''_a, over the teeth, is obtained by adding h_a, from Table XVIII. to d'_a, from formula (30).

The *number of the slots*, n'_c, since practice has shown, in accordance with the theoretical considerations, that better results are obtained from deep and narrow slots than from shallow and wide ones, should be taken as large as possible, from the mechanical and economical standpoint. In the following Table XIII. good practical limits of the number of slots for armatures of different diameters are given:

TABLE XIII.—NUMBER OF SLOTS IN TOOTHED ARMATURES.

DIAMETER OF ARMATURE. d''_a		NUMBER OF SLOTS. n'_c
Inches.	Centimetres.	
5	12.5	25 to 40
10	25	40 " 70
15	27.5	50 " 100
20	50	60 " 150
30	75	80 " 200
50	125	100 " 250
100	250	150 " 300
150	375	200 " 400
200	500	300 " 500

As to the *width of the slots*, b_s, Fig. 49, a number of conflicting conditions governs its relation to that of the teeth: On account of the tendency of the teeth to create eddy currents

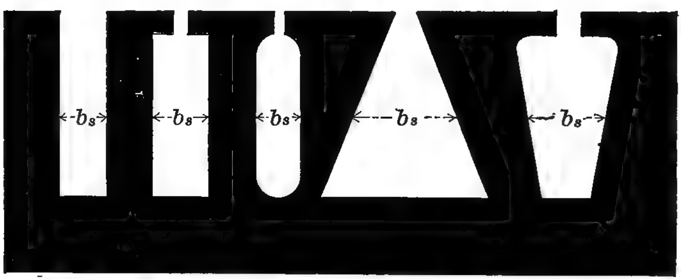


Fig. 49.—Various Types of Slots for Toothed Armatures.

in the pole faces, their width ought to be small compared with that of the slots, or shallow slots and narrow teeth should be used; in order to reduce the hysteresis loss in the teeth and the heat caused by the same to a minimum, the mass of the teeth should be small and their area perpendicular to the flow of the lines, hence their width should be large, that is, on this account narrow slots and wide teeth should be employed; for the sake of effectively reducing the magnetic reluctance of the circuit, the area at the bottom of the teeth should be large, hence the slots narrow and the teeth wide.

L. Baumgardt[1] proposes to calculate the hysteresis heat per unit volume of the teeth for a variety of values for the width of the slot, also to find that width of slot for which the density in the teeth for given armature-diameter, number, and sectional area of slots becomes a minimum, and to compare the values so found, choosing a practical width that is not too far from giving minimum hysteresis heat and minimum tooth density. For the purpose of calculating the relative values of the hysteresis heat for a given width of the slot he gives a formula which, when reduced to its simplest form, becomes:

$$Y = \frac{b_s^2 \times \left\{A^{\frac{2}{3}} - \left(A - \frac{S_s}{b_s}\right)^{\frac{2}{3}}\right\}}{S_s \times (2b_s \times A - S_s)},$$

the symbol A standing for the expression:

$$A = \frac{d''_a \pi - n'_c \times b_s}{2n'_c \times \tan \frac{180^\circ}{n'_c}}, \qquad \ldots\ldots\ldots\ldots (32)$$

in which Y = hysteresis heat per unit volume of teeth divided by a constant that depends upon the machine under consideration;

d''_a = external diameter of armature (in millimetres);

n'_c = number of slots;

b_s = width of slots (in millimetres);

S_s = sectional area of slot $= b_s \times h_a$ (in millimetres).

In order to save the trouble of employing this rather complicated formula in every single instance, the author has calculated the values of Y for $b_s = .75$ to 25 mm. (1/32 to 1 inch) for a variety of cases ranging from $d''_a = 100$ mm. (= 4″), $n'_c = 24$, $S_s = 90$ mm.^{2} (= .14 square inch) to $d''_a = 5{,}000$ mm. (= 197½″), $n'_c = 320$, $S_s = 2{,}500$ mm.^{2} (=3.875 square inches), and in taking the minimum value of Y in each armature as unity, has, for every case, plotted a curve with the various widths of the slot as abscissæ and the value of

$$\frac{Y}{Y_{\text{min}}}$$

[1] "On the Dimensioning of Toothed Armatures," by Ludwig Baumgardt, *Elektrotechn. Zeitschr.*, vol. xiv. p. 497 (September 1, 1893); *Electrical World*, vol. xxii. p. 234 (September 23, 1893).

as ordinates. In Fig. 50 these curves are arranged in four groups with reference to the size of the armature, only the two limiting curves of each group being drawn. They show that the specific hysteresis heat at first diminishes slightly as the width of the slot increases and arrives at a minimum point

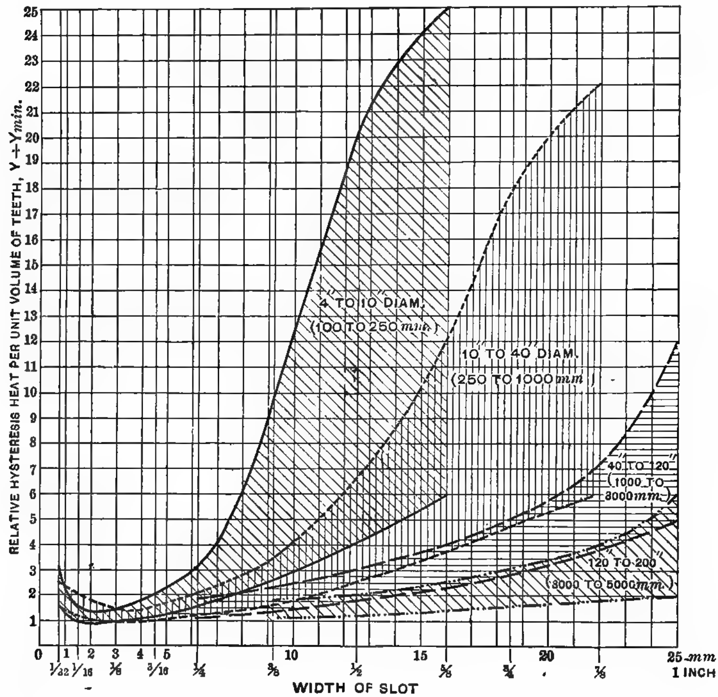


Fig. 50.—Variation of Hysteresis Heat per Unit Volume of Teeth with Increasing Width of Slot, for different Sizes of Toothed Armatures.

which over the whole range lies between the narrow limits of 1/16 and 3/16 inch (1.5 and 5 mm.) width of slot, after which it increases very rapidly in case of small armatures, and more or less slowly in case of large ones. Since a slot of 1/16 inch (1.5 mm.) width is too small for even the smallest armature and one of 3/16 inch (4.5 mm.) is too small for anything but a very small machine, it follows that the minimum of hysteresis heating cannot be reached in practice, but by making the slots narrow and deep the hysteresis effect can be kept within prac-

tical limits. The width of the slot having been chosen, the limits of the specific hysteresis heat, expressed as multiples of the minimum value, can then be obtained from the curves in Fig. 50, or from the following Table XIV., which has been compiled from the curves given:

TABLE XIV.—SPECIFIC HYSTERESIS HEAT IN TOOTHED ARMATURES, FOR DIFFERENT WIDTHS OF SLOTS.

Width of Slot.		Relative Specific Hysteresis Heat in Teeth.			
Inch.	Millimetres.	4" to 10" (100 to 250 mm.) Armature.	10" to 40" (250 to 1,000 mm.) Armature.	40" to 120" (1,000 to 3,000 mm.) Armature.	120" to 200" (3,000 to 5,000 mm.) Armature.
1/32	0.75	1½ to 3	1½ to 2½	1¼ to 2½	1¼ to 2
1/16	1.5	1 " 1½	1¼ " 2	1⅛ " 2	1⅛ " 1¾
1/8	3	1 " 1½	1 " 1½	1 " 1½	1⅛ " 1½
3/16	4.5	1¼ " 2	1 " 1½	1 " 1½	1 " 1½
1/4	6	1½ " 3	1¼ " 2	1⅛ " 1¾	1 " 1½
3/8	9.25	2½ " 10	1½ " 3½	1¼ " 2¼	1⅛ " 1¾
1/2	12.5	4 " 20	2½ " 6½	1¾ " 3	1¼ " 2
5/8	16	6 " 25	3¾ " 12	2¼ " 4	1⅜ " 2½
3/4	18.5	......	4¾ " 18	2¾ " 5	1½ " 3
7/8	22	......	6 " 22	3¾ " 7	1¾ " 4
1	25	......	......	5 " 12	2 " 6

According to this table the specific heat due to hysteresis, if, for instance, a ¾ inch (18.5 mm.) slot is used in a 100 inch (2,500 mm.) armature, is from 2¾ to 5 times as high as in case of a ⅛ inch (3 mm.) slot for which, in that group, it is a minimum.

The value of b_s, for which the magnetic density in the teeth becomes a minimum, is found by making the circumferential width at the bottom of the teeth,

$$\pi\,(d''_a - 2h_a) - n'_c \times b_s = \pi\left(d''_a - \frac{2\,S''_s}{b_s}\right) - n'_c \times b_s$$

a maximum; and the value of b_s which does the latter is

$$b''_s = \sqrt{\frac{2\pi \times S''_s}{n'_c}}, \qquad \ldots\ldots\ldots\ldots(33)$$

where b''_s = width of slot for minimum tooth density, in inches or in centimetres;

S''_s = cross-section of slot, in square inches, or in square centimetres;

n'_c = number of slots.

While formula (33) in connection with Table XIV. is very useful for the determination of the best width of the slots in case their cross-section is given, ordinarily the problem is to be attacked by first selecting the number of teeth, then determining the width, and finally the depth of the slot. Considering all the adverse conditions, the author has found it a good practical rule to make the width of the slots

$$b_s = \frac{d''_a \times \pi}{2 \times n'_c}, \qquad \ldots\ldots\ldots\ldots (34)$$

TABLE XV.—DIMENSIONS OF TOOTHED ARMATURES, IN ENGLISH MEASURE.

DIAMETER OF ARMATURE, IN INCHES. d''_a	DIMENSIONS OF SLOTS. Depth, in inches. h_a	Width, in inches. b_s	Ratio of Depth to Width. $h_a : b_s$	NUMBER OF SLOTS. $n'_c = \frac{d''_a \pi}{2b_s}$	CORE DIAMETER, IN INCHES. $d_a = d''_a - 2h_a$	WIDTH AT BOTTOM OF TOOTH, IN INCHES. $\frac{d_a \pi}{n'_c} - b_s$
5	$\frac{5}{8}$	$\frac{1}{4}$	2.50	30	$3\frac{3}{4}$	.14
6	$\frac{11}{16}$	$\frac{17}{64}$	2.59	36	$4\frac{5}{8}$	.14
8	$\frac{3}{4}$	$\frac{9}{32}$	2.66	44	$6\frac{1}{2}$	.18
10	$\frac{7}{8}$	$\frac{19}{64}$	2.95	52	$8\frac{1}{4}$	.20
12	1	$\frac{5}{16}$	3.20	60	10	.21
15	$1\frac{1}{8}$	$\frac{21}{64}$	3.43	72	$12\frac{3}{4}$	.23
18	$1\frac{1}{4}$	$\frac{11}{32}$	3.64	80	$15\frac{1}{2}$	.26
21	$1\frac{3}{8}$	$\frac{3}{8}$	3.66	88	$18\frac{1}{4}$	.28
25	$1\frac{1}{2}$	$\frac{13}{32}$	3.69	98	22	.30
30	$1\frac{5}{8}$	$\frac{7}{16}$	3.71	108	$27\frac{3}{4}$	.37
40	$1\frac{3}{4}$	$\frac{15}{32}$	3.73	136	$36\frac{1}{2}$	.38
50	$1\frac{7}{8}$	$\frac{1}{2}$	3.75	160	$46\frac{1}{4}$	.41
60	2	$\frac{17}{32}$	3.76	180	56	.45
70	$2\frac{1}{8}$	$\frac{9}{16}$	3.78	196	$65\frac{3}{4}$	.51
80	$2\frac{1}{4}$	$\frac{19}{32}$	3.79	212	$75\frac{1}{2}$	.52
90	$2\frac{1}{2}$	$\frac{5}{8}$	4	228	85	.55
100	$2\frac{3}{4}$	$\frac{11}{16}$	4	232	$94\frac{1}{2}$	.59
125	3	$\frac{3}{4}$	4	264	119	.67
150	$3\frac{1}{2}$	$\frac{7}{8}$	4	272	143	.78
200	4	1	4	320	192	.89

that is to say, to make the width of the slots equal to half their pitch on the outer circumference, for the special case of a straight-tooth core, then the width of the slots is equal to the top width of the teeth.

The proper sectional area S''_s of the slots to accommodate a sufficient amount of armature winding is obtained by making the depth of the slot from 2½ to 4 times its width, according to the size of the armature, the minimum value referring to very small and the maximum value to the largest machines.

Applying these rules to armatures of various sizes, the accompanying Tables XV. (see page 70) and XVI. have been calculated, giving the dimensions of toothed armatures, the former in English and the latter in metric measure:

TABLE XVI.—DIMENSIONS OF TOOTHED ARMATURES, IN METRIC MEASURE.

DIAMETER OF ARMATURE, IN CM. d''_a	DIMENSIONS OF SLOTS.			NUMBER OF SLOTS. $n'_c = \frac{d''_a \pi}{2 b_s}$	CORE DIAMETER, IN CM. $d_a = d''_a - 2h_a$	WIDTH AT BOTTOM OF TOOTH, IN CM. $\frac{d_a \pi}{n'_c} - b_s$
	Depth, in cm. h_a	Width, in cm. b_s	Ratio of Depth to Width. $h_a . b_s$			
10	1.5	.6	2.50	24	7	.32
15	1.75	.65	2.69	36	11.5	.36
20	2	.7	2.86	44	16	.44
25	2.25	.75	3.00	52	20.5	.49
30	2.5	.8	3.13	60	25	.51
40	3	.9	3.34	70	36	.72
50	3.5	1.0	3.50	78	43	.75
60	4	1.1	3.64	86	52	.80
75	4.5	1.2	3.75	98	66	.92
100	5	1.3	3.85	120	90	1.06
150	5.5	1.4	3.95	168	139	1.20
200	6	1.5	4.0	210	188	1.32
250	7	1.75	4.0	224	236	1.56
300	8	2.0	4.0	236	286	1.81
400	9	2.25	4.0	288	382	1.92
500	10	2.5	4.0	320	480	2.21

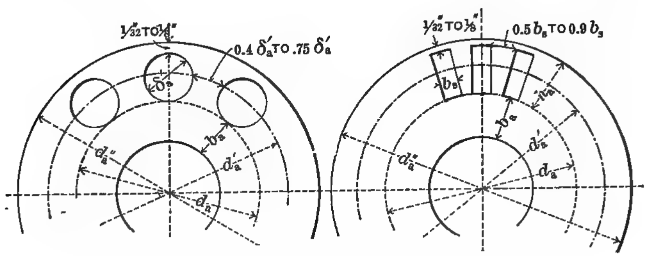
b. Perforated Armatures.

The same considerations that prevailed in determining the number and the width of the slots in toothed armatures are also decisive for the dimensioning of perforated cores. The

number of perforations, for this reason, can be taken in the same limits as the number of slots for toothed cores. See Table XIII.

In case of *round* holes, Fig. 51, the thickness of the iron between two adjacent perforations should be taken between 0.4 and 0.75 times the diameter of the hole.

For *rectangular* holes, Fig. 52, the thickness of the iron



Figs. 51 and 52.—Dimensions of Perforated-Core Discs.

between them is to be taken somewhat greater than for round holes, namely, from 0.5 to 0.9 times the width of the channel.

The distance of the holes from the outer periphery is to be made as small as possible, and may vary between 1/32 and 1/8 inch, according to the size of the armature.

23. Length of Armature Core.

The number of wires that can be placed in one layer around the armature circumference, and the depth of the winding space, determine the total number of conductors on the armature, and the latter, together with the length of active wire, gives the length of the armature core.

a. Number of Wires per Layer.

For smooth armatures the number of wires per layer is obtained in dividing the available core circumference by the thickness of the insulated armature wire. If the whole circumference is to be filled by the winding, then

$$n_w = \frac{d_a \times \pi}{\delta'_a}, \qquad \text{.............} (35)$$

where n_w = number of armature wires per layer;
d_a = diameter of armature core, in inches;
δ'_a = width of *insulated* armature conductor, in inches. See § 24.

If, in the metric system, d_a is given in cm. and δ'_a in mm., the above formula becomes:

$$n_w = \frac{10 \times d_a \times \pi}{\delta'_a}. \quad \ldots\ldots\ldots\ldots(36)$$

In case the winding is to consist in separated coils, the sum of the separating spaces is to be deducted from the armature circumference.

The value of n_w is to be rounded off to the nearest lower even and easily divisible number, and, in the case of a drum armature, allowance for division strips or driving horns is to be made according to Table XVII. This table gives the average circumferential space occupied by the division strips in drum armatures of different sizes and various voltages, in per cent. of the core circumference. After fixing the number of armature divisions, which will be shown in § 25, this table may also be used to determine the thickness of the driving horns, which are usually made of hard wood, or fibre, and sometimes of iron:

TABLE XVII.—ALLOWANCE FOR DIVISION STRIPS IN DRUM ARMATURES.

Diameter of Armature Core.		Percentage of Core Circumference Occupied by Division Strips.		
Inches.	Centimetres.	Up to 300 Volts.	400 to 750 Volts.	800 to 2000 Volts.
Up to 3	Up to 7.5	12 %	15 %	..
" 6	" 15	10	12	15 %
" 12	" 30	8	10	12
" 20	" 50	7	9	10
" 30	" 75	6	8	9

Denoting one-hundredth of these percentages by k_1, the core circumference being unity, the formula for the number of wires per layer in a drum armature in English measure becomes:

$$n_w = \frac{d_a \times \pi \times (1 - k_1)}{\delta'_a}. \quad \ldots\ldots\ldots(37)$$

In metric measure the same value of n_w is obtained by multiplying the numerator of (37) by 10, thus deriving the metric formula similarly as (36) is derived from (35).

In toothed armatures the number of wires in one layer is found from the number, n'_c, and the available width, b'_s, of the slots by the equation:

$$n_w = n'_c \times \frac{b'_s}{\delta'_a}. \qquad \ldots\ldots\ldots\ldots\ldots (38)$$

In this formula the value of b'_s, is to be derived from the actual width, b_s, of the armature slots (§ 22), by deducting the thickness of insulation used for lining their sides, data for the latter being given in § 24.

For calculation in metric system the factor 10 is to be employed, as before.

b. Height of Winding Space. Number of Layers.

In dividing the available height, h'_a, of the winding space by the height δ''_a, of the insulated armature conductor, the number of layers of wire on the armature is found:

$$n_l = \frac{h'_a}{\delta''_a}; \qquad \ldots\ldots\ldots\ldots\ldots (39)$$

n_l = number of layers of armature wire;
h'_a = available height of winding space, in inches;
δ''_a = height of *insulated* armature conductor, in inch.

The height of the insulated armature conductor, δ''_a, in the case of round or square wire, is identical with its width, δ'_a.

If h_a is expressed in cm. and δ''_a in mm., the right-hand side of (39) must be multiplied by 10 in order to correct the formula for the metric system.

The available height, h'_a, of the winding space is obtained from its total height, h_a, averages for which are given in Table XVIII. (page 75) by deducting from 1/32 to 1/4 inch (see § 24), according to size and voltage of machine, for the insulation of the armature core, insulation between the layers, thickness of binding wires, etc.

The nearest whole number is to be substituted for the value of n_l.

TABLE XVIII.—HEIGHT OF WINDING SPACE IN ARMATURES.

ENGLISH MEASURE.				METRIC MEASURE.			
Diameter of Armature, in inches.	Height of Winding Space, in inches.			Diameter of Armature, in cm.	Height of Winding Space, in centimetres.		
	Smooth Armature Core.		Toothed Armature Core.		Smooth Armature Core.		Toothed Armature Core.
	Drum Armature.	Ring Armature.			Drum Armature.	Ring Armature.	
2	$\frac{1}{4}''$	..	..	5	.6	..	..
4	$\frac{5}{16}$	$\frac{7}{32}''$	..	10	.8	.5	..
6	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{8}''$	15	1	.6	1.5
10	$\frac{1}{2}$	$\frac{9}{32}$	$\frac{3}{4}$	25	1.2	.7	2
15	$\frac{5}{8}$	$\frac{5}{16}$	1	35	1.5	.8	2.5
20	$\frac{3}{4}$	$\frac{3}{8}$	$1\frac{1}{4}$	50	2	1	3
30	$\frac{7}{8}$	$\frac{7}{16}$	$1\frac{1}{2}$	75	2.2	1.2	3.5
50	..	$\frac{1}{2}$	$1\frac{3}{4}$	100	..	1.4	4
75	..	$\frac{5}{8}$	2	200	..	1.6	5
100	..	$\frac{3}{4}$	$2\frac{1}{2}$	300	..	1.8	6
150	..	$\frac{7}{8}$	$3\frac{1}{4}$	400	..	2.1	8
200	..	1	4	500	..	2.5	10

The approximate radial height taken up by the *armature-binding* in *smooth* armatures may be taken from the following table:

TABLE XVIIIa.—DATA FOR ARMATURE BINDING.

Capacity of Dynamo in Kilowatts.	Thickness of Armature Binding.	Size of Binding Wire.	Thickness of Mica Insulating Strip.	Average Width of Bands.
1	.030″	No. 24 B. & S.	.010″	$\frac{1}{4}''$
5	.035	" 22 "	.010	$\frac{3}{8}$
10	.040	" 21 "	.012	$\frac{1}{2}$
50	.050	" 19 "	.014	$\frac{5}{8}$
100	.060	" 17 "	.015	$\frac{3}{4}$
200	.070	" 16 "	.016	$\frac{7}{8}$
500	.080	" 14 "	.018	1
1000	.090	" 13 "	.018	$1\frac{1}{2}$
2000	.100	" 12 "	.020	2

These figures, besides allowing for the binding wires, which range from No. 24 B. & S. (.020″) to No. 12 B. & S. gauge (.080″) respectively, as indicated, include the insulation of the

bands, the thickness of which, therefore, varies from .010 to .020 inch, according to the size of the armature. The bands usually consist of from 12 to 25 convolutions of phosphor bronze or steel wire, their width varying from ¼ inch to 2 inches. They are insulated from the winding by strips of mica from ⅛ to 1 inch wider than themselves, and are placed at distances apart equal to about twice the width of a band.

In *straight-tooth* armatures recesses are usually turned to receive a few light bands, while armatures with *projecting teeth* and with *perforated cores* need, of course, no binding at all.

c. Total Number of Armature Conductors. Length of Armature Core.

The product of the number of layers and the number of conductors per layer gives the total number of conductors on the armature; and this, divided into the total length of active armature conductor, furnishes the active length of one conductor, that is, the length of the armature body:

$$l_a = \frac{12 \times L_a}{\frac{n_w \times n_l}{n_\delta}} = \frac{12 \times n_\delta \times L_a}{n_w \times n_l}, \quad \ldots\ldots (40)$$

where l_a = length of armature core parallel to pole faces, in inches;

L_a = length of active armature conductor, in feet, from formula (26);

n_w = number of wires per layer, from formula (35), (37), or (38), respectively;

n_l = number of layers of armature wire, from formula (39);

n_δ = number of wires stranded in parallel to make up one armature conductor of area $\delta_a{}^2$, formula (27);

$\frac{n_w \times n_l}{n_\delta}$ = total number of conductors on armature.

In the metric system, L_a being expressed in metres, the length l_a is found in centimetres by replacing the factor 12 in (40) by 100.

For preliminary calculations an approximate value of the

number of conductors, N_c, all around the polefacing circumference of the armature, may be obtained by dividing the conductor area found from formula (27) into the net area of the winding space. Taking .6 of the total area of the winding space as an average for its net area in smooth armatures with winding filling the entire circumference, we obtain:

$$\frac{n_w \times n_1}{n_\delta} = N_c = \frac{1,000,000 \times .6 \times d_a \times \pi \times h_a}{\delta_a{}^2}$$

$$= 1,885,000 \times \frac{d_a \times h_a}{\delta_a{}^2}. \quad \ldots\ldots\ldots (41)$$

This result is to be correspondingly reduced for windings filling only part of circumference, or to be multiplied by $(1 - k_1)$, see formula (37), in case of a drum armature, respectively.

In toothed armatures the average net height of the winding space is about three-fourths of the total depth of the slot, hence the approximate number of armature conductors:

$$\frac{n_w \times n_1}{n_\delta} = N_c = \frac{1,000,000 \times n'_c \times b'_s \times \frac{3}{4} h_a}{\delta_a{}^2}$$

$$= 750,000 \times \frac{n'_c \times b'_s \times h_a}{\delta_a{}^2}. \quad \ldots\ldots\ldots (42)$$

In (41) and (42) the values of d_a, h_a, and b'_s are given in inches, and $\delta_a{}^2$ in circular mils.

For metric calculations formula (41) takes the form:

$$N_c = \frac{100 \times .6 \times d_a \pi \times h_a}{(\delta_a)^2_{mm.}}$$

$$= 190 \times \frac{d_a \times h_a}{(\delta_a)^2_{mm.}}, \quad \ldots\ldots\ldots\ldots\ldots (43)$$

and formula (42) is replaced by:

$$N_c = \frac{100 \times n'_c \times b'_s \times \frac{3}{4} h_a}{(\delta_a)^2_{mm.}}$$

$$= 75 \times \frac{n'_c \times b'_s \times h_a}{(\delta_a)^2_{mm.}}. \quad \ldots\ldots\ldots\ldots (44)$$

In (43) and (44) the dimensions d_a, h_a, and b'_s are expressed in centimetres, and $(\delta_a)^2_{mm.}$ in square millimetres.

24. Armature Insulations.

a. Thickness of Armature Insulations.

According to the size and the voltage of a dynamo the thicknesses of the insulations in its armature vary in very wide limits.

The coating of the armature conductor, if single wire is employed, usually is effected by a double cotton covering ranging in diametral thickness from .012 to .020 inch (0.3 to 0.5 mm.), according to the size of the wire and the voltage, see Table XXVI., § 28. If stranded cable is used for winding the armature, either bare or single cotton-covered wire is used to make up the cable, and the whole is covered with two or three layers of cotton. The thickness of the single cotton insulation in this case varies from .005 to .010 inch (0.125 to 0.25 mm.) in diameter. For very thin wires, from No. 20 B. W. G. (.035 inch = 0.9 mm.) down, a double silk covering from .004 to .005 inch (0.1 to 0.125 mm.) diametral thickness is applied.

In case of rectangular or wedge-shaped conductors, according to their size and to the voltage of the machine, either a double cotton covering, as with wires, is used, or oiled paper, cardboard, asbestos, or mica is employed for their enwrapping or separation. The thickness of the insulation in the latter case varies between .010 and .0125 inch (0.25 and 3 mm.) each side, see columns *e* of the following Table XIX.

Besides this coating of the single conductors, sometimes—particularly in high voltage machines—one or more sheets of insulating material are employed to separate the layers from one another. The thickness of this insulation, for which either oiled paper, rubber tape, silk, or mica is used, ranges from .004 to .030 inch (0.1 to 0.75 mm.), columns *f*, Table XIX.

The insulation of the conductors from the iron body in smooth armatures is effected by serving the core with one or two coatings of enamel or japan and then covering it by either oiled paper, cardboard, canvas, silk, tape, sheet rubber, cotton cloth, asbestos, or mica, varying in radial thickness between .010 and .200 inch (0.25 and 5 mm.), columns *a*, Table XIX.

In *drum armatures* the complete core insulation, besides this circumferential coating, *a*, consists of coverings, *b*, over the core edges, of core-face insulations, *c*, and of shaft-insulations,

d, all overlapping each other as shown in Fig. 53. The *edge insulation* is effected by first covering the core with layers of oiled paper, oiled muslin, or canvas, then winding with rubber tape, and finally adding a layer of mica or asbestos; according to the voltage and size of machine this edge insulation varies

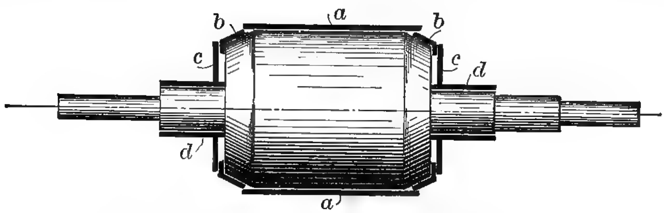


Fig. 53.—Core Insulations on Drum Armature.

from .020 to .250 inch (0.5 to 6.4 mm.), see columns *b*, Table XIX. The *core-face insulation* is made up of circular sheets of oiled paper, muslin, linen, cardboard, asbestos, vulcanized fibre, or leatheroid, in thicknesses varying from .030 to .400 inch (0.75 to 10 mm.), columns *c*, Table XIX. The *shaft-insulation*, finally, usually consists of rubber tape in connection with oiled paper, muslin, or mica; its radial thickness ranges from .050 to .300 inch (1.25 to 7.5 mm.), see columns *d*,

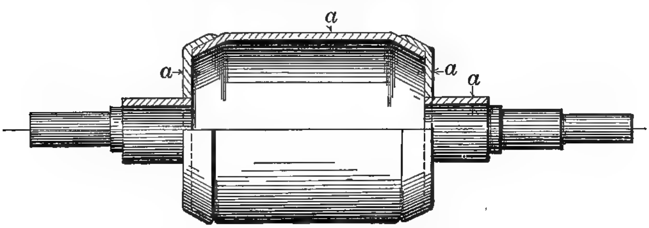


Fig. 54.—Core Insulation of High-Voltage Drum Armature.

Table XIX. In modern high-voltage drum armatures mica is used exclusively for insulating the core, the edge- and face-insulations being united into the form of flanged micanite discs, and micanite cylinders or tubes being used around the circumference and over the shaft, see Fig. 54. In this case of all-mica insulation the thicknesses of the coatings at the various parts of the core can all be made alike and equal in amount to that of the circumferential covering, columns *a*, Table XIX.

In *ring armatures* the core insulation, *a*, is extended so as to include also the inner circumference and the faces of the body as well. To prevent grounding of the winding at the edges, their insulation is thickened by coatings inserted beneath the circumferential covering. In small rings, Fig. 55, the insula-

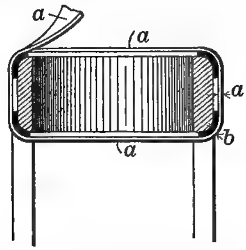


Fig. 55.—Core Insulations on Small Ring Armature.

tion, *a*, usually is applied in form of a narrow band, and is simply wrapped around the core, the reinforcements at the edges being laid upon the core, as the enwrapping proceeds, in the shape of short strips of oiled material or mica of such gauge as to make the total thickness of insulation at the edges equal to the edge-insulation given in columns *b* of Table XIX. In large ring machines, Fig. 56, the core faces are often insu-

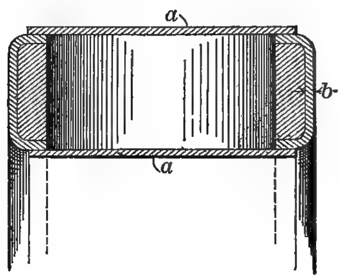


Fig. 56.—Core Insulations on Large Ring Armature.

lated by means of curved vulcabeston, pressboard, or micanite discs, *b*, fitting over the end rings; these discs are pressed or molded in special forms, and are of a thickness ranging from .060 to .250 inch (1.5 to 6.4 mm.), see columns *b*, Table XIX.

For *toothed* and *perforated* armatures, Fig. 57, the core-circumference insulation is carried out in form of channels or tubes of paper, or cardboard, or vulcanized fibre, fitted into the grooves, or, especially in large toothed-core machines, by

means of micanite troughs lining the bottom and the sides of the slots. The thickness of this lining ranges from .010 to .125 inch (0.25 to 3 mm.), proportional to the size of the slots,

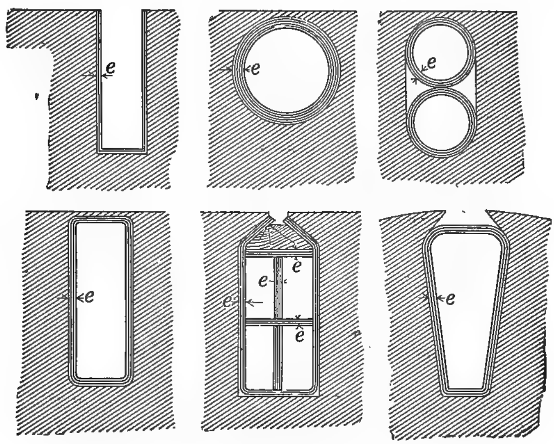


Fig. 57.—Various Forms of Slot Insulation.

and according to the voltage of the dynamo, columns *e*, Table XIX. The core faces of toothed armatures are insulated in a similar manner as those of a smooth armature. Fig. 58 shows

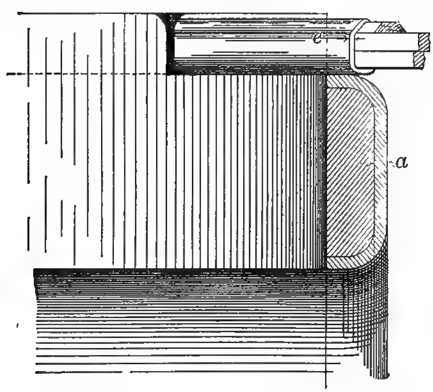


Fig. 58.—Core Insulation of Large Toothed-Ring Armature.

a well-insulated armature core of a large toothed-ring machine, micanite troughs being used in the slots and micanite caps over the end rings.

TABLE XIX.—THICKNESS OF ARMATURE INSULATIONS FOR DYNAMOS OF VARIOUS SIZES AND VOLTAGES.

CAPACITY IN KILO-WATTS.	UP TO 150 VOLTS.						250 TO 500 VOLTS.						1,000 TO 3,000 VOLTS.						5,000 TO 10,000 VOLTS.					
	Core Circumference.	Core Edges.	Core Faces.	Shaft Insulation.	Slot Lining.	Insulation between Layers.	Core Circumference.	Core Edges.	Core Faces.	Shaft Insulation.	Slot Lining.	Insulation between Layers.	Core Circumference.	Core Edges.	Core Faces.	Shaft Insulation.	Slot Lining.	Insulation between Layers.	Core Circumference.	Core Edges.	Core Faces.	Shaft Insulation.	Slot Lining.	Insulation between Layers.
*	a †	b ‡	c §	d ‡	e ‖	f ¶	a †	b ‡	c §	d ‡	e ‖	f ¶	a †	b ‡	c §	d ‡	e ‖	f ¶	a †	b ‡	c §	d ‡	e ‖	f ¶
	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Up to .25	.010	.020	.030	.050	..	.004	.015	.030	.050	.060		.005	..	..	..	..	..	..	..	..	..	..	..	..
" 1	.012	.025	.040	.060	..	.005	.020	.040	.060	.070	..	.007	.030	.050	.075	.080	..	.010	..	..	..	..	..	..
" 5	.015	.030	.050	.070	.010	.005	.030	.050	.075	.080	.015	.007	.040	.060	.100	.090	.020	.010	.050	.080	.125	.100	.030	.012
" 10	.020	.040	.075	.080	.012	007	.040	.060	.100	.090	.020	.010	.050	.070	.125	.100	.025	.012	.060	.090	.150	.110	.040	.015
" 20	030	.050	.100	.090	.015	.007	.050	.070	.125	.100	.025	.010	.060	.080	.150	.110	.030	.012	.070	.100	.200	.125	.050	.015
" 50	.040	.060	.125	.100	.020	.010	.060	.080	.150	.110	.030	.012	.070	.090	.200	.125	.035	.015	.080	.110	.250	.150	.060	.020
" 100	.050	.070	.150	.110	.025	.010	.070	.090	.200	.125	.035	.012	.080	.100	.250	.150	.040	.015	.090	.125	.300	.200	.070	.020
" 200	.060	.080	.200	.125	.030	.012	.080	.100	.250	.150	.040	.015	.090	.110	.300	.200	.050	.020	.100	.150	.350	.250	.080	.025
" 300	.070	.090	.250	.150	.035	.012	.090	.110	.300	.200	.045	.015	.100	.125	.350	.250	.060	.020	.125	.175	.400	.300	.090	.025
" 500	.080	.100	..	..	.040	.015	.100	.120	..	..	.050	.020	.110	.150	..	..	.070	.025	.150	.200	..	..	.100	.030
" 1,000	.090	.110	..	..	.045	.015	.110	.130	..	..	.060	.020	.125	.175	..	..	.080	.025	.175	.225	..	..	.110	.030
" 2,000	.100	.125	..	..	050	.015	.125	.150	..	..	.070	.020	.150	.200	..		.090	.025	.200	.250	..	..	.125	.030

* The letters at the heads of the columns refer to Figs. 53 to 58.
† Also thickness of edge, face, and shaft-covering, in the case of armatures insulated by mica throughout.
‡ In case of all-mica insulation, see columns *a*.
§ For drum armatures only; for rings, see columns *b*; in case of all-mica insulation, columns *a*.
‖ Also insulation between bare conductors.
¶ For winding with insulated conductors only; for insulation between bare conductors, see columns *e*.

In the preceding Table XIX. (see page 82) the thicknesses of armature core insulations are compiled for machines of various sizes and for different voltages.

b. Selection of Insulating Material.

Armature insulations must not only possess high insulating resistance, but also great disruptive strength, that is, the ability to withstand rupturing or puncturing by electric pressure. Besides these two main properties, successful insulating materials must also be perfectly flexible and elastic, must be non-absorptive, and unaffected by heat. Unfortunately there is no material that in a very high degree possesses all these properties together, and, in selecting armature insulators, such a material is to be chosen in every case which best fulfills the particular conditions, having as its prominent property that which is most desired without being objectionable in other respects.

Mica ranks highest in disruptive strength, has a high insulating resistance, is non-absorptive and unaffected by heat, but it very easily breaks in bending, and therefore, in spite of being the most perfect armature insulator, cannot be used in places where the insulation is required to be flexible.

Paraffined materials are distinguished by their enormous insulation resistance, and have a high disruptive strength; but they cannot stand much bending, and are seriously affected by heat.

Rubber has good insulating qualities, and is extremely flexible, but is injured by temperatures above 65° Centigrade (= 150° Fahr.).

Insulating materials prepared by treating certain fabrics, such as cotton, linen, silk, and paper, with *linseed oil*, and oxidizing the oil at the proper temperature to expel any moisture, although not being of marked disruptive strength or of extremely high insulating resistance, yet make very satisfactory armature insulation, as they can be made to possess all the properties required of a perfect insulator in a practically sufficient degree. By using pure linseed oil, properly treated, and by exercising special care in preparing the surfaces, a comparatively high insulation value, both in resistance and in disruptive strength, can be obtained, while the materials are

perfectly flexible, practically non-absorptive, and affected only by temperatures far above that which entirely destroys the cotton or silk insulation on the armature wires. In using these materials, care should be taken that their surfaces are perfectly uniform, for, if the oil is not evenly distributed, the disruptive strength and the insulation resistance fall off considerably. The greatest thickness of an unevenly coated, oil-insulating material determines the number of layers of it that can be placed into a certain space, while the smallest thickness determines the insulation-value, which often runs as much as fifty per cent. below that of an evenly covered sheet of the average thickness if the surfaces were uniform. Oil insulations made of pure linseed oil are preferable to those in which the ordinary commercial oil is used, since to give the latter its oxidizing properties certain metallic oxides are employed which, although being classed as insulators, have an insulating value far below that of oil. With commercial linseed oil there is, therefore, never any certainty that some of these oxides may not be held in suspension, but it is essential for a high insulation resistance that an insulating material shall not contain any other substance having a lower insulating value than itself.

Micanite, which is made of pure India sheet mica cemented together with a cement of very high resistance, can be molded in any desired shape, or in combination with certain other materials can be rendered more or less pliable, thus combining the excellent qualities of mica with the property of flexibility, and making a most perfect armature insulating material. *Micanite cloth*, *micanite paper*, and *micanite plate* are varieties of this material. The latter is a combination of sheet mica with pure gum or solution of guttapercha, or with a special cement, the office of the gum or cement being to hold the laminæ together but to allow them to slide upon each other when the plate is bent.

Vulcanized fibre is comparatively low both in resistance and in disruptive strength, and is seriously affected by exposure to moisture.

Vulcabeston, an insulating substance composed of asbestos and rubber, is not affected seriously by high temperatures, and has the advantage that it can be molded like micanite, but

TABLE XX.—RESISTIVITY AND SPECIFIC DISRUPTIVE STRENGTH OF VARIOUS INSULATING MATERIALS.

MATERIAL.	THICKNESS USED FOR ARMATURE INSULATION.		AVERAGE RESISTIVITY AT 30° CENT.		SPECIFIC DISRUPTIVE STRENGTH.		
	inch.	mm.	Megohms per square inch-mil.	Megohms per cm.²-mm.	Limits, in Volts per mil Thickness.	Practical Average. Volts per mil.	Practical Average. Volts per mm.
Asbestos	.004–.020	.1–.5	7	1,800	100–180	125	5,000
" oiled	.008–.025	.2–.6	680 *	173,000	225–490	300	12,000
" and Muslin, oiled	.010–.030	.25–.75	850 *	216,000	330–500	375	15,000
Bristol Board	.012–.050	.3–1.25	120	31,000	150–250	175	7,000
Cotton, Single Covering (on wires)	.005–.012	.125–.3	10	2,500	260–340	275	11,000
Cotton, Single Covering, shellacked	.006–.015	.15–.4	25	6,400	340–370	350	14,000
Cotton, Single Covering, boiled in paraffine	.006–.015	.15–.4	11,800,000	3,000,000,000	380–480	400	16,000
Cotton, Double Covering	.012–.020	.3–.5	10	2,500	210–240	225	9,000
Cotton, Double Covering, shellacked	.015–.025	.4–.6	25	6,400	250–300	275	10,000
Fibre, vulcanized, red	.030–.075	.75–2.0	470	120,000	150–325	200	8,000
Hard Rubber	1/16–3	1.5–75	600	150,000	900–1,300	1,000	40,000
Leatheroid	.015–.040	.35–1.0	6	1,500	150–250	175	7,000
Linseed Oil, pure, oxidized	....	....	10,000	2,540,000	750–900	800	32,000
Mica, pure, white	.001–.125	.025–3.0	33,000	8,400,000	2,000–8,000	3,000	120,000
Micanite, Cloth	.012–.016	.3–.4	310,000 †	79,000,000	240–490	300	12,000
" " flexible	.008–.020	.2–.5	440,000 †	112,000,000	175–310	200	8,000
" Paper	.005–.012	.125–.3	490,000 ‡	124,000,000	390–510	425	17,000
" " flexible	.010–.025	.25–.6	500,000 ‡	127,000,000	280–390	300	12,000
" Plate	.010–.075	.25–.2	980,000 §	250,000,000	940–1,120	1,000	40,000
" " flexible, "A" ‖	.010–.015	.25–.4	620,000 †	158,000,000	830–1,040	900	36,000
" " " "B" ¶	.010–.020	.25–.5	320,000 **	81,000,000	575–790	600	24,000
Oiled Cloth (Cotton, Linen, or Muslin)	.005–.030	.125–.75	650 ††	165,000	450–650	500	20,000
Oiled Paper, single coat	.004–.006	.1–.15	1,350 ‡‡	340,000	550–700	600	24,000
" " double "	.006–.010	.15–.25	1,600 ‡‡	400,000	600–960	700	28,000
Paper, white writing	.003–.006	.075–.15	5	1,270	200–275	225	9,000
" yellow	.004–.008	.1–.2	3	760	190–250	200	8,000
" brown	.005–.010	.125–.25	2	510	160–200	175	7,000
Paraffined Paper	.002–.008	.05–.2	11,800,000	3,000,000,000	800–1,000	900	36,000
Parchment, oiled	.010–.020	.25–.5	180	45,700	830–950	850	34,000
Press Board	.025–.075	.6–2.0	100	25,400	100–420	150	6,000
Rubber, Sheet	.015–.060	.35–1.5	3,000,000	760,000,000	350–600	400	16,000
Shellacked Cloth	.006–.012	.15–.3	30	7,600	30–60	40	1,600
Silk, Single Covering (on wires)	.001–.0025	.025–.065	50	12,000	350–565	475	19,000
" " " shellacked.	.0015–.004	.04–.1	75	18,000	500–570	525	21,000
" Double "	.0015–.005	.04–.125	50	12,000	320–420	375	15,000
" " " shellacked.	.002–.007	.05–.175	75	18,000	420–510	450	18,000
Varnished Cheese Cloth	.006–.012	.15–.3	35	9,000	240–265	250	10,000
Vulcabeston	.040–.100	1.0–2.5	15	3,800	60–110	75	3,000
Wood, Mahogany	1/8–4	3.0–100	.06 §§	15	10–25	15	600
" Pine	1/8–4	3.0–100	.3 §§	75	5–20	10	400
" Walnut	1/8–4	3.0–100	.6 §§	150	15–40	20	800

* Insulation resistance at 50° C. is about 3/8, at 70° C. about 1/10, and at 100° C. about 1/80 of that at 30° C.
† " " " " " " 3/4, " " " 1/2, " " " 1/8 " " "
‡ " " " " " " 1/2, " " " 1/8, " " " 1/8 " " "
§ " " " " " " 7/10, " " " 3/10, " " " 1/10 " " "
‖ Mica Laminæ, put together by solution of guttapercha (Mica Insulator Co.).
¶ " " " " " patent cement (Mica Insulator Co.).
** The insulating properties of this material (one of the products of the Mica Insulator Co.) is affected but very little by temperature, its specific resistivity at 50° C. being about .9, at 70° C. about .95, and at 100° C. about .85 of the average resistivity at 30° C.
†† Resistivity at 50° C. is about 3/8, at 70° C. about 1/8, and at 100° C about 1/25 of that at 30° C.
‡‡ " " " " " 1/4, " " " 1/10, " " " 1/30 " " "
§§ Calculated from tests made by Addenbrooke, see Munroe & Jamieson's "Pocket-Book," tenth edition (1894), page 251.

both its resistivity and its specific disruptive strength are very small comparatively.

The preceding Table XX. gives the insulating properties of the various insulating materials commonly used, and is averaged from information contained in writings by Steinmetz,[1] and by Canfield and Robinson,[2] from a report by Herrick and Burke,[3] and from tests expressly made for the purpose. The values of the disruptive strength are those between parallel surfaces, and, since for the same material the break-down voltage per mil varies with the thickness—in some cases decreasing, in others increasing (according to the nature of the material), as much as 50 per cent. when varying the thickness of the sample from .005 to .025 inch—are averaged from tests with different thicknesses.

Since the insulation resistance varies considerably with temperature[4] (see notes to Table XX.), and since readings taken with identically the same samples at the same temperature but at different times showed large deviations—presumably owing to differences in moisture—the figures for the resistivity have, chiefly, a comparative value, but may with sufficient accuracy be taken as averages for the computation of the insulation resistance of armatures, commutators, etc., of dynamo-electric machines.

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[1] "Note on the Disruptive Strength of Dielectrics," paper read before the American Institute of Electrical Engineers by Charles P. Steinmetz. *Transactions A. I. E. E.*, vol. x. p. 85 (February 21, 1893); *Electrical Engineer*, vol. xv. p. 342 (April 5, 1893).

[2] "The Disruptive Strength of Insulating Materials," engineering thesis by M. C. Canfield and F. Gge. Robinson, Columbia College, *Electrical Engineer*, vol. xvii. p. 277 (March 28, 1894).

[3] "Report on Tests of Insulating Materials manufactured by The Mica Insulator Co., Schenectady, N. Y.," by Albert B. Herrick and James Burke, electrical engineers, New York, August 13, 1896.

[4] "Effect of Temperature on Insulating Materials," by Geo. F. Sever, A. Monell, and C. L. Perry, *Transactions A. I. E. E.*, vol. xiii. p. 225 (May 20, 1896); *Electrical World*, vol. xxvii. p. 642 (May 30, 1896), vol. xxviii. p. 41 (July 11, 1896); *Electrical Engineer*, vol. xxi. p. 556 (May 27, 1896).

CHAPTER V.

FINAL CALCULATION OF ARMATURE WINDING.

25. Arrangement of Armature Winding.

By "arrangement" of the armature winding is understood the grouping of the conductors into a number of armature coils, each containing a certain number of turns, or convolutions, of the armature wire, and each one corresponding to a division of the collector or commutator.

a. Number of Commutator Divisions.

The E. M. F. generated by the combination of a series of convolutions, or by a coil, while under the commutator brushes, is not constant, but fluctuates with the rate of its cutting lines of force in the different positions during that period. This fluctuation of the E. M. F. of a dynamo, consequently, increases with the angle which is embraced by each coil of the armature, and can be mathematically determined from the measure of this angle. This is extensively treated in § 9, and Table I. contained therein shows that in a 12-coil armature, in which the angle inclosed by each coil is 30°, the fluctuations of the E. M. F. amount to $\pm$ 1.7 per cent. of the maximum E. M. F. generated; that in an 18-coil armature, in which the coil-angle is 20°, they are $\pm$ ¾ per cent.; for 24 divisions, corresponding to an angle of 15°, about $\pm$ ½ of 1 per cent.; for 36 coils, embracing an angle of 10° each, $\pm \frac{3}{5}$ of 1 per cent.; for 48 divisions of 7½° each, $\pm \frac{1}{10}$ of 1 per cent.; for 90 divisions with coil-angle of 4°, $\pm \frac{3}{100}$ of 1 per cent.; and that for a 360 division commutator, finally, for which the angle inclosed by each coil is 1°, they are reduced to but $\frac{3}{1000}$ of 1 per cent.

From these figures it is apparent that the fluctuations become practically insignificant, or the potential of the machine practically steady, if, for bipolar dynamos, armature coils of an angular breadth of less than 10°, or what amounts to the same thing, if commutators with from 36 divisions

upward are used. For low potential machines—up to 300 volts—it has been found good practice to provide, per pair of armature circuits, from 40 to 60 divisions in the commutator.

For high potential dynamos the voltage itself determines the number of commutator bars. For, in these, the self-induction set up in the separate coils, and the sparking at the commutator caused by the potential of this self-induction between two adjacent commutator divisions, are more important considerations than the fluctuation of the E. M. F.

No potential below 20 volts is able to maintain an arc across even the slightest distance between two copper points. The potentials above this figure necessary to carry an arc over a certain distance depend upon the intensity of the current. In order to maintain, between two copper conductors, an arc of .040 inch length,—the usual thickness of the commutator insulation for high voltage machines,—according to actual experiments made by the author, imitating as nearly as possible the conditions of a commutator, a current of

100	amperes	takes	20	volts
50	"	"	21	"
20	"	"	23	"
10	"	"	25	"
5	"	"	30	"
2	"	"	40	"
1	"	"	50	"

From this it can be concluded that, in order to prevent the commutator of a high voltage machine from becoming unnecessarily expensive, allowances have to be made as follows:

TABLE XXI.—DIFFERENCES OF POTENTIAL BETWEEN COMMUTATOR DIVISIONS.

CURRENT INTENSITY PER ARMATURE CIRCUIT.	DIFFERENCE OF POTENTIAL BETWEEN COMMUTATOR DIVISIONS.
Over 100 amperes	10 to 20 volts
100 to 50 "	12 " 21 "
50 " 20 "	15 " 23 "
20 " 10 "	20 " 25 "
10 " 5 "	25 " 30 "
5 " 2 "	30 " 40 "
2 " 1 "	35 " 50 "

The respective minimum numbers of commutator divisions, consequently, are:

$$\left.\begin{array}{llll}
\text{For over 100 A. p. circuit:} & & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{20} = \dfrac{E \times n'_p}{10} \\
\text{“ 100 to 50 A.} & \text{“} & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{21} = \dfrac{E \times n'_p}{10.5} \\
\text{“ 50 to 20 A.} & \text{“} & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{23} = \dfrac{E \times n'_p}{11.5} \\
\text{“ 20 to 10 A.} & \text{“} & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{25} = \dfrac{E \times n'_p}{12.5} \\
\text{“ 10 to 5 A.} & \text{“} & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{30} = \dfrac{E \times n'_p}{15} \\
\text{“ 5 to 2 A.} & \text{“} & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{40} = \dfrac{E \times n'_p}{20} \\
\text{“ 2 to 1 A.} & \text{“} & (n_c)_{\min} = \dfrac{E \times 2\,n'_p}{50} = \dfrac{E \times n'_p}{25}
\end{array}\right\} \quad \textbf{(45)}$$

Having thus determined the minimum number of divisions that can be used in the commutator without excessive sparking, the actual number, n_δ, to be employed has to be chosen by comparing this value of $(n_c)_{\min}$ with the total number of conductors on the armature, found by multiplying the rounded result of equation (35), (37) or (38), respectively, with that of formula (39), and dividing the product by the number, n_δ, of armature wires stranded in parallel.

b. Number of Convolutions per Commutator Division.

The number of turns, n_a, of armature conductors per commutator division, or the number of convolutions in each armature coil, is then readily obtained by dividing the total number of armature convolutions by the number of coils, n_c. The number of armature convolutions, in ring armatures, is identical with the number of armature conductors, while in drum armatures it takes two conductors to make one turn, and, therefore, the number of turns is but one-half the number of conductors. Hence we have for *ring* armatures:

$$n_a = \frac{n_w \times n_1}{n_c \times n_\delta}; \quad \ldots\ldots\ldots\ldots\ldots \textbf{(46)}$$

and for *drum* armatures or *drum-wound ring* armatures:

$$n_a = \frac{n_w \times n_1}{2 \times n_c \times n_\delta}. \quad \ldots\ldots\ldots\ldots \textbf{(47)}$$

c. Number of Armature Divisions.

If the armature is to have spacing strips, or driving horns, the number of the armature divisions for this purpose depends upon the number of armature coils, n_c, the number of turns per armature coil, n_a, and the number of conductors in parallel, n_δ.

In ordinary machines the number of armature divisions is usually made equal to the number of coils, n_c, and sometimes —especially in drum armatures—double the number of coils, $2n_c$, is taken. For high current output machines often a greater number of armature divisions than that given by the number of coils is chosen. In such a case the total number of single wires, or cables, $n_c \times n_a \times n_\delta$, is to be suitably arranged in groups. The number of these groups is to be a multiple of the number of coils, n_c, and since the number of turns per coil, n_a, in high current dynamos, is usually $= 1$, the problem of grouping, in this case, amounts to subdividing the number of parallel wires, n_δ.

26. Radial Depth of Armature Core. Density of Magnetic Lines in Armature Body.

Diameter and length of the armature core being determined, its proper radial depth can be readily found by the cross-section to be provided for the passage of the magnetic lines of force.

The density of lines permitted per unit area of armature cross-section is limited by the heating of the armature due to hysteresis and eddy current losses. The heat, generated by either of these causes, increases with the density of the lines, and with the number of magnetic reversals per second. The latter number is the product of the number of revolutions per second and the number of magnet poles, and therefore it is obvious that, in order to keep the temperature increase of the armature in its practical limits, in dynamos where this product is great, larger specific sectional areas of the armature core are to be allowed than in machines having a small number of magnetic reversals, or a low "frequency," as half the number of reversals, or the number of complete magnetic "cycles," is called. The former—high frequency—is the case for high

speed and multipolar dynamos, the latter—low frequency—for low speed and bipolar ones. On the other hand, large multipolar machines generally have well-ventilated ring armatures, and in these an even considerably larger amount of heat generated will produce a smaller temperature increase than in drum armatures of equal output.

TABLE XXII.—CORE-DENSITIES FOR VARIOUS KINDS OF ARMATURES.

Species of Dynamo.	Type of Machine.		Kind of Armature.	Flux Density in Minimum Cross-Section of Armature. Lines per sq. inch. $\mathfrak{B}''_a$	Lines per cm.² $\mathfrak{B}_a$
Incandescent Dynamos; Railway Generators; Machines for Power-Transmission and Distribution; Stationary and Railway Motors.	Bi-polar	High Speed	Drum	50,000 to 70,000	8,000 to 11,000
			Ring	60,000 " 80,000	9,000 " 12,500
		Low Speed	Drum	60,000 " 80,000	9,000 " 12,500
			Ring	70,000 " 100,000	11,000 " 15,500
	Multi-polar	High Speed	Drum	35,000 " 50,000	5,500 " 8,000
			Ring	50,000 " 70,000	8,000 " 11,000
		Low Speed	Drum	40,000 " 60,000	6,000 " 9,000
			Ring	60,000 " 80,000	9,000 " 12,500
Series Arc Lighting Dynamos.	Bi-polar	High Speed	Ring	110,000 " 130,000	17,000 " 20,000
	Multi-polar	High Speed	Ring	100,000 " 120,000	15,500 " 18,500
Electroplating and Metallurgical Dynamos.	Bi-polar	High Speed	Drum	40,000 " 60,000	6,000 " 9,000
			Ring	50,000 " 70,000	8,000 " 11,000
	Multi-polar	High Speed	Ring	35,000 " 50,000	5,500 " 8,000
		Low Speed	Ring	40,000 " 60,000	6,000 " 9,000
Accumulator Charging Dynamos; Battery Motors.	Bi-polar	High Speed	Drum	35,000 " 50,000	5,500 " 8,000
			Ring	40,000 " 60,000	6,000 " 9,000
	Multi-polar	High Speed	Drum	30,000 " 45,000	4,500 " 7,000
			Ring	35,000 " 50,000	5,500 " 8,000

With dynamos for special purposes still other points have to be considered: In arc lighting dynamos, in order to keep

the magnetic flux constant at varying load, it is necessary to make the magnetic circuit insensitive to considerable changes in exciting power, and this is achieved by working the entire circuit at a very high saturation; on the other hand, in machines used exclusively for charging accumulators, the saturation of the circuit should be very low, for then, during charging, when the counter E. M. F. of the cells gradually rises, the voltage of the charging dynamo also rises automatically instead of remaining nearly constant, as it would do if the magnetism were incapable of further increase. Again, in dynamos for electroplating, electrotyping, electrolytic precipitation of metals and for electro-smelting, and in motors driven by accumulators or primary batteries, on account of the very low terminal-voltage, the field density in the gaps must be kept low (compare § 18), and therefore a low saturation through the entire circuit is required.

According to these considerations, the values given in Table XXII., page 91, for the flux-densities in the radial cross-section of the armature core are recommended (see § 91).

The cross-section of the magnetic circuit in the armature core is the product of the net length and the net radial depth of the core, and of the number of poles; and this product, divided into the total magnetic flux through the armatures, gives the density of lines per unit area. In order to obtain the net radial depth, or breadth of the cross-section of any armature core, therefore, the total armature flux is to be divided by the product of the armature density, of the net length of the body, and of the number of poles:

$$b_a = \frac{\Phi}{2\, n_p \times \mathfrak{B}''_a \times l_a \times k_2} \quad \ldots\ldots\ldots\ldots (48)$$

The symbols used in this formula denote:

b_a = radial depth, or breadth of cross-section of armature core, in inches;

Φ = useful flux, in maxwells, or number of useful lines of force cutting armature conductors, the calculation of which is the subject of § 56.

$\mathfrak{B}''_a$ = flux density in minimum area of armature core, in lines per square inch, given in Table XXII.;

n_p = number of pairs of magnet poles.

l_a = length of armature core, in inches, from formula (40);
k_2 = ratio of net iron section to total cross-section of armature core, see Table XXIII.

For calculation in metric measure $\mathfrak{B}''_a$ is to be replaced by $\mathfrak{B}_a$ (Table XXII.) and l_a to be expressed in centimetres; formula (48), then, will furnish b_a in centimetres.

The constant k_2 depends upon the material, and the manner of building up, of the armature core. In order to prevent excessive losses and resulting heating of the armature due to eddy currents in the iron, it is necessary to laminate the body perpendicular to the direction of the active armature conductors. In case the active pole faces embrace either the outer, or the inner, or both circumferences of the armature, the active conductors are those parallel to the shaft, and the lamination of the core is to be effected perpendicularly to the shaft; while in case of the poles being at the sides of the armature (flat ring type), the active conductors run perpendicular to the shaft, and the lamination is to take place parallel to the shaft. In case of the polepieces embracing three sides of the armature section, finally, the active conductors are partly parallel and partly perpendicular to the shaft, and the lamination, in consequence, is also to be carried out in both directions. The materials for effecting these various laminations are iron *discs*, iron *ribbon*, and iron *wire*, respectively. The insulation of these laminæ, in the majority of machines, has been, and is yet, effected by inserting sheets of thin paper, asbestos, etc., between them, although it has been repeatedly shown by practical experiments [1] that such an insulation is not only entirely unnecessary, but, on the contrary, even disadvantageous. For, in order not to lose too much of the available sectional area of the body, the lamination in such armature is usually made rather coarse, but it is just the fineness of the lamination, and not the thickness of its insulation, that avoids in a higher degree the generation of eddy currents. The oxide coating created by heating the iron is a very effective and suitable insulation of the armature core laminæ.

[1] Ernst Schulz, *Elektrotechn. Zeitschr.*, December 29, 1893. See "Iron in Armatures," *Electrical World*, vol. xxiii. p. 91 (January 20, 1894), and p. 248 (February 24, 1894).

In the following Table XXIII. values of the ratio k_2 of the net to the total core section are given for various thicknesses of the iron, and for the different modes of insulation now in use:

TABLE XXIII.—RATIO OF NET IRON SECTION TO TOTAL CROSS SECTION OF ARMATURE CORE.

MATERIAL OF ARMATURE CORE.	THICKNESS OF IRON.		INSULATION BETWEEN LAMINÆ.	VALUE OF RATIO k_2.
	inch.	mm.		
Sheet Iron (Discs or Ribbon).	.080	2	Paper or Asbestos	0.95 to 0.90
	.040	1	" " "	.92 " .88
	.020	0.5	" " Enamel	.90 " .85
	.010	0.25	" " "	.85 " .80
	.010	0.25	Oxide Coating	.95 " .90
Square Wire.	.080	2	Cotton Covering	.90 " .80
	.040	1	" "	.85 " .75
	.020	0.5	Enamel or Varnish	.80 " .70
	.020	0 5	Oxide Coating	.90 " .85
Round Wire.	.080	2	Cotton Covering	.80 " .70
	.040	1	" "	.75 " .65
	.020	0.5	Enamel or Varnish	.70 " .60
	.020	0.5	Oxide Coating	.85 " .80

In large ring armatures, for the sake of ventilation, air spaces are provided in modern machines by means of light brass frames inserted in certain intervals between the core discs. In calculating the radial depth, b_a, of the body, the sum of these distance-pieces is to be deducted from the total length, l_a, of the core, the reduced length so found being used instead of l_a in formula (48).

27. Total Length of Armature Conductor.

The amount of inactive, or "dead," wire required to join the active portions of the armature conductors into continuous turns depends upon the shape of the armature, the height of the winding space, and the manner of winding. In an armature, for instance, having a core section of great length parallel to the pole faces, and but a small thickness perpendicular to

the same, this inactive addition to the generating wire will be comparatively small, while in short armatures with great core depth the proportion of the dead to the active length will be considerably greater. Furthermore, if two armatures of same length, same core diameter, and same radial depth have equal lengths of active conductor, but are wound with different heights of the winding space,—as will be the case if the one has a smooth core with winding covering the entire circumference and the other a toothed body,—then the external diameters, and consequently the lengths necessary to join the active conductors, are greater in the latter case, and therefore it is evident that the armature with the higher winding space requires a greater total length of armature conductor. If, finally, two otherwise entirely equivalent armatures are wound by different systems, a considerable difference may be found in the total lengths of wire required to produce equal lengths of active conductor.

a. Drum Armatures.

The *active* length of the armature conductor being known, the simplest method of expressing its *total* length, for a drum armature, is

$$L_t = k_3 \times L_a, \quad \ldots\ldots\ldots\ldots\ldots (49)$$

where L_t = total length of armature conductor (wires in parallel considered as one conductor);

L_a = active length of armature conductor, from formula (26);

k_3 = constant, depending upon shape of armature and system of winding. See Table XXIV.

The ratio, k_3, of the total to the active length of the armature conductor, in a drum armature, depends chiefly upon the ratio of length to diameter of the armature core. In modern machines the lengths of drum armatures usually vary between one and two diameters; in special designs, however, a value as low as 0.75 or as high as 2.5 may be taken for this ratio. The following Table XXIV. gives average values of the constant k_3 for various shape-ratios for smooth as well as for toothed drum armatures:

TABLE XXIV.—RATIO BETWEEN TOTAL AND ACTIVE LENGTH OF WIRE ON DRUM ARMATURES.

Ratio of Length to Diameter of Armature Core. $l_a : d_a$	Value of k_3.	
	Smooth Armature.	Toothed Armature.
0.75	2.50	3.10
0.8	2.45	3.05
0.9	2.40	3.00
1.0	2.35	2.95
1.1	2.30	2.90
1.2	2.25	2.85
1.3	2.20	2.80
1.4	2.15	2.75
1.5	2.10	2.70
1.6	2.05	2.65
1.7	2.00	2.60
1.8	1.95	2.55
1.9	1.90	2.50
2.0	1.85	2.45
2.25	1.75	2.35
2.5	1.70	2.25

Another form often used for expressing the total length of wire on a drum armature is indicated by the formula:

$$L_t = N_c \times (l_a + k_4 \times d_a) = L_a \times \left(1 + k_4 \times \frac{d_a}{l_a}\right); \quad (50)$$

N_c = total number of conductors all around armature circumference;

k_4 = constant, depending upon system of winding.

This formula (50) has the advantage over (49) that no special table is required for its constant, since for a certain type of armature, and a certain system of winding, k_4 is very nearly constant for all sizes and shapes. The value of k_4 for the smooth drum armatures considered in Table XXIV. lies between 1.3 and 1.7, and as an average 1.5 may be taken, thus making the formula for the total length of conductor for *smooth drum* armatures:

$$L_t = L_a \times \left(1 + 1.5 \times \frac{d_a}{l_a}\right). \quad \ldots\ldots(51)$$

Formula (51), then, means that the additional length of dead wire to every conductor of a smooth drum armature is

one and one-half times its core diameter, or that the total length of one conductor in a smooth drum is equal to the length of the body plus one and a half times the core diameter. The reason why k_4, even for the same type and same winding method, has not the same value for all sizes, is that the proportion of the core diameter to the thickness of the armature shaft is very much different for different sizes. In a small drum, for instance, the shaft takes up considerably much more room than in a large one, and therefore the dead lengths are comparatively larger in a small machine. In fact $k_4 = 1.5$ gives too high values for small ratios of length to diameter, which occur in large drums, while the values found by (51) for large ratios of length to diameter, which are met in small armatures, are below those given in Table XXIV.

For $l_a : d_a = 1$, or $d_a : l_a = 1$, for instance, we obtain, by comparison of (49) with (50) and (51):

$$k_3 = \frac{L_t}{L_a} = 1 + k_4 \times \frac{d_a}{l_a} = 1 + 1.5 \times 1 = 2.5,$$

while Table XXIV., which is averaged from actual practice, gives $k_3 = 2.35$ for this ratio, to which number would correspond the small value of

$$k_4 = \frac{k_3 - 1}{\left(\frac{d_a}{l_a}\right)} = \frac{2.35 - 1}{1} = 1.35.$$

On the other hand, if $l_a : d_a = 2$, or $d_a : l_a = 0.5$, we get:

$$k_3 = 1 + 1.5 \times 0.5 = 1.75;$$

the table-value for k_3 is 1.85 for this shape-ratio, and therefore the high value of

$$k_4 = \frac{1.85 - 1}{.5} = 1.70$$

would answer in this case.

For toothed-drum armatures the numerical value of k_4, in the practical limits of the ratio of shape, lies between 1.9 and 3, the average being about 2.5; hence formula (50) for *toothed-drum* armatures becomes:

$$L_t = L_a \times \left(1 + 2.5 \times \frac{d_a}{l_a}\right); \quad \ldots\ldots\ldots(52)$$

that is to say, the average length to be added to each active conductor in a toothed-drum armature is two and one-half times the core diameter of the drum, or the average dead length in each turn (consisting of two active conductors) is five times the diameter at the bottom of the slots.

b. Ring Armatures.

In a helically or spirally wound ring armature of a core section of given dimensions, the ratio of the total to the active length of the conductor depends upon the arrangement of the

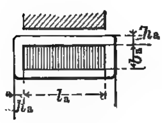
Fig. 59.

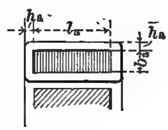
Fig. 60.

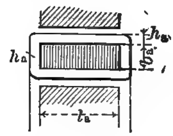
Fig. 61.

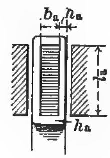
Fig. 62.

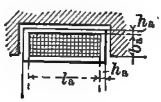
Fig. 63.

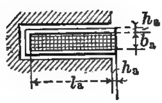
Fig. 64.

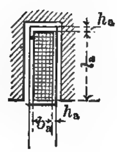
Fig. 65.

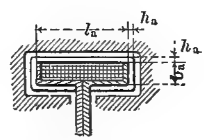
Fig. 66.

Figs. 59 to 66.—Various Arrangements of Field-Magnet Frame around Ring Armature.

field-magnet frame. There are, altogether, eight different possibilities of arranging the poles around a ring armature; these eight methods, however, can be classified into but five principally different cases, viz.:

Case I. Polepieces facing *one* long armature surface, see Figs. 59 and 60;

" II. " " *two* long armature surfaces, see Figs. 61 and 62;

" III. " " *one* long and *two* short armature surfaces, see Fig. 63;

" IV. " " *two* long and *one* short armature surface, see Figs. 64 and 65;

" V " " *one* long and *two* short, and *part* of second long surface, see Fig. 66.

Denoting with l_a the length, and with b_a the breadth of the cross-section, and assuming the winding-height, h_a, to be the same all around the section, we obtain the following formulæ for calculating the total length of conductor on a ring armature:

$$\text{Case I.:}\quad L_t = \frac{2\,(l_a + b_a) + h_a\,\pi}{l_a} \times L_a \quad \ldots\ldots\ldots(53)$$

$$\text{" II.:}\quad L_t = \frac{l_a + b_a + h_a\,\frac{\pi}{2}}{l_a} \times L_a \quad \ldots\ldots\ldots(54)$$

$$\text{" III.:}\quad L_t = \frac{2\,(l_a + b_a) + h_a\,\pi}{l_a + 2\,b_a + h_a\,\frac{\pi}{2}} \times L_a \quad \ldots\ldots\ldots(55)$$

$$\text{" IV.:}\quad L_t = \frac{2\,(l_a + b_a) + h_a\,\pi}{2\,l_a + b_a + h_a\,\frac{\pi}{2}} \times L_a \quad \ldots\ldots\ldots(56)$$

In these formulæ l_a, b_a, and L_a are known by virtue of equations (40), (48) and (26), respectively, and h_a can be taken from Table XVIII., if the actual winding depth is not already known by having previously determined the winding and its arrangement.

A formula for Case V. is not given, because, in the first place, the arrangement shown in Fig. 66 is not at all practical, and the makers who first introduced the same have long since discarded it, and, second, because the distance of the internal pole projections depends upon the construction and manner of supporting of the armature core, and, consequently, cannot be definitely expressed.

c. Drum-Wound Ring Armatures.

In modern ring armatures of the types indicated by Figs. 59 and 60, the conductors facing two adjacent poles of opposite polarity are often connected in the fashion of a bipolar drum, by completing their turns across the end surfaces of the armature body, thus converting the multipolar ring armature into the combination of as many bipolar drum armatures as there are pairs of poles in the field frame; see § 43. By this arrangement, which is illustrated in Fig. 67, not only a con-

siderable saving of dead wire is experienced, but also the exchanging of conductors in case of repair is rendered much more convenient, especially when *formed coils* are used, which is the almost universal practice now.

The total length of the armature conductor can, in this case, be calculated by applying, for both smooth and toothed bodies, the above formula (51), replacing in the same the core diameter, d_a, by the chordal distance of two neighboring poles, measured from centre to centre along the circumference of the armature over the winding. The formula for the total length of conductor on a drum-wound ring armature, therefore, is (see Fig. 67, page 101):

$$L_t = \left(1 + 1.5 \times \frac{d_a'' \times \sin \frac{180°}{2\,n_p}}{l_a}\right) \times L_a \quad \dots\dots\dots\dots (57)$$

Inserting in this formula the numerical value for the size of half the pole angle, we obtain the following set of formulæ for the various pole numbers that may be used in practice:

TABLE XXV.—TOTAL LENGTH OF CONDUCTOR ON DRUM WOUND RING ARMATURES.

NUMBER OF POLES. $2n_p$	HALF POLE-ANGLE. $\frac{180°}{2n_p}$	LENGTH OF POLE-CHORD (DIAMETER = 1) $\sin\frac{180°}{2n_p}$	TOTAL LENGTH OF ARMATURE CONDUCTOR (FORMULA 57). $L_t = \left\{1 + \left(1.5 \times \sin\frac{180}{2n_p}\right) \times \frac{d''_a}{l_a}\right\} \times L_a$
4	45°	0.707	$L_t = \left(1 + 1.161 \times \frac{d''_a}{l_a}\right) \times L_a$
6	30	.500	0.750
8	$22\frac{1}{2}$	.383	.574
10	18	.309	.464
12	15	.259	.388
14	$12\frac{6}{7}$	.222	.333
16	$11\frac{1}{4}$	.195	.293
18	10	.174	.261
20	9	.156	.235
24	$7\frac{1}{2}$	.131	.196
30	6	.105	.157

28. Weight of Armature Winding.

A copper wire of 1 circular mil

$$\left(= \frac{1}{1{,}000{,}000} \times \frac{\pi}{4} \text{ square inch}\right)$$

area weighs .00000303 pound per foot of length; our armature conductor of δ_a^2 circular mils sectional area, therefore, has a weight per foot of .00000303 $\times$ δ_a^2 pound. And the total L_t feet of it, used in winding the armature, will weigh:

$$wt_a \doteq .00000303 \times \delta_a^2 \times L_t \,; \quad \ldots\ldots(58)$$

wt_a = weight of bare armature winding, in pounds;
δ_a^2 = sectional area of armature conductor, in circular mils, from formula (27);
L_t = total length of armature conductor, in feet, formulæ (49) to (57), respectively.

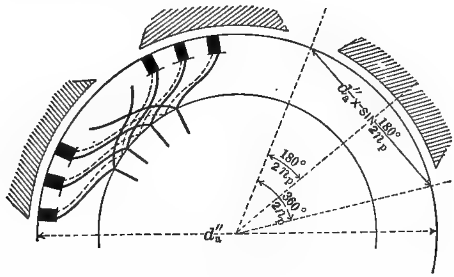

Fig. 67.—Face Connections of Drum-Wound Ring Armature.

In case of round gauge wires, the product .00000303 $\times$ δ_a^2 is contained in the gauge table under the heading "lbs. per foot," and consequently the bare weight of the winding is found by simply multiplying the respective table-value by the total length, L_t, and, eventually, by the number of wires, n_δ stranded in parallel.

If, in case of heavy rectangular or trapezoidal armature bars, the cross-section, δ_a^2, is given in square inches, the numerical constant in the above formula (58) should be replaced by 3.858, this being the weight per foot of a copper bar of 1 square inch sectional area.

When the length of the wire is given in metres and its sectional area in square millimetres, formula (58) will give the weight of the armature-winding in kilogrammes, if the factor .0089 is used as the numerical constant, 8.9 being the specific gravity of copper, and .0089, therefore, the weight in kilogrammes of one metre of copper wire having a cross-section of one square millimetre area.

When standard gauge wire is to be employed in winding the armature, it is desirable to know the weight of the winding, including its covering, particularly in the case when insulated wire, such as is obtainable from wire manufacturers, is to be used. This covered weight of the winding can be expressed as a multiple of the bare weight, by the equation:

$$wt'_a = k_6 \times wt_a \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(59)}$$

in which k_6 is a constant depending upon the ratio of the bare diameter of the wire to the thickness of its insulation. In

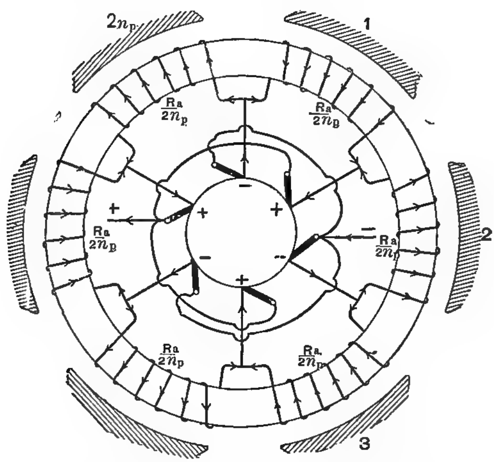

Fig. 68.—Armature-Circuits of Multipolar Dynamo.

Table XXVI., page 103, these ratios and the corresponding values of k_6 are given for all standard gauge wires likely to be used for winding armatures, for single and for double cotton covering.

29. Armature Resistance.

The electrical resistance of the armature winding can be determined by the total length of wire wound on the armature, and by the sectional area of the conductor. If R_a denotes the total resistance of the armature wire, all in one continuous length, and if there are n'_p bifurcations in the armature, and, therefore, $2\,n'_p$ electrically parallel armature portions, then the

armature forms the combination of $2\,n'_p$ parallel branches of

$$\frac{R_a}{2\,n'_p}\text{ ohms}$$

resistance each.

TABLE XXVI.—WEIGHT OF INSULATION ON ROUND COPPER WIRE.

Gauge of Wire.		Diameter of Wire (bare).		Single Cotton Insulation.				Double Cotton Insulation.			
B. W. G.	B. & S.	inch	mm	Thickness of insulation. Inch.	Ratio of bare diameter to thickness of insulation.	Weight of insulation per 100 lbs. of covered wire.	Weight of covered wire per lb. of bare wire, k_5.	Thickness of insulation. Inch.	Ratio of bare diameter to thickness of insulation.	Weight of insulation per 100 lbs. of covered wire.	Weight of covered wire per lb. of bare wire, k_5.
1	..	.300	7.62	..	..	..	..	.020	15	2.28	1.0228
..	1	.289	7.34	..	..	..	..	.020	14.45	2.32	1.0232
2	..	.284	7.21	..	..	..	..	.020	14.2	2.33	1 0233
[illegible]	..	.259	6.58	..	..	..	..	.020	12.95	2.40	1.024
..	2	.258	6.55	..	..	..	..	.020	12.9	2.40	1.024
4	..	.238	6.04	..	..	..	..	.020	11.9	2.50	1.025
..	3	.229	5.82	..	..	..	..	.020	11.45	2.55	1.0255
5	..	.220	5.59	..	..	..	..	.020	11	2.65	1.0265
..	4	.204	5.18	.012	17	2.20	1.022	.020	10.2	2.85	1.0285
6	..	.203	5.16	.012	16.9	2 20	1.022	.020	10.15	2.86	1.0286
..	5	.182	4.62	.012	15.15	2.27	1 0227	.018	10.1	2.87	1.0287
7	..	.180	4.57	.012	15	2.28	1.0228	.018	10	2.90	1.029
8	.	.165	4.19	.012	13.75	2.33	1.0233	.018	9.17	3.20	1.032
..	6	.162	4.12	.010	16.2	2.24	1.0224	.018	[illegible]	3.25	1.0325
9	..	.148	3.76	.010	14.8	2.30	1.023	.016	9.25	3.15	1.0315
..	7	.144	3 66	.010	14.4	2 32	1.0232	.016	[illegible]	3.25	1.0325
10	..	.134	3.40	.010	13.4	2.36	1 0236	.016	8.4	3.55	1.0355
..	8	.1285	3.27	.010	12.85	2.40	1.024	.016	8	3.75	1.0375
11	..	.120	3.05	.010	12	2.50	1.025	.016	7.5	4.10	1 041
..	9	.1144	2.91	.010	11.4	2.55	1.0255	.016	7.1	4.35	1 0435
12	..	.109	2.77	.010	10.9	2.66	1.0266	.016	6.8	4.60	1.046
..	10	.102	2.59	.010	10.2	2 85	1.0285	.016	6.4	5.00	1.05
13	..	.095	2.41	.010	9.5	3 10	1.031	.016	5.9	5.55	1.0555
..	11	.091	2.31	.010	9.1	3.25	1.0325	.016	5.7	5.85	1.0585
14	..	.083	2.11	.007	12	2.50	1.025	.016	5.2	6.60	1.066
..	12	.081	2.06	.007	11.6	2.54	1.0254	.016	5.1	6.80	1.068
15	13	.072	1.83	.007	10.3	2.80	1.028	.016	4.5	7.80	1.078
16	..	.065	1.65	.007	9.3	3.15	1.0315	.016	4.1	8.60	1.086
..	14	.064	1.63	.007	9 1	3.25	1.0325	.016	4	8.80	1.088
17	..	.058	1.47	.007	8.3	3.60	1.036	.014	4 1	8.60	1.086
..	15	.057	1.45	.007	8.1	3.70	1 037	.014	4 1	8.60	1.086
..	16	.051	1.30	.007	7.3	4.20	1.042	014	3.6	9.60	1.096
18	..	.049	1.25	.007	7	4.40	1.044	.014	3.5	9.80	1.098
..	17	.045	1.15	.005	[illegible]	3.25	1.0325	.012	3.75	9.30	1.093
19	..	.042	1.07	.005	8.4	3.55	1.0355	.012	3 5	9.80	1.098
..	18	.040	1 02	.005	8	3.75	1.0375	.012	3 33	10.10	1.101
..	19	.036	0.91	.005	7.2	4.30	1.043	.005*	7.2	5 60	1.056
20	..	.035	0 89	.005	7	4.40	1.044	.005*	7	6.00	1.06
21	20	.032	0 81	.005	6.4	5.00	1.05	.005*	6.4	6.60	1.066
22	21	.028	0.71	.005	5.6	6.00	1 06	.004*	7	6.00	1.06
23	22	.025	0.64	.005	5	7.00	1.07	.004*	6.25	7.00	1.07
24	23	.022	0.56	.005	4.4	8.00	1.08	.004*	5.5	8.00	1.08
25	24	.020	0.51	.005	4	8.80	1.088	.004*	5	8.80	1.088
26	25	.018	0.46	.005	3.6	9.60	1.096	.004*	4.5	9.60	1.096
27	26	.016	0.41	.005	3.2	10.40	1.104	.004*	4	10.40	1.104
28	27	.014	0.36	.005	2.8	11.25	1.1125	.004*	3.5	11.25	1.1125
29	28	.013	0.33	.005	2 6	11.65	1.1165	.004*	3.25	11.65	1.1165
30	..	.012	0.31	.005	2.4	12.05	1.1205	.004*	3	12.05	1.1205
..	29	.011	0.28	.005	2.2	12.45	1.1245	.004*	2.75	12 45	1.1245

* Double silk: 1 mil of silk insulation taken equal in weight to 1.25 mil of cotton covering.

In case of a multipolar dynamo with parallel grouping the number of parallel armature branches, $2\,n'_p$, is equal to the number of poles $2\,n_p$, and the resistance of each branch becomes

$$\frac{R_a}{2\,n'_p} = \frac{R_a}{2\,n_p},$$

see Fig. 68, page 102.

The joint resistance of these $2\,n'_p$ circuits, that is, the actual armature resistance, will consequently be

$$r_a = \frac{R_a}{(2\,n'_p)^2} = \frac{R_a}{4 \times (n'_p)^2}.$$

The total resistance, R_a, of all the armature wire in series can be calculated from the total length, L_t, and the sectional area, δ_a^2, of the conductor by the formula

$$R_a = L_t \times \frac{10.5}{\delta_a^2},$$

where 10.5 is the resistance, in ohms, at 15.5° C. (= 60° Fahr.) of a copper wire of 1 circular mil sectional area and 1 foot length, and of a conductivity of about 98 per cent. of that of pure copper. The quotient

$$\frac{10.5}{\delta_a^2},$$

for commercial copper, or

$$\frac{10.32}{\delta_a^2},$$

for chemically pure copper, represents the resistance per foot of the armature conductor, and can, for every standard size of wire, be taken from the wire gauge table.

Introducing the value of R_a into the above equation, we obtain the following formula for the resistance at 15.5° C. (= 60° Fahr.) of any armature having a single conductor:

$$r_a = \frac{1}{4 \times (n'_p)^2} \times L_t \times \left(\frac{10.5}{\delta_a^2}\right). \quad \ldots\ldots\ldots \textbf{(60)}$$

r_a = resistance of armature winding at 15.5° C., in ohms;
n'_p = number of bifurcations of current in armature; for special values of n'_p in the usual cases see symbols of formula (24), § 16;

L_t = total length of armature conductor, in feet, formulæ (49) to (57), respectively;

δ_a^2 = sectional area of armature conductor, in circular mils, formula (27).

In an armature, each conductor of which consists of n_δ parallel strands of wire of $\delta_{a_1}^2$ circular mils sectional area, there are $2\,n'_p$ parallel circuits of n_δ wires each, or altogether $2 \times n'_p \times n_\delta$ parallel circuits of

$$\frac{R_a}{2 \times n'_p \times n_\delta} \text{ ohms}$$

resistance each; the joint resistance, therefore, is

$$r_a = \frac{R_a}{4 \times (n'_p)^2 \times n_\delta^2},$$

and since in this case the total resistance of the whole armature wire in series is

$$R_a = L_t \times n_\delta \times \left(\frac{10.5}{\delta_{a_1}^2}\right),$$

we obtain for the resistance at 15.5° C. (= 60° Fahr.) of any armature winding consisting of n_δ strands of wire of $\delta_{a_1}^2$ circular mils sectional area, the general formula

$$r_a = \frac{1}{4 \times (n'_p)^2 \times n_\delta^2} \times L_t \times n_\delta \times \left(\frac{10.5}{\delta_{a_1}^2}\right)$$

$$= \frac{1}{4 \times n_\delta \times (n'_p)^2} \times L_t \times \left(\frac{10.5}{\delta_{a_1}^2}\right). \quad \ldots\ldots(61)$$

The resistance of a commercial copper wire of 1 metre length and 1 square millimetre sectional area being .017 ohm at 15.5° C., the formula for the armature resistance at 15.5° C., when dimensions are given in metric units, becomes:

$$r_a = \frac{1}{4 \times n'^2_p \times n_\delta} \times L_t \times \left(\frac{.017}{(\delta_{a_1})_{mm}^2}\right), \quad \ldots(62)$$

where L_t = total length of armature conductor, in metres;

n'_p = number of parallel armature branches;

n_δ = number of armature conductors wound in parallel;

$(\delta_{a_1})_{mm^2}$ = area of single wire, in square millimetres, $n_\delta \times (\delta_{a_1})_{mm^2}$ being equal to $(\delta_a)_{mm^2}$, as obtained from formula (28).

In order to obtain the armature resistance at any other temperature, higher than 15.5° C., add 1 per cent. for every $2\frac{1}{2}°$ centigrade over 15.5°. The resistance r_a, at 15.5° C., being known, the armature resistance at $\theta°$ Centigrade consequently will be

$$(r'_a)_{\theta°\,C.} = r_a + \frac{1}{100} \times r_a \left(\frac{\theta°\,C. - 15.5°}{2.5}\right)$$

$$= r_a \times \left(1 + \frac{\theta°\,C. - 15.5°}{250}\right). \qquad \textbf{(63)}$$

If the temperatures are measured by the Fahrenheit scale, 1 per cent. is to be added to the resistance for every $4\frac{1}{2}°$ over 60° Fahr., and the formula becomes:

$$(r'_a)_{\theta°\,F.} = r_a + \frac{1}{100} \times r_a \left(\frac{\theta°\,F. - 60°}{4.5}\right)$$

$$= r_a \left(1 + \frac{\theta°\,F. - 60°}{450}\right) \qquad \textbf{(64)}$$

In both (63) and (64), r_a is the resistance at 15.5° C. (= 60° Fahr.) found from formula (61) or (62), respectively.

CHAPTER VI.

ENERGY LOSSES IN ARMATURE. RISE OF ARMATURE-TEMPERATURE.

30. Total Energy Loss in Armature.

There are three sources of energy-dissipation in the armature which cause a portion of the energy generated to be wasted, and which give rise to injurious heating of the armature. These sources are (1) overcoming of electrical resistance of armature winding, (2) overcoming of magnetic resistance of iron, and (3) generation of electric currents in the armature core. The energy spent for the first cause, that is, the energy spent by the current in overcoming the ohmic resistance of the conductors, is often called the *C^2R loss* ($C =$ current, $R =$ resistance), for reasons evident from § 31. The energy consumed from the second cause, or spent in continually reversing the magnetism of the iron core, as the armature revolves in the field, is called the *hysteresis loss* (see § 32), and the energy spent from the third cause, in setting up useless currents in the iron and, in a small degree, also in the armature conductors, is styled the *eddy current loss*, or *Foucault current loss* (see § 33).

The total energy transformed into heat in the armature of a dynamo-electric machine is the sum of the C^2R loss, of the hysteresis loss, and of the eddy current loss, and can be expressed by the formula:

$$P_A = P_a + P_h + P_e, \quad \ldots\ldots\ldots\ldots \textbf{(65)}$$

in which $P_A =$ total watts absorbed in armature;
$P_a =$ watts consumed by armature winding, formula (68);
$P_h =$ watts consumed by hysteresis, formula (73);
$P_e =$ watts consumed by eddy currents, formula (75).

31. Energy Dissipated in Armature Winding.

The energy required to pass an electric current through any resistance is given, in watts, by the product of the square of the current intensity, in amperes, into the resistance, in ohms. The energy absorbed by the armature winding, therefore, is:

$$P_a = (I')^2 \times r'_a, \quad \ldots\ldots\ldots\ldots (66)$$

where P_a = energy dissipated in armature winding, in watts;
I' = total current generated in armature, in amperes;
r'_a = resistance of armature winding, hot, in ohms; see formulæ (60) to (64), respectively.

The total current, I', in series-wound dynamos, is identical with the current output I; in shunt- and compound-wound dynamos, however, I' consists of the sum of the external current, and the current necessary to excite the shunt magnet winding. The amount of current passing through the shunt winding is the quotient of the potential difference, E, at the terminals of the machine, by the resistances of the shunt circuit, r_m, that is the sum of the resistance of the shunt winding and of the regulating rheostat, in series with the shunt winding.

For the resistance, r'_a, of the armature winding, when hot,—in order to be on the safe side in determining the armature losses,—we will take that at, say 65.5° C. (= 150° Fahr.), or, according to formula (63), the resistance, r_a, at 15.5° C. (= 60° Fahr.), multiplied by

$$\left(1 + \frac{65.5^\circ - 15.5^\circ}{250}\right) = 1.2\,.$$

The energy dissipated in overcoming the resistance of the armature winding, consequently, for shunt- and compound-dynamos can be obtained from the formula:

$$P_a = 1.2 \times \left(I + \frac{E}{r_m}\right)^2 \times r_a \quad \ldots\ldots (67)$$

I = current-output of dynamo, in amperes;
E = E. M. F. output of dynamo, in volts;
r_a = resistance of armature, at 15.5° C. (= 60° Fahr.), in ohms;

r_m = resistance of shunt-circuit (magnet resistance + regulating resistance) at 15.5° C. (for series dynamos $\frac{E}{r_m} = 0$.)

If P_a is to be computed before the field calculations are made, that is to say, before r_m is known, it is sufficiently accurate for practical purposes to express, from experience, the total armature current, I', as a multiple of the current output, I; and, therefore, we have approximately

$$P_a = 1.2 \times (k_6 \times I)^2 \times r_a \quad \ldots\ldots\ldots (68)$$

and in this the coefficient k_6 for series dynamos is $k_6 = 1$, and for shunt- and compound-wound dynamos can be taken from the following Table XXVII.:

TABLE XXVII.—TOTAL ARMATURE CURRENT IN SHUNT- AND COMPOUND-WOUND DYNAMOS.

CAPACITY IN KILOWATTS.	SHUNT CURRENT IN PER CENT. OF CURRENT OUTPUT.	TOTAL CURRENT, AS MULTIPLE OF CURRENT OUTPUT. k_6
.1	15 %	1.15
.25	12	1.12
.5	10	1.10
1	8	1.08
2.5	7	1.07
5	6	1.06
10	5	1.05
20	4	1.04
30	3.5	1.035
50	3	1.03
100	2.75	1.0275
200	2.5	1.025
300	2.25	1.0225
500	2	1.02
1,000	1.75	1.0175
2,000	1.5	1.015

32. Energy Dissipated by Hysteresis.

The iron of the armature core is subjected to successive magnetizations and demagnetizations. Owing to the molecular friction in the iron, a lag in phase is caused of the effected magnetization behind the magnetizing force that produces it, and energy is dissipated during every reversal

of the magnetization. The name of "*Hysteresis*" (from the Greek ὑστερέω, to lag behind) was given by Ewing, in 1881, to this property of paramagnetic materials, by virtue of which the magnetizing and demagnetizing effects lag behind the causes that produce them.

Although Warburg,[1] Ewing,[2] Hopkinson,[3] and others have made numerous researches about the nature of this property of paramagnetic substances, it was not until recently that a definite *Law of Hysteresis* was established. In an elaborate paper presented to the American Institute of Electrical Engineers on January 19, 1892, Charles Proteus Steinmetz[4] gave the results of his experiments, showing that the energy dissipated by hysteresis is proportional to the 1.6th power of the magnetic density, directly proportional to the number of magnetic reversals and directly proportional to the mass of the iron. This law he expressed by the empirical formula:

$$P'_h = \eta_1 \times \mathfrak{B}_a^{1.6} \times N_1 \times M'_1,$$

where P'_h = energy consumed by hysteresis, in ergs;

η_1 = constant depending upon magnetic hardness of material ("Hysteretic Resistance");

$\mathfrak{B}_a$ = density of lines per square centimetre of iron;

N_1 = frequency, or number of complete cycles of 2 reversals each, per second;

M'_1 = mass of iron, in cubic centimetres.

The values of the hysteretic resistance found by Steinmetz for various kinds of iron are given in Table XXVIII., page 111.

For the materials employed in building up the armature core, according to this table, we can take the following average values of the hysteretic resistance:

Sheet iron : $\eta_1 = .0035$,
Iron wire : $\eta_1 = .040$.

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[1] Warburg, *Wiedem. Ann.*, vol. xiii. p. 141 (1881); Warburg and Hoenig, *Wiedem. Ann.*, vol. xx. p. 814 (1884).

[2] Ewing, *Proceed. Royal Soc.*, vol. xxxiv. p. 39, 1882; *Philos. Trans.*, part ii. p. 526 (1885).

[3] J. Hopkinson, *Philos. Trans. Royal Soc.*, part ii. p. 455 (1885).

[4] Steinmetz, *Trans. A. I. E. E.*, vol. ix. p. 3; *Electrical World*, vol. xix. pp. 73 and 89 (1892); vol. xx. p. 285 (1892).

TABLE XXVIII.—HYSTERETIC RESISTANCE FOR VARIOUS KINDS OF IRON.

KIND OF IRON.	HYSTERETIC RESISTANCE. η_1
Sheet Iron, magnetized lengthwise	.0025 to .005
" .0165″ thick (= .42 mm.)	.0035
" .015″ " (= .38 ")	.004
" .006″ " (= .15 ")	.005
" magnetized across Lamination	.007
Iron Wire, length-magnetization	.0035
" cross- "	.040
Wrought Iron, Norway Iron	.0023
" " ordinary, mean	.0033
Cast Iron, ordinary, mean	.013
" containing $\frac{1}{8}$ % Aluminium	.0137
" " $\frac{1}{2}$ % "	.0146
Mitis Metal	.0043
Tool Steel, glass hard	.070
" oil hardened	.027
" annealed	.0165
Cast Steel, hardened	.012 to .028
" annealed	.003 to .009

Inserting the average values given on page 110 into Steinmetz's equation, and reducing the latter to our practical units, we obtain for the energy loss by hysteresis in any armature having core built of *discs* or *ribbon:*

$$P_h = 10^{-7} \times .0035 \times \left(\frac{\mathfrak{B}''_a}{6.45}\right)^{1.6} \times N_1 \times 28{,}316 \times M$$
$$= 5 \times 10^{-7} \times \mathfrak{B}''^{1.6}_a \times N_1 \times M, \quad \dots\dots\dots(69)$$

and in any armature with core of iron *wire:*

$$P_h = 5.7 \times 10^{-6} \times \mathfrak{B}''^{1.6}_a \times N_1 \times M, \quad \dots\dots\dots(70)$$

where P_h = energy absorbed by hysteresis, in watts; 1 watt = 10^7 ergs;

$\mathfrak{B}''_a$ = density, in lines per square inch, corresponding to average specific magnetizing force of armature core, see § 91;

N_1 = frequency, in cycles per second, $= \frac{N}{60} \times n_p$;

N = number of revs. per min., n_p = number of pairs of poles;

M = mass of iron in armature core, in cubic feet; 1 cu. ft. = 28,316 cm.^{3}

The mass, in cubic feet, for both drum and ring armatures with *smooth* core is:

$$M = \frac{d'''_a \times \pi \times b_a \times l_a \times k_2}{1,728}; \quad \ldots\ldots(71)$$

d'''_a = mean diameter of armature core, in inches,
= $d_a - b_a$, see Fig. 45, page 58;
l_a = length of armature core, in inches;
b_a = radial depth of armature core, in inches;
k_2 = ratio of net iron section to total cross-section, see Table XXI., § 26;
1,728 = multiplier to convert cubic feet into cubic inches.

And for *toothed* and *perforated* armatures:

$$M = \frac{(d'''_a \pi \times b_a - n'_c S''_s) \times l_a \times k_2}{1,728}; \quad \ldots.(72)$$

d'''_a = mean core-diameter, in inches = $d''_a - (b_a + h_a)$, see Fig. 48, page 65;
n'_c = number of slots;
S''_s = sectional area of slots, in square inches.

Formulæ (69) and (70) can be rendered more convenient for practical use by uniting the terms $5 \times 10^{-7} \times \mathfrak{B}''^{1.6}_a$ and $5.7 \times 10^{-6} \times \mathfrak{B}''^{1.6}_a$, respectively, into one factor, η, the *factor of hysteresis;* that is, the energy absorbed by hysteresis in one cubic foot of iron, when subjected to magnetization and demagnetization at the rate of one complete cycle (two reversals) per second.

For convenience, the author, in Table XXIX., has calculated the numerical values of these hysteresis factors, η, for all core densities from 10,000 to 125,000 lines per square inch, thus simplifying the equation for the hysteresis loss into the formula:

$$P_h = \eta \times N_1 \times M. \quad \ldots\ldots\ldots\ldots (73)$$

In Table XXIX., columns headed $\eta \div 480$, are added for the case the hysteresis loss is to be calculated for an armature, of which the weight, in pounds, of the iron core is known:

TABLE XXIX.—HYSTERESIS FACTORS FOR DIFFERENT CORE DENSITIES, IN ENGLISH MEASURE.

MAGNETIC DENSITY IN ARMATURE CORE. LINES OF FORCE PER SQ. IN. $\mathcal{B}''_a$	WATTS DISSIPATED AT A FREQUENCY OF ONE COMPLETE MAGNETIC CYCLE PER SECOND.				MAGNETIC DENSITY IN ARMATURE CORE. LINES OF FORCE PER SQ. IN. $\mathcal{B}''_a$	WATTS DISSIPATED AT A FREQUENCY OF ONE COMPLETE MAGNETIC CYCLE PER SECOND.			
	Sheet Iron.		Iron Wire.			Sheet Iron.		Iron Wire.	
	per cu. ft.	per lb.	per cu. ft.	per lb.		per cu. ft.	per lb.	per cu. ft.	per lb.
	η	$\eta \div 480$	η	$\eta \div 480$		η	$\eta \div 480$	η	$\eta \div 480$
10,000	1.25	.0026	14 3	.030	66,000	25.72	.0537	294.0	.613
15,000	2.40	.0050	27.4	.057	67,000	26.34	.0550	301.0	.628
20,000	3.79	.0079	43.3	.090	68,000	26.97	.0563	308.2	.643
25,000	5.42	.0113	62.0	.129	69,000	27.61	.0576	315.5	.658
30,000	7.30	.0152	83.5	.174	70,000	28.26	.0589	322.8	.673
31,000	7.70	.0160	88.0	.183	71,000	28.91	.0603	330.1	.688
32,000	8.10	.0168	92.6	.192	72,000	29.56	.0617	337.6	.704
33,000	8.50	.0177	97.2	.202	73,000	30.22	.0631	345.1	.720
34,000	8.91	.0186	101.8	.212	74,000	30.89	.0645	352.9	.736
35,000	9.33	.0195	106.5	.222	75,000	31.56	.0659	360.7	.752
36,000	9.76	.0204	111.5	.232	76,000	32.23	.0673	368.5	.768
37,000	10.20	.0213	116.5	.242	77,000	32.91	.0687	376.3	.784
38,000	10.65	.0222	121.6	.253	78,000	33.60	.0701	384.2	.800
39,000	11.10	.0231	126.8	.264	79,000	34.29	.0715	392.1	.817
40,000	11.55	.0240	132.0	.275	80,000	34.99	.0730	400.0	.834
41,000	12.01	.0250	137.2	.286	81,000	35.69	.0745	408.0	.851
42,000	12.48	.0260	142.5	.297	82,000	36.40	.0760	416.0	.868
43,000	12.96	.0270	148.0	.308	83,000	37.11	.0775	424.0	.885
44,000	13.45	.0280	153.7	.320	84,000	37.82	.0790	432.4	.902
45,000	13.95	.0290	159.4	.332	85,000	38.54	.0805	440.8	.919
46,000	14.45	.0300	165.1	.344	86.000	39.27	.0820	449.2	.936
47,000	14.95	.0311	170.8	.356	87,000	40.01	.0835	457.6	.954
48.000	15.45	.0322	176.6	.368	88,000	40.75	.0850	466 0	.972
49,000	15.96	.0333	182.4	.380	89,000	41.50	.0865	474.5	.990
50,000	16.48	.0344	188.3	.392	90,000	42.25	.0881	483.0	1.008
51,000	17.01	.0355	194.3	.405	91,000	43.00	.0897	491.5	1.023
52,000	17.55	.0366	200.6	.418	92,000	43 76	.0913	500.0	1.042
53,000	18.10	.0377	206.9	.431	93,000	44.53	.0929	509.0	1.064
54,000	18.65	.0388	213.2	.444	94,000	45.30	.0945	518.0	1.080
55,000	19.21	.0400	219.5	.457	95,000	46.07	.0961	527.0	1.098
56,000	19.78	.0412	226.0	.470	96,000	46.85	.0977	536.0	1.116
57.000	20.35	.0424	232.6	.484	97.000	47.63	.0993	545.0	1.135
58,000	20.92	.0436	239.2	.498	98,000	48.41	.1009	554.0	1.154
59,000	21.50	.0448	245.8	.512	99 000	49.20	.1025	563.0	1.173
60,000	22.09	.0460	252.5	.526	100,000	50.00	.1041	572.0	1.192
61,000	22.69	.0472	259.4	.530	105,000	54.06	.1127	618.0	1.290
62,000	23.29	.0485	266.3	.554	110.000	58.23	.1215	666.0	1.388
63.000	23.89	.0498	273.0	.568	115,000	62.53	.1305	715.0	1.490
64,000	24.50	.0511	280.0	.583	120,000	66.95	.1400	765.0	1.595
65,000	25.11	.0524	287.0	.598	125,000	71.50	.1500	817.5	1.705

The values of η contained in this table are graphically represented in Fig. 69, two different scales, one ten times the other, being used for the ordinates in plotting the curves, as designated.

For the metric system, in formula (73) the mass M in cubic

feet is to be replaced by M_1 in cubic metres, from the formula:

$$M_1 = \frac{d'''_a \times \pi \times b_a \times l_a \times k_2}{1,000,000} - \frac{n'_c \times S_s \times l_a \times k_2}{1,000,000}, \quad (74)$$

the second term of which refers to toothed and perforated armatures only, and in which

M_1 = mass of iron in armature body, in cubic metres;
d'''_a = mean diameter of armature core, in centimetres;

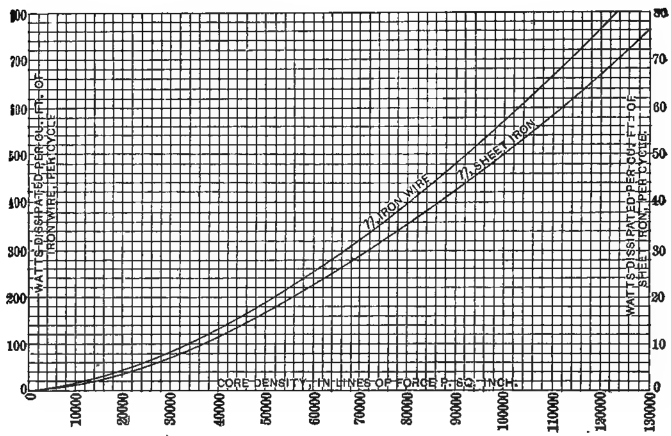


FIG. 69.—Hysteresis Factor for Sheet Iron and Iron Wire, at Different Core Densities.

$d'''_a = d_a - b_a$, for *smooth* armatures;
$= d''_a - (b_a + h_a)$, for *toothed* armatures;
l_a = length of armature core, in centimetres;
b_a = radial depth of armature core, in centimetres;
n'_c = number of slots;
S_s = slot-area, in square centimetres;
k_2 = ratio of magnetic to total length of armature core, Table XXI., § 26.

Then, formula (73) will give the hysteresis loss in watts, if the factor of hysteresis η is replaced by η' from the following Table XXX., η' being calculated from $3.5 \times 10^{-4} \times \mathfrak{B}_a^{1.6}$, in case of sheet-iron, and from $4 \times 10^{-3} \times \mathfrak{B}_a^{1.6}$, in case of iron wire:

TABLE XXX.—HYSTERESIS FACTORS FOR DIFFERENT CORE DENSITIES, IN METRIC MEASURE.

Magnetic Density in Armature Core. Lines of Force per cm.² (Gausses) $\mathfrak{B}_a$	Watts Dissipated at a Frequency of One Complete Magnetic Cycle per Second. Sheet Iron. per cu. m. η'	Sheet Iron. per kg. $\eta' \div 7{,}700$	Iron Wire. per cu. m. η'	Iron Wire. per kg. $\eta' \div 7{,}700$	Magnetic Density in Armature Core. Lines of Force per cm.² (Gausses) $\mathfrak{B}_a$	Watts Dissipated at a Frequency of One Complete Magnetic Cycle per Second. Sheet Iron. per cu. m. η'	Sheet Iron. per kg. $\eta' \div 7{,}700$	Iron Wire. per cu. m. η'	Iron Wire. per kg. $\eta' \div 7{,}700$
2,000	67.0	.0087	765.1	.0994	12,000	1,177.0	.1529	13,451.0	1.7469
3,000	128.1	.0166	1,467.1	.1905	12,250	1,216.5	.1580	13,902.3	1.8054
3,500	163.9	.0213	1,873.1	.2432	12,500	1,256.4	.1632	14,359.0	1.8648
4,000	202.9	.0264	2,319.3	.3012	12,750	1,296.9	.1685	14,821.0	1.9248
4,500	245.0	.0318	2,800.2	.3637	13,000	1,337.8	.1737	15,288.7	1.9855
5,000	290.0	.0377	3,314.6	.4305	13,250	1,379.2	.1791	15,761.7	2.0470
5,250	313.6	.0407	3,583.6	.4654	13,500	1,421.0	.1845	16,240.0	2.1091
5,500	337.8	.0439	3,860 5	.5014	13,750	1,463.4	.1901	16,724.0	2.1730
5,750	362.7	.0471	4,145.1	.5383	14,000	1,506.2	.1952	17,213.0	2.2355
6,000	388.3	.0504	4,437.1	.5763	14,250	1,549.4	.2012	17,708.0	2.2997
6,250	414.5	.0538	4.848.0	.6151	14,500	1,593.2	.2069	18,207.4	2.3646
6,500	441.3	.0573	5,043.3	.6550	14,750	1,637.3	.2126	18,712.0	2.4301
6,750	468.8	.0609	5,357.3	.6958	15,000	1,681.9	.2179	19,222.0	2.4964
7,000	496.9	.0645	5,678.3	.7375	15,250	1,727.0	.2243	19,742.0	2.5639
7.250	525.6	.0683	6,006.1	.7800	15,500	1.792.6	.2302	20,257.4	2.6309
7,500	554.8	.0721	6,355.3	.8254	15,750	1,818.6	.2362	20,783.0	2.6991
7.750	584.7	.0759	6,682.4	.8679	16,000	1,864.9	.2422	21,313.5	2.7681
8,000	615.2	.0799	7,030.7	.9131	16,250	1,911.8	.2484	21,848.5	2.8375
8,250	646.2	.0839	7,385.5	.9592	16,500	1,959.0	.2544	22,389.0	2.9076
8,500	677.9	.0880	7,747.0	1.0061	16,750	2,006.7	.2606	22,934.0	2.9785
8,750	710.1	.0922	8,114.8	1.0539	17,000	2,054.9	.2669	23,484.5	3.0499
9,000	742.8	.0965	8,488.6	1.1024	17,250	2,103.5	.2732	24,039.5	3.1220
9,250	776.1	.1101	8,869.2	1.1152	17,500	2,152.5	.2795	24,599.0	3.1947
9,500	809.9	.1105	9,255.6	1.1202	17,750	2,201.9	.2860	25,164.0	3.2681
9,750	844.2	.1110	9.649.0	1.2532	18,000	2,251.7	.2924	25,733.6	3.3420
10,000	879.2	.1142	10,047.7	1.3049	18,250	2,301.9	.2990	26,307.6	3.4165
10,250	914.6	.1188	10,452.2	1.3574	18,500	2,352.6	.3055	26,886.0	3.4918
10,500	950.5	.1234	10,863.2	1.4108	18,750	2,403.7	.3122	27,470.0	3.5676
10,750	987.0	.1282	11,284.0	1.4650	19,000	2,455 1	.3189	28,058 6	3.6440
11,000	1,024.0	.1330	11,702.5	1.5198	19,250	2,507.1	.3256	28,652.0	3.7210
11,250	1,061.5	.1379	12,131.0	1.5755	19,500	2,559.3	.3324	29,248 6	3.7986
11,500	1,099.5	.1428	12,565.3	1.6319	19.750	2,636.2	.3424	29,814.6	3.8760
11,750	1,138.0	.1478	13,005.3	1.6890	20,000	2,665.1	.3461	30,458.7	3 9556

With regard to the exponent of $\mathfrak{B}''_a$, in formulæ (69) and (70), Steinmetz's value, which in the preceding is given as 1.6 over the whole range of magnetization, has been attacked by Professor Ewing,[1] who by recent investigations has found it to vary with the density of magnetization. In the case of sheet

[1] J. A. Ewing and Miss Helen G. Klaassen, *Philos. Trans. Roy. Soc.; Electrician* (London), vol. xxxii. pp. 636, 668, 713 ; vol. xxxiii. pp. 6, 38 (April and May, 1894); *Electrical World*, vol. xxiii. pp. 569, 573, 614, 680, 714, 740 (April and May, 1894); *Electrical Engineer*, vol. xvii. p. 647 (May 9, 1894).

iron of .0185 inch (=.47 mm.) thickness, for instance, the hysteretic exponent ranged as follows:

TABLE XXXI.—HYSTERETIC EXPONENTS FOR VARIOUS MAGNETIZATIONS.

DENSITY OF MAGNETIZATION.		HYSTERETIC EXPONENT.
Lines of Force per Square Inch. $\mathfrak{B}''_a$	Lines per cm.² (Gausses.) $\mathfrak{B}_a$	
1,300 to 3,000	200 to 500	1.9
3,000 " 6,500	500 " 1,000	1.68
6,500 " 13,000	1,000 " 2,000	1.55
13 000 " 50,000	2,000 " 8,000	1.475
50,000 " 90,000	8,000 " 14,000	1.7

Although Ewing thus has shown that no formula with a constant exponent can represent the hysteretic losses within anything like the limits of experimental accuracy, he concludes that Steinmetz's exponent 1.6 gives values which are nowhere so grossly divergent from the truth as to unfit them for use in practical calculations. This conclusion holds particularly good for the densities applied in dynamo-electric machinery, as from the above Table XXXI. can be seen that for densities between 4 and 14 kilogausses (25,000 and 90,000 lines per square inch, respectively), compare Table XXII., § 26, the hysteretic exponent, according to Ewing's experiments, varies from 1.475 to 1.7, the average of which is 1.59, indeed a good agreement with Steinmetz's value.

Experiments on the variation of the hysteretic loss per cycle as function of the temperature have been made by Dr. W. Kunz,[1] for the temperatures up to 800° C. (= 1,472° Fahr.). They show that with rising temperature the hysteresis loss decreases according to a law expressed by the formula

$$P'_h = a + b\,\theta,$$

where P'_h = hysteresis loss per cycle, in ergs;
θ = temperature, in centigrade degrees;
a and b = constants for the material, depending upon the temperature and on the maximal density of magnetization.

[1] Dr. W. Kunz, *Elektrotechn. Zeitschr.*, vol. xv. p. 194 (April 5, 1894); *Electrical World*, vol. xxiii. p. 647 (May 12, 1894).

The decrease of the hysteretic loss, consequently, consists of two parts: one part, $b\,\theta$, which is proportional to the increase of the temperature, and another part, a, which becomes permanent, and seems to be due to a permanent change of the molecular structure, produced by heating. This latter part, in soft iron, is also proportional to the temperature, thus

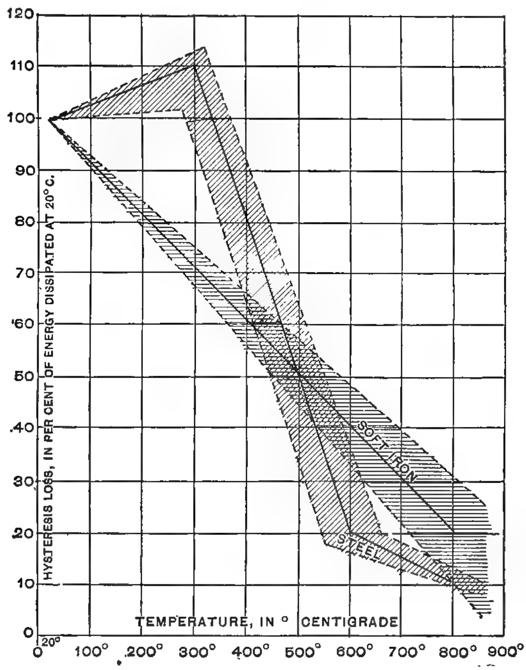


Fig. 70.—Influence of Temperature upon Hysteresis in Iron and Steel.

making the hysteretic loss of soft iron a linear function of the temperature, but is irregular in steel.

The curves in the latter case show a slightly ascending line to about 300° C. (= 572° Fahr.), then change into a rapidly descending straight portion to about 600° C. (= 1,112° Fahr.), when a second "knee" occurs, and the descension becomes more gradual.

The author has refigured all of Kunz's test results, basing the same upon the hysteresis loss at 20° C. (= 68° Fahr.) as unity

in every set of observations. In Fig. 70 dotted lines have then been drawn, inclosing all the values thus obtained, for soft iron and for steel, respectively, and two full lines, one for each quality of iron, are placed centrally in the planes bounded by the two sets of dotted lines, thus indicating the average values of the hysteretic losses, in per cent. of the energy loss at 20° C. Arranging the same in form of a table, the following law is obtained:

TABLE XXXII.—VARIATION OF HYSTERESIS LOSS WITH TEMPERATURE.

Temperature.		Energy Dissipated by Hysteresis in Per Cent. of Hysteresis Loss at 20° C. (= 68° Fahr.)	
In Centigrade Degrees.	In Fahrenheit Degrees.	Soft Iron.	Steel.
20°	68°	100%	100%
100	212	90	103
200	392	80	106
300	572	70	110
400	752	60	80
500	932	50	50
600	1,112	40	20
700	1,292	30	15
800	1,472	20	10
20	68	70	40

The last row of this table, which gives the hysteresis loss at 20° C., at the end of the test, shows that the energy required to overcome the hysteretic resistance is reduced to about 70 per cent. in case of soft iron and to about 40 per cent. in case of steel, after having been subjected to magnetic cycles at high temperatures. Kunz further found that the hysteretic energy loss can thus be considerably reduced by repeatedly applying high temperatures while iron is under cyclic influence.

For soft iron a set of straight lines was obtained in this way, each following of which had a lower starting point, and descended less rapidly than the foregoing one, until, finally, after the fourth repetition of the heating process, a stationary condition was reached.

For steel, already the second set of tests with the same sample did not show the characteristic form of the, at first

ascending, then rapidly, and finally slowly descending steel curve, but furnished a rapidly descending straight line. For every further repetition, the corresponding line becomes less inclined, and for the fifth test is parallel to the axis of abscissæ. Steel, therefore, after heating it but once as high as 800° C. (= 1,472° Fahr.), loses its characteristic properties, and with every further repetition becomes a softer, less carbonaceous iron.

33. Energy Dissipated by Eddy Currents.

From his experiments Steinmetz also derived that the energy consumed in setting up induced currents in a body of iron increases with the square of the magnetic density, with the square of the frequency, and in direct proportion with the mass of the iron:

$$P'_e = \varepsilon' \times \mathfrak{B}_a^2 \times N_1^2 \times M'_1;$$

P'_e = energy dissipated by eddy currents, in ergs;

$\mathfrak{B}_a$ = density of lines of force, per square centimetre of iron;

N_1 = frequency, in cycles per second, $= \frac{N}{60} \times n_p$;

M'_1 = mass of iron, in cubic centimetres;

ε' = eddy current constant, depending upon the thickness and the specific electric conductivity of the material; for the numerical value of this constant Steinmetz gives the formula:

$$\varepsilon' = \frac{\pi^2}{6} \times \delta^2 \times \gamma \times 10^{-9} = 1.645 \times \delta^2 \times \gamma \times 10^{-9}.$$

δ = thickness of material, in centimetres.

γ = electrical conductivity, in mhos;

for iron : γ = 100,000 mhos;

for copper: γ = 700,000 mhos.

Inserting the value of ε' with reference to iron, into the above equation expressing the Eddy Current Law, and transforming into practical units, the eddy current loss in an armature, in watts, is obtained:

$$P_e = 10^{-7} \times 1.645 \times (2.54\, \delta_i)^2 \times 10^{-4} \times \left(\frac{\mathfrak{B}''_a}{6.45}\right)^2 \times N_1^2 \times 28,316$$
$$\times M = 7.22 \times 10^{-6} \times \delta_i^2 \times \mathfrak{B}''^2_a \times N_1^2 \times M. \quad ..(75)$$

δ_i = thickness of iron laminæ in armature core, inch;

$\mathfrak{B}''_a$ = density, in lines per square inch, corresponding to average specific magnetizing force of armature core, see § 91;

N_1 = frequency, in cycles per second;

M = mass of iron, in cubic feet.

Uniting again $7.22 \times 10^{-5} \times \delta_i^2 \times \mathfrak{B}''^2_a$ into one factor, in this case the *Eddy Current Factor*, ε, we have the simplified formula:

$$P_e = \varepsilon \times N_1^2 \times M. \qquad \ldots\ldots\ldots\ldots(76)$$

TABLE XXXIII.—EDDY CURRENT FACTORS FOR DIFFERENT CORE DENSITIES AND FOR VARIOUS LAMINATIONS, ENGLISH MEASURE.

MAGNETIC DENSITY IN ARMATURE CORE. LINES OF FORCE PER SQ. IN.	WATTS DISSIPATED PER CUBIC FOOT OF IRON AT A FREQUENCY OF 1 CYCLE PER SECOND, ε. Thickness of Lamination, δ_i.				MAGNETIC DENSITY IN ARMATURE CORE. LINES OF FORCE PER SQ. IN.	WATTS DISSIPATED PER CUBIC FOOT OF IRON AT A FREQUENCY OF 1 CYCLE PER SECOND, ε. Thickness of Lamination, δ_i			
$\mathfrak{B}''_a$	.010"	.020"	.040"	.080"	$\mathfrak{B}''_a$	.010"	.020"	.040"	.080"
10,000	.0007	.003	.012	.046	66,000	.0315	.126	.503	2.013
15,000	.0016	.007	.026	.104	67,000	.0325	.130	.519	2.075
20,000	.0029	.012	.046	.185	68,000	.0335	.134	.534	2.137
25,000	.0045	.018	.072	.288	69,000	.0345	.138	.550	2.200
30,000	.0065	.026	.104	.416	70,000	.0355	.142	.566	2.265
31,000	.0070	.028	.111	.444	71,000	.0365	.146	.582	2.330
32,000	.0074	.030	.118	.472	72,000	.0375	.150	.599	2.396
33,000	.0079	.032	.126	.503	73,000	.0385	.154	.616	2.463
34,000	.0084	.034	.134	.534	74,000	.0396	.158	.633	2 530
35,000	.0089	.036	.142	.567	75,000	.0407	.163	.650	2.600
36,000	.0094	.038	.150	.600	76,000	.0418	.167	.668	2.670
37,000	.0099	.040	.158	.633	77,000	.0429	.171	.685	2.740
38,000	.0104	.042	.167	.667	78,000	.0440	.176	.703	2.810
39,000	.0110	.044	.176	.703	79,000	.0451	.180	.721	2.883
40,000	.0116	.046	.185	.740	80,000	.0462	.185	.740	2.958
41,000	.0122	.049	.194	.777	81,000	.0474	.190	.758	3.033
42,000	.0128	.051	.204	.815	82,000	.0486	.194	.777	3.108
43,000	.0134	.054	.214	.855	83,000	.0498	.199	.796	3.184
44,000	.0140	.056	.224	.896	84,000	.0510	.204	.815	3.260
45,000	.0146	.059	.234	.937	85,000	.0523	.209	.835	3.340
46,000	.0153	.061	.245	.979	86,000	.0535	.214	.855	3.420
47,000	.0160	.064	.256	1.022	87,000	.0548	.219	.875	3 500
48,000	.0167	.067	.267	1.066	88,000	.0560	.224	.895	3.580
49,000	.0174	.070	.278	1.110	89,000	.0573	.229	.916	3.662
50,000	.0181	.072	.289	1.155	90,000	.0586	.234	.937	3.745
51,000	.0188	.075	.300	1.200	91,000	.0599	.240	.958	3.830
52 000	.0195	.078	.312	1.248	92 000	.0612	.245	.979	3.915
53,000	.0202	.081	.324	1.297	93,000	.0625	.250	1.000	4.000
54,000	.0210	.084	.337	1.346	94,000	.0638	.255	1.021	4.085
55,000	.0218	.087	.349	1.397	95,000	.0651	.261	1.043	4.170
56,000	.0226	.091	.362	1.448	96,000	.0665	.266	1.064	4.257
57,000	.0234	.094	.375	1.500	97,000	.0679	.272	1.086	4.345
58,000	.0242	.097	.389	1.555	98,000	.0693	.277	1.109	4.436
59,000	.0251	.101	.403	1.610	99,000	.0707	.283	1.132	4.528
60,000	.0260	.104	.416	1.665	100,000	.0722	.289	1.156	4.622
61,000	.0269	.108	.430	1.720	105,000	.0797	.319	1.274	5.095
62,000	.0278	.111	.444	1.776	110,000	.0875	.350	1.398	5.593
63,000	.0287	.115	.458	1.833	115,000	.0955	.382	1.528	6.113
64,000	.0296	.118	.473	1.891	120,000	.1040	.416	1.664	6.655
65,000	.0305	.122	.486	1.951	125,000	.1128	.451	1.806	7.222

The values of ε for core-densities from 10,000 to 125,000 lines per square inch, and for laminations of thickness $\delta_i = .010''$, $.020''$, $.040''$, and $.080''$, are given in the foregoing Table XXXIII., page 120.

Curves corresponding to the value of ε in the above table are plotted in Fig. 71.

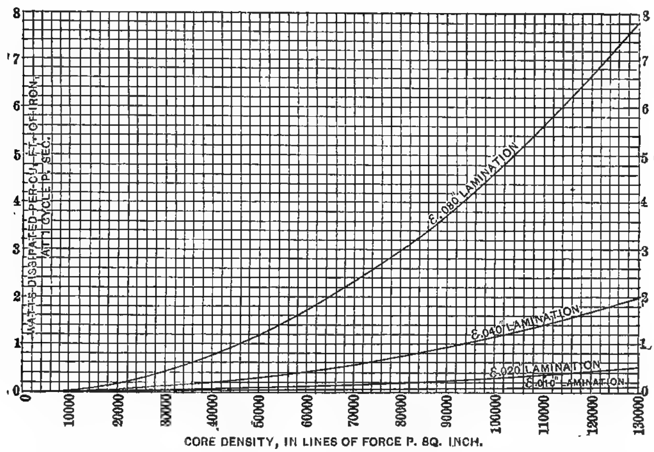


Fig. 71.—Eddy Current Factors for Various Densities and Different Laminations.

The eddy current factors ε' for the metric system, in watts per cubic metre of iron, as calculated from $1.645 \times 10^{-7} \times \delta'^2_i \times \mathfrak{B}_a^2$, for densities from $\mathfrak{B}_a = 2{,}000$ to $20{,}000$ gausses, and for laminations of $\delta'_i = 0.25$ mm., 0.5 mm., 1 mm., and 2 mm. thickness, are given in Table XXXIV., page 122.

Prof. Thompson[1] gives for the calculation of the eddy current loss the following formula by Fleming which is much used by English engineers:

$$P_e = \delta'^2_i \times \mathfrak{B}_a^2 \times N_1^2 \times M'_1 \times 10^{-16},$$

where P_e = eddy current loss, in watts;
δ'_i = thickness of iron laminæ, in mils;
$\mathfrak{B}_a$ = magnetic density, in lines per square centimetre;
N_1 = frequency, in cycles per second;
M'_1 = mass of iron, in cubic centimetres.

[1] "Dynamo Electric Machinery," 5th edition, p. 137.

TABLE XXXIV.—EDDY CURRENT FACTORS FOR DIFFERENT CORE DENSITIES AND FOR VARIOUS LAMINATIONS, METRIC MEASURE.

MAGNETIC DENSITY IN ARMATURE CORE. LINES OF FORCE PER CM.² (GAUSSES). $\mathfrak{B}_a$	WATTS DISSIPATED PER CUBIC METRE OF IRON, AT A FREQUENCY OF 1 CYCLE PER SECOND, ϵ'. Thickness of Lamination, δ'_i. 0.25 mm	0.5 mm	1 mm	2 mm	MAGNETIC DENSITY IN ARMATURE CORE. LINES OF FORCE PER CM.² (GAUSSES). $\mathfrak{B}_a$	WATTS DISSIPATED PER CUBIC METRE OF IRON, AT A FREQUENCY OF 1 CYCLE PER SECOND, ϵ'. Thickness of Lamination, δ'_i. 0.25 mm	0.5 mm	1 mm	2 mm
2,000	.041	.165	.658	2.632	12,000	1.481	5.922	23.688	94.752
3,000	.093	.370	1.481	5.922	12,250	1.543	6.172	24.687	98.746
3,500	.126	.504	2.015	8.061	12,500	1.607	6.426	25 704	102.815
4,000	.165	.658	2.632	10.528	12,750	1.671	6.685	26.741	106.965
4,500	.208	.833	3.331	13.325	13,000	1.738	6.950	27.801	111.203
5,000	.257	1.028	4.113	16.450	13,250	1.805	7.220	28.880	115.520
5,250	.283	1.134	4.534	18.136	13,500	1.874	7.495	29.980	119.921
5,500	.311	1.244	4.976	19.904	13,750	1.944	7.775	31.100	124.401
5,750	.340	1.360	5.441	21.766	14,000	2.015	8.061	32.242	128.968
6,000	.370	1.481	5.922	23.689	14,250	2.088	8.351	33.404	133.615
6.250	.402	1.607	6.426	25.704	14,500	2.162	8.647	34.586	138.345
6,500	.434	1.738	6.950	27.801	14,750	2.237	8.947	35.789	143.156
6,750	.468	1.874	7.495	29.980	15,000	2.313	9.253	37.013	148.050
7,000	.504	2.015	8.061	32.242	15,250	2.391	9.564	38.257	153.026
7,250	.542	2.167	8.667	34.666	15,500	2.470	9.880	39.521	158.085
7.500	.578	2.313	9.253	37.013	15,750	2.550	10.202	40.806	163.225
7.750	.619	2.476	9.903	39.613	16,000	2.632	10.528	42.112	168.448
8.000	.658	2.632	10.528	42.112	16,250	2.715	10.860	43.438	173.753
8,250	.700	2.799	11.196	44.785	16,500	2.799	11.196	44.785	179.141
8,500	.743	2,971	11.885	47.545	16,750	2.885	11.538	46.153	184.610
8,750	.787	3.149	12.595	50.379	17,000	2.971	11.885	47.541	190.162
9,000	.833	3.331	13.325	53.298	17,250	3.059	12.237	48.949	195.796
9,250	.880	3.519	14.075	56.300	17,500	3.149	12.595	50.378	201.512
9.500	.930	3.712	14.846	59.384	17,750	3.239	12.957	51.828	207.311
9,750	.977	3.909	15.638	62.550	18,000	3.331	13.325	53.298	213.192
10.000	1.028	4.113	16.450	65.800	18,250	3.424	13.697	54.789	219.155
10,250	1.080	4.321	17.283	69.130	18.500	3.519	14.075	56.300	225.201
10,500	1.134	4.534	18.136	72.545	18,750	3.615	14.458	57.832	231.328
10,750	1.188	4.753	19.011	76.042	19.000	3.712	14.846	59.385	237.538
11,000	1.244	4.976	19.905	79.618	19,250	3.810	15.240	60.958	243.830
11,250	1.301	5.205	20.820	83.278	19,500	3.910	15.638	62.551	250.205
11,500	1.360	5.440	21.756	87.022	19,750	4.010	16.041	64.165	256.661
11,750	1.420	5.678	22.712	90.846	20,000	4.113	16.450	65.800	263.200

Transforming this formula into English units, we obtain:

$$P_e = \left(\frac{\delta_i}{1{,}000}\right)^2 \times \left(\frac{\mathfrak{B}''_a}{6.45}\right)^2 \times N_1^2 \times (28{,}316 \times M) \times 10^{-16}$$

$$= 6.81 \times 10^{-8} \times \delta_i^2 \times \mathfrak{B}''^2_a \times N_1^2 \times M,$$

which is practically the same as formula (75), the results by Fleming's formula being about 5.7 per cent. smaller than by the former.

34. Radiating Surface of Armature.

The radiating surface, or cooling surface, of an armature is that portion of its superficial area which is in direct contact

with the surrounding air, and which consequently gives off the heat generated in the winding and in the iron core. It is evident that the shape and the construction of the armature and the arrangement of the field determine the size of this radiating portion of the armature surface. In drum armatures, for instance, only the external surface is liberating heat, while in ring armatures, according to design, either the external surface only or any two or three sides of the cross-section, or even the entire superficial area may act as cooling surface.

a.—Radiating Surface of Drum Armatures.

In drum armatures the dead portion of the winding forms two *heads* at the ends of the cylindrical body, and the ex-

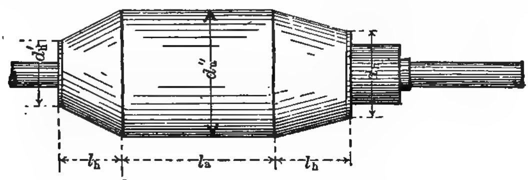


Fig. 72.—External Dimensions of Drum Armature.

ternal area extending over the cylindrical part as well as over these two conical heads, is the radiating surface of the armature. In order to calculate the cooling area of a drum armature, it is therefore necessary to first determine the size of the armature heads.

The length of the heads, l_h, Fig. 72, depends upon the diameter of the armature, the size of the shaft and the height of the winding space, and can be found from the empirical formula:

$$l_h = k_7 \times d''_a + 2 \times h_a \,, \quad \ldots\ldots\ldots\ldots (77)$$

where: l_h = length of armature heads, in inches, or in centimetres;

k_7 = constant, depending upon the size of the armamature (see Table XXXV.);

d''_a = external diameter of armature, in inches, or in centimetres;

h_a = height of winding space, in inches, or in centimetres.

If d''_a and h_a are given in inches, (77) gives the length of the heads in inches; and if both d''_a and h_a are expressed in centimetres, also l_h will be obtained in centimetres.

The coefficient k_7 in this formula varies with the slope of the head, and this, in turn, depends upon the ratio between the diameter of the armature and the thickness of the shaft. For, in large machines the shaft bears a smaller proportion to the armature diameter than in small ones, and therefore in large armatures there is comparatively much more room between the shaft circumference and the core periphery than in small armatures, and since the diameter of the head must never exceed that of the armature itself, it is evident that the slope of the head is smaller, and consequently its relative length is larger in the smaller armatures. The following Table XXXV. gives the values of this coefficient for the various sizes of drum armatures:

TABLE XXXV.—LENGTH OF HEADS IN DRUM ARMATURES.

EXTERNAL DIAMETER OF ARMATURE. d''_a		VALUE OF k_7	AVERAGE LENGTH OF HEADS. l_h
Inches.	Cm.		
Up to 6	Up to 15	.60 to .50	$l_h = .55 \times d''_a + 2\,h_a$
" 12	" 30	.55 to .45	$= .50 \times d''_a + 2\,h_a$
" 18	" 45	.50 to .40	$= .45 \times d''_a + 2\,h_a$
" 24	" 60	.45 to .35	$= .40 \times d''_a + 2\,h_a$
" 30	" 75	.40 to .30	$= .35 \times d''_a + 2\,h_a$

As to the diameters at the ends of the heads, that of the front head, d_h, at commutator end of armature, is generally made from 0.75 d''_a to 1.0 a''_a, while the diameter of the end washer of the back head, d'_h, ranges in size from 0.5 d''_a to 0.75 d''_a. Taking $d_h = 0.9\, d''_a$ as the average diameter of the front head, and $d'_h = 0.6\, a''_a$ as that on the back head (Figs. 73 and 74, page 125), we obtain the following formula for the radiating surface of a drum armature:

$$S_A = d''_a \times \pi \times \left(l_a + 1\tfrac{3}{4} \times \sqrt{l_h^2 + \left(\frac{d''_a}{8}\right)^2}\right),$$

or approximately:

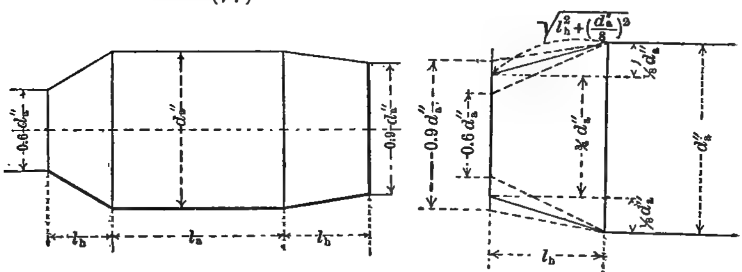
$$S_A = d''_a \times \pi \times (l_a + 1.8 \times l_h); \quad \ldots\ldots(78)$$

S_A = radiating surface of armature, in square inches, or in square centimetres;

d''_a = external diameter of armature, in inches, or in centimetres, $= d_a + 2\ h_a$;

l_a = length of armature body, in inches, or in centimetres, formula (40);

l_h = length of armature head, in inches, or centimetres; from formula (77) and Table XXXV.



Figs. 73 and 74.—Size of Heads in Drum Armatures.

b.—Radiating Surface of Ring Armature.

In ring armatures the construction and mounting of the core may be such that either one, two, or three, or all four sides of the cross-section are in contact with the air, but in modern

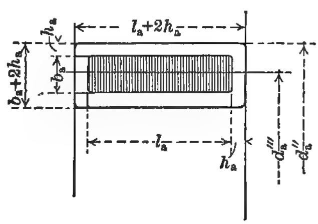


Fig. 75.—Dimensions of Ring Armature.

machines almost without exception, all four, or at least three of the surfaces constituting the ring are radiating areas.

Fig. 75 shows the cross-section of a ring armature.

In the first mentioned case (four sides) we have the formula:

$$S_A = 2 \times d'''_a \times \pi \times (l_a + b_a + 4 \times h_a), \quad \ldots(79)$$

and in the latter case (three sides):

$$S_A = d''_a \times \pi \times (l_a + 2\,h_a) + 2 \times d'''_a \times \pi \times (b_a + 2\,h_a); \quad \ldots\ldots\ldots (80)$$

S_A = radiating surface of armature, in square inches, or in square centimetres;

d''_a = external diameter of armature, in inches, or in centimetres;

d'''_a = mean diameter of armature core, in inches, or in centimetres;

l_a = length of armature core, in inches, or in centimetres;

b_a = radial depth of armature core, in inches, or in centimetres;

h_a = height of winding space, in inches, or in centimetres.

35. Specific Energy Loss. Rise of Armature Temperature.

While the amount of the total energy consumed, P_A, formula (65), determines directly the quantity of heat generated in the armature, the amount of heat liberated from it depends upon the size of its radiating surface, upon its circumferential velocity, and upon the ratio of the pole area to the radiating surface.

The most important of these factors in the heat conduction from an armature naturally is the size of the radiating surface, while the speed and the ratio of polar embrace are of minor influence only; and it is, therefore, the ratio of the energy consumed in the armature to the size of the cooling surface, that is, the *specific energy loss*, which limits the proportion of heat generated to heat radiated, and which consequently affords a measure for the degree of the temperature increase of the armature.

A. H. and C. E. Timmermann,[1] of Cornell University, who made the armature radiation the subject of their paper read before the American Institute of Electrical Engineers, in May, 1893, from a series of elaborate experiments drew the following conclusions:

(1) An increase of the temperature of the armature causes an increased radiation of heat per degree rise in tempera-

[1] A. H. and C. E. Timmermann, *Transactions Am. Inst. of Elec. Eng.*, vol. x. p. 336 (1893).

ture, but the ratio of increase diminishes as the temperature increases, and an increase of the amount of heat generated in the armature increases the temperature of the armature, but less than proportionately.

(2) As the peripheral velocity is increased, the amount of heat liberated per degree rise in temperature is increased, but the rate of increase becomes less with the higher speeds.

(3) The effect of the field-poles is to prevent the radiation of heat; as the percentage of the polar embrace is increased, the amount of heat radiated per degree rise in temperature becomes less.

Combining these results with the data and tests of various dynamos, the author finds the following values, given in Table XXXVI., of the temperature increase per unit of specific energy loss, that is, for every watt of energy dissipated per square inch of radiating surface, under various conditions of peripheral velocity and polar embrace:

TABLE XXXVI.—SPECIFIC TEMPERATURE INCREASE IN ARMATURES.

PERIPHERAL VELOCITY.		RISE OF TEMPERATURE PER UNIT OF SPECIFIC ENERGY LOSS, IN DEGREES CENTIGRADE, θ'_a.						
		Ratio of Pole Area to Total Radiating Surface.						
Feet per sec.	Metres per sec.	.8	7	.6	.5	.4	.3	.2
0	0	110°	100°	95°	90°	86°	83°	80°
10	3	80	74	70	67	64	62	60
20	6	64	61	58	56	54	52	50
30	9	55	53	51	49½	48	46½	45
40	12	50	48½	47	46	45	44	43
50	15	48	47	46	45	44	43	42
75	22.5	47	46	45	44	43	42	41
100	30	46	45	44	43	42	41	40
150	45	45	44	43	42	41	40	40

In Fig. 76 these temperatures are represented graphically; Curves I., II. . . VII., corresponding to Columns 3, 4, . 9, of Table XXXVI., respectively.

Multiplying this specific temperature increase by the respective specific energy loss, the rise of temperature can be found from:

$$\theta_a = \theta'_a \times \frac{P_A}{S_A}, \quad \ldots\ldots\ldots\ldots\ldots (81)$$

where θ_a = rise of temperature in armature, in degrees Centigrade;

θ'_a = specific temperature increase, or rise of armature temperature, per unit of specific energy loss, from Table XXXVI., or Fig. 76;

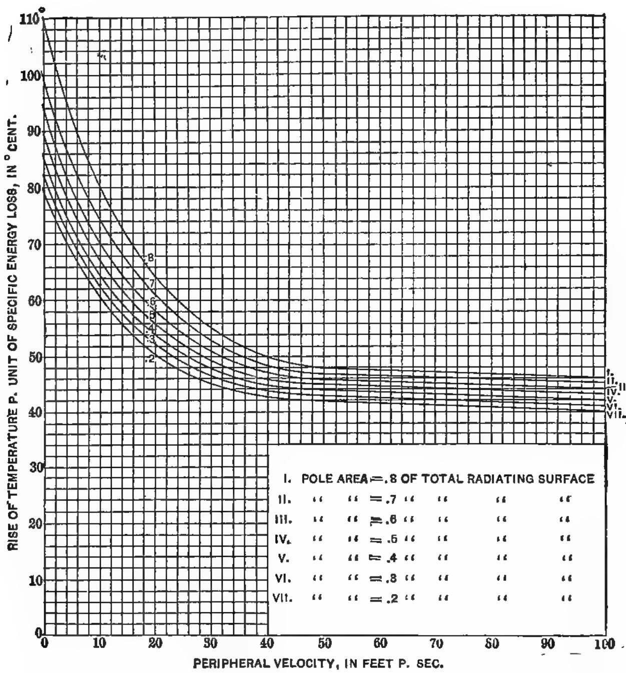


Fig. 76.—Specific Temperature Increase in Armatures.

P_A = total energy consumed in armature, in watts, formula (65);

S_A = radiating surface of armature, in square inches, from formula (78), (79), or (80), respectively;

$\frac{P_A}{S_A}$ = specific energy loss, *i. e.*, watts energy loss per square inch of radiating surface.

In order to obtain the temperature increase in *Fahrenheit* degrees, the result obtained by (81) is to be multiplied by

$$\frac{9}{5} = 1.8;$$

and if S_A is expressed in *square centimetres*, the factor 6.45 must be adjoined.

36. Empirical Formula for Heating of Drum Armatures.

From tests made with drum armatures, Ernst Schulz[1] derived the following empirical formula:

$$\theta_a = 0.012 \times \frac{\sigma \times n_p \times N \times M'_1}{S'_A}$$

$$= \frac{2}{3} \times 10^{-6} \times \frac{\mathfrak{B}_a \times n_p \times N \times M'_1}{S'_A}, \quad \dots\dots\dots(82)$$

in which θ_a = rise of armature temperature, in degrees Centigrade;

σ = factor of magnetic saturation in smallest cross-section of armature core,

$$= \frac{\Phi}{2\,n_p \times b_a \times l_a \times k_2} \div 18{,}000 = \mathfrak{B}_a \div 18{,}000.$$

Φ = useful flux through armature, in maxwells;

$2\,n_p$ = number of poles;

$b_a \times l_a \times k_2$ = net area of least cross-section of armature core, in square centimetres;

$\mathfrak{B}_a$ = flux-density in armature core, in gausses;

N = number of revolutions per minute;

M'_1 = mass of armature core, in cubic centimetres;

S'_A = surface of armature-core, in square centimetres,

$$= d_a\,\pi \times l_a + \frac{d_a^2\,\pi}{2}, \text{ for } \textit{smooth} \text{ armatures,}$$

$$= d''_a\,\pi \times l_a + \frac{d''^2_a\,\pi}{2}, \text{ for } \textit{toothed} \text{ and } \textit{perforated}$$

armatures.

The numerical constant in this formula is averaged from values ranging between .008 and .012 for smooth-core machines, and between .010 and .0125 for toothed armatures.

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[1] Ernst Schulz, *Elektrotechn. Zeitschr.*, vol. xiv. p. 367 (June 30, 1893); *Electrical World*, vol. xxii. p. 118 (August 12, 1893).

Translating (82) into the English system of measurement, we obtain the formula:

$$\theta_a = .00045 \times \frac{\mathfrak{B}''_a \times n_p \times N \times M}{S'_A}; \quad \ldots.(83)$$

θ_a = rise of armature temperature, in degrees Centigrade;
$\mathfrak{B}''_a$ = density of magnetization, in lines per square inch;
n_p = number of pairs of poles;
N = number of revolutions per minute;
M = mass of iron, in cubic feet;
S'_A = armature core surface, in square inches.

The value of the constant in the English system, for the type of machines experimented upon by Schulz, varies between the limits .0003 and .0005.

The numerical factor depends upon the units chosen, upon the ventilation of the armature, upon the quality of the iron, and upon the thickness of the lamination, and consequently varies considerably in different machines. For this reason it is advisable not to use formula (82) or (83), respectively, except in case of calculating an armature of an existing type for which this constant is known by experiment. In the latter case, Schulz's formula, although not as exact, is even more convenient than the direct equation (81) which necessitates the separate calculation of the energy losses, while (82) and (83) contain the factors determining these losses, and therefore will give the result quicker, provided that the numerical factor has been previously determined from similar machines.

Another empirical formula for the temperature increase of drum armatures, which, however, requires the specific energy-loss to be calculated, and which therefore is not as practical as that of Schulz, and which cannot give as accurate results as can be obtained by the use of Table XXXVI. in connection with formula (81), has recently been given by Ernest Wilson.[1]

37. Circumferential Current Density of Armature.

An excellent check on the heat calculation of the armature, and in most cases all that is really necessary for an examina-

[1] Ernest Wilson, *Electrician* (London), vol. xxxv. p. 784 (October 11, 1895); *Elektrotechn. Zeitschr.*, vol. xvi. p. 712 (November 7, 1895).

tion of its electrical qualities, is the computation of the circumferential current density of the armature. This is the sum of the currents flowing through a number of active armature conductors corresponding to unit length of core-periphery, and is found by dividing the total number of amperes all around the armature by the core circumference:

$$i_c = \frac{N_c \times \frac{I'}{2\,n'_p}}{d_a \times \pi}; \quad \ldots\ldots\ldots\ldots\ldots (84)$$

i_c = circumferential current density, in amperes per inch length of core periphery, or in amperes per centimetre;

N_c = total number of armature conductors, all around periphery;

I' = total current generated in armature, in amperes;

$2\,n'_p$ = number of electrically parallel armature portions, eventually equal to the number of poles;

$\frac{I'}{2\,n'_p}$ = current flowing through each conductor, in amperes;

$N_c \times \frac{I'}{2\,n'_p}$ = total number of amperes all around armature; this quantity is called "*volume of the armature current*" by W. B. Esson, and "*circumflux of the armature*" by Silvanus P. Thompson;

d_a = diameter of armature core, in inches; in case of a toothed armature, on account of the considerably greater winding depth, the external diameter, d''_a, is to be taken instead of d_a, in order to bring toothed and smooth armatures to about the same basis; for a similar reason, for an inner-pole dynamo, the mean diameter, d'''_a, should be substituted for d_a.

By comparing the values of i_c found from (84) with the averages given in the following Table XXXVII., the rise of the armature temperature can be approximately determined, and thus a measure for the electrical quality of the armature be gained. The degree of fitness of the proportion between the armature winding and the dimensions of the core is indicated by

the amount of increase of the armature temperature. If the latter is too high, it can be concluded that the winding is proportioned excessively, and either should be reduced or divided over a larger armature surface:

TABLE XXXVII.—RISE OF ARMATURE TEMPERATURE, CORRESPONDING TO VARIOUS CIRCUMFERENTIAL CURRENT DENSITIES.

Circumferential Current Density, i_c.		Rise of Armature Temperature, θ_a.			
		High Speed (Belt-Driven) Dynamos.		Low Speed (Direct-Driven) Dynamos.	
Amperes per inch.	Amperes per cm.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.
50 to 100	20 to 40	15° to 25°	27° to 45°	10° to 20°	18° to 36°
100 " 200	40 " 80	20 " 35	36 " 63	15 " 25	27 " 45
200 " 300	80 " 120	30 " 50	54 " 90	20 " 35	36 " 63
300 " 400	120 " 160	40 " 60	72 " 108	25 " 40	45 " 72
400 " 500	160 " 200	50 " 70	90 " 126	30 " 45	54 " 81
500 " 600	200 " 240	60 " 80	108 " 144	35 " 50	63 " 90
600 " 700	240 " 280	70 " 90	126 " 162	40 " 60	72 " 108
700 " 800	280 " 320	80 " 100	144 " 180	50 " 70	90 " 126

The difference in the temperature-rise at same circumferential current density for high-speed and low-speed dynamos (columns 3 and 5, or 4 and 6, respectively, of the above table) is due to the fact that, other conditions being equal, in a low-speed machine less energy is absorbed by hysteresis and eddy currents; that, consequently, less total heat is generated in the armature, and, therefore, more cooling surface is available for the radiation of every degree of heat generated.

38. Load Limit and Maximum Safe Capacity of Armatures.

From Table XXXVII. also follows that, according to the temperature increase desired, the load carried by an armature varies between 50 and 800 amperes per inch (= 20 to 320 amperes per centimetre) of circumference, or between about 150 and 2,500 amperes per inch (= 60 to 1,000 amperes per centimetre) of armature diameter. As a limiting value for safe working, Esson[1] gives 1,000 amperes per inch diameter (= 600 amperes per centimetre) for ring armatures, and 1,500

[1] Esson, *Journal I. E. E.*, vol. xx. p. 142. (1890.)

amperes (= 400 amperes per centimetre) for drums. Kapp[1] allows 2,000 amperes per inch (= 800 amperes per centimetre) diametral current density for diameters over 12 inches as a safe load.

Taking 1,900 amperes per inch diameter (= 600 amperes per inch circumference) as the average limiting value of the armature-load in high-speed dynamos, corresponding to a temperature rise of about 70° to 80° Centigrade (= 126° to 144° Fahrenheit), compare Table XXXVII., we have:

$$N_c \times \frac{I'}{2\,n'_p} = 1{,}900 \times d_a, \quad \ldots\ldots\ldots \quad (85)$$

and since for the total electrical energy of the armature we can write, see formula (136), § 56,

$$P' = E' \times I' = \frac{N_c \times \Phi \times N}{n'_p \times 10^8 \times 60} \times I', \quad \ldots (86)$$

in which P' = total electrical energy generated in dynamo, in watts;
E' = total E. M. F., generated in armature, in volts;
I' = total current generated in armature, in amperes;
N_c = number of armature conductors;
Φ = number of useful lines of force;
N = speed, in revolutions per minute;
n'_p = half number of parallel armature circuits (eventually also number of pairs of poles);

we obtain for the limit of the capacity, by inserting (85) into (86):

$$P' = \frac{1{,}900 \times d_a \times \Phi \times N}{10^8 \times 30} = 63 \times 10^{-8} \times d_a \times N \times \Phi. \quad (87)$$

But the useful flux, Φ, is the product of gap area and field density, or, approximately,

$$\Phi = \frac{d_a \times \pi}{2} \times \beta'_1 \times l_a \times \mathcal{H}'',$$

[1] Kapp, S. P. Thompson's "Dynamo-Electric Machinery," 4th edition, p. 439.

and consequently the safe capacity of a *high-speed* dynamo:

$$P' = 63 \times 10^{-8} \times d_a \times N \times \frac{d_a \times \pi}{2} \times \beta'_1 \times l_a \times \mathcal{H}''$$

$$= 10^{-6} \times d_a^2 \times l_a \times \beta'_1 \times N \times \mathcal{H}''. \quad \ldots\ldots\ldots (88)$$

For *low-speed* machines, 2,500 amperes per inch diameter, or 800 amperes per inch circumference, can safely be allowed, hence, *in order to obtain the safe capacity of a direct-driven machine, the factor 1.33 must be adjoined to formula* (88).

In (88), P' = maximum safe capacity of armature, in watts;
d_a = diameter of armature core, in inches;
l_a = length of armature core, in inches;
β'_1 = percentage of useful gap circumference; to be taken somewhat higher than the percentage of polar arc, to allow for circumferential spread of the lines of force, see Table XXXVIII.;
$\mathcal{H}''$ = field density, in lines of force per square inch;
N = speed, in revolutions per minute.

Inserting into (85) the equivalent limit current density in metric units, of 240 amperes per centimetre circumference (= 765 amperes per centimetre diameter), the maximum safe capacity, in watts, of a *high-speed* armature given in *metric* measure is obtained:

$$P' = \frac{765 \times d_a \times N \times \Phi}{10^8 \times 30}$$

$$= 4 \times 10^{-7} \times d_a^2 \times l_a \times \beta'_1 \times N \times \mathcal{H}, \quad \ldots (89)$$

wherein all dimensions are expressed in centimetres. For *low-speed* machines the factor 4 in this formula must be replaced by 5.33. Average values for β'_1, taken from practice, are given in Table XXXVIII. on the opposite page.

In this table the percentages given for toothed armatures refer to *straight tooth* cores only; for *projecting teeth* a value between the straight tooth and the perforated armature should be taken, proportional to the size of the opening between the tooth projections.

TABLE XXXVIII.—PERCENTAGE OF EFFECTIVE GAP CIRCUMFERENCE FOR VARIOUS RATIOS OF POLAR ARC.

Percentage of Polar Arc. β_1	Percentage of Effective Gap Circumference. β'_1							
	2 Poles.		4 to 6 Poles.		8 to 12 Poles.		14 to 20 Poles.	
	Smooth or Perforated Armature.	Toothed Armature.	Smooth or Perforated Armature.	Toothed Armature.	Smooth or Perforated Armature.	Toothed Armature.	Smooth or Perforated Armature.	Toothed Armature.
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.95	.98	.96	.97	.955	.965	.955	.96	.955
.90	.96	.915	.94	.91	.93	.91	.92	.91
.85	.94	.87	.905	.865	.89	.865	.88	.865
.80	.91	.825	,87	.82	.85	.82	.84	.82
.75	.88	.78	.835	.775	.815	.775	.80	.775
.70	.85	.735	.80	.73	.78	.725	.76	.725
.65	.82	.69	.765	.685	.74	.68	.72	.675
.60	.78	.645	.73	.635	.70	.63	.68	.625
.55	.74	.60	.69	.59	.665	.58	.64	.575
.50	.70	.55	.65	.54	.625	.53	.60	.525

39. Running Value of Armature.

In order to form an idea of the efficiency of an armature as an inductor, its "*running value*" has to be determined.

In forming the quotient of the total energy induced by the product of the weight of copper on the armature and the field density, the number of watts generated per pound of copper at unit field density is obtained, an expression which indicates the relative inducing power of the armature:

$$P'_a = \frac{E' \times I'}{wt_a \times \mathcal{H}''}; \quad \ldots\ldots\ldots\ldots (90)$$

P'_a = running value of armature in watts per unit weight of copper, at unit field density;

E' = total E. M. F. generated in armature, in volts;

I' = total current generated in armature, in amperes;

wt_a = weight of copper in armature, in pounds or in kilogrammes, formula (58);

$\mathcal{H}''$ = field density, in lines of force per square inch, or per square centimetre, respectively.

The value of P'_a for a newly designed armature being found, its relative inductor efficiency can then be judged at by comparison with other machines. The running value of modern dynamos, according to the type of machine and the kind of armature, varies between very wide limits, and the following are the averages for well-designed machines:

TABLE XXXIX.—RUNNING VALUES OF VARIOUS KINDS OF ARMATURES.

TYPE OF MACHINE.		KIND OF ARMATURE.	RUNNING VALUE, P'_a (Watts per unit weight of copper at unit field density.) English Measure. Watts per lb. at 1 line per sq. inch.	Metric Measure. Watts per kg. at 1 line per cm.²
High Speed	Bipolar	Drum	.015 to .03	.045 to .09
		Ring	.01 " .02	.03 " .06
	Multipolar	Drum	.01 " .02	.03 " .06
		Ring	.0075 " .015	.022 " .045
Low Speed	Bipolar	Drum	.0075 " .015	.022 " .045
		Ring	.005 " .01	.015 " .03
	Multipolar	Drum	.005 " .01	.015 " .03
		Ring	.00375 " .0075	.011 " .022

CHAPTER VII.

MECHANICAL EFFECTS OF ARMATURE WINDING.

40. Armature Torque.

The work done by the armature of a dynamo can be expressed in two ways: electrically, as the product of E. M. F. and current strength,

$$P' = E' \times I' \text{ watts;}$$

and mechanically, as the product of circumferential speed and turning moment, or torque,

$$P' = 2\,\pi \times N \times \tau \times \frac{746}{33{,}000} = .142 \times N \times \tau \text{ watts;}$$

P' = total energy developed by machine, in watts;
E' = total E. M. F. generated in armature, in volts;
I' = total current generated in armature, in amperes;
N = speed, in revolutions per minute;
τ = torque, in foot-pounds;
746 = number of watts making one horse power;
33,000 = number of foot-pounds per minute making one horse power.

Equating the above two expressions, we obtain:

$$E' \times I' = .142 \times N \times \tau,$$

from which follows:

$$\tau = \frac{E' \times I'}{.142 \times N} = 7.042 \times \frac{E' \times I'}{N} \text{ foot-pounds. ..}(\mathbf{91})$$

Or, in metric system, 1 kilogramme-metre being = 7.233 foot-pounds,

$$\tau = \frac{7.042}{7.233} \times \frac{E' \times I'}{N} = .974\,\frac{E' \times I'}{N} \text{ kg.-metres. ....}(\mathbf{92})$$

Inserting into (91) and (92) the expression for the E. M. F. from § 56, viz.:

$$E' = \frac{N_c \times \Phi \times N}{n'_p \times 10^8 \times 60},$$

the equation for the torque becomes:

$$\tau = 7.042 \times \frac{N_c \times \Phi \times N}{n'_p \times 10^8 \times 60} \times \frac{I'}{N}$$

$$\left.\begin{aligned} &= \frac{11.74}{10^{10}} \times \frac{I'}{n'_p} \times N_c \times \Phi \text{ foot-pounds} \\ &= \frac{1.625}{10^{10}} \times \frac{I'}{n'_p} \times N_c \times \Phi \text{ kg.-metres} \end{aligned}\right\} \ldots\ldots (93)$$

from which follows that in a given machine the torque depends in nowise upon the speed, but only upon the current flowing through the armature, and upon the magnetic flux.

41. Peripheral Force of Armature Conductors.

By means of the armature torque we can now calculate the drag of the armature conductors in a generator, respectively the pull exerted by the armature conductors in a motor.

The torque divided by the mean radius of the armature winding gives the total peripheral force acting on the armature; and the latter, divided by the number of effective conductors, gives the peripheral force acting on each armature conductor. In English measure, if the torque is expressed in foot-pounds and the radius of the winding in inches, the peripheral force of each conductor is:

$$f_a = \frac{\tau}{\frac{\frac{1}{2}d'_a}{12} \times N_c \times \beta'_1} = \frac{24 \times \tau}{d_a \times N_c \times \beta'_1} \text{ pounds } \textbf{(94)}$$

Inserting into this equation the value of τ from formula (91), we obtain:

$$f_a = \frac{24 \times 7.042 \times \frac{E' \times I'}{N}}{d'_a \times N_c \times \beta'_1}$$

$$= \frac{2 \times 7.042 \times \pi}{60} \times \frac{E' \times I'}{\left(\frac{N}{60} \times \frac{d'_a \times \pi}{12}\right) \times N_c \times \beta'_1},$$

or,

$$f_a = .7375 \times \frac{E' \times I'}{v_c \times N_c \times \beta'_1} \text{ pounds; } \ldots . \textbf{(95)}$$

f_a = peripheral force per armature conductor, in pounds;
$E' \times I'$ = total output of armature, in watts;
v_c = mean conductor velocity, in feet per second;
N_c = total number of armature conductors;
β'_1 = percentage of effective armature conductors, see Table XXXVIII., § 38.

A second expression for the peripheral force can be obtained by substituting in the original equation (94) the value of τ from formula (93), thus:

$$f_a = \frac{24 \times 11.74}{10^{10}} \times \frac{I'}{n'_p} \times \frac{N_c}{N_c \times \beta'_1} \times \frac{\Phi}{d'_a}$$

$$= \frac{2.82}{10^8} \times \frac{I' \times \Phi}{n'_p \times d'_a \times \beta'_1} \text{ pounds.} \quad \ldots\ldots\ldots\ldots (96)$$

Replacing in this the useful flux Φ by its equivalent, the product of gap area and field density, we find a third formula for the peripheral force:

$$f_a = \frac{2.82}{10^8} \times \frac{I' \times d'_a \times \frac{\pi}{2} \times \beta'_1 \times b_a \times \mathcal{H}''}{n'_p \times d'_a \times \beta'_1}$$

$$= \frac{4.43}{10^8} \times \frac{I'}{n'_p} \times l_a \times \mathcal{H}'' \text{ pounds;} \quad \ldots\ldots\ldots\ldots (97)$$

$\frac{I'}{2\,n'_p}$ = total current flowing in each armature conductor, in amperes;
l_a = length of armature core, in inches;
$\mathcal{H}''$ = field density, in lines of force per square inch.

If the dimensions of the armature are given in centimetres, the conductor velocity in metres per second, and the field density in gausses, the peripheral force is obtained in kilogrammes from the following formulæ:

$$f_a = .102 \times \frac{E' \times I'}{v_c \times N_c \times \beta'_1} \text{ kilogrammes,} \quad \ldots (98)$$

$$f_a = \frac{3.25}{10^8} \times \frac{I' \times \Phi}{n'_p \times d'_a \times \beta'} \text{ kilogrammes,} \quad \ldots (99)$$

and

$$f_a = \frac{5.1}{10^8} \times \frac{I'}{n'_p} \times l_a \times \mathcal{H} \text{ kilogrammes,} \quad \ldots (100)$$

which correspond to (95), (96) and (97), respectively.

It is on account of this peripheral force exerted by the magnetic field upon the armature conductors that there is need of a good positive method of conveying the driving power from the shaft to the conductors, or *vice versa;* in the generator it is the conductors, and not the core discs, that have to be driven; in the motor it is they that drive the shaft. Thus the construction of the armature is aggravated by the condition that, while the copper conductors must be mechanically connected to the shaft in the most positive way, yet they must be electrically insulated from all metallic parts of the core. In drum armatures the centrifugal force still more complicates matters in tending to lift the conductors from the core; in smooth drum armatures it has therefore been found necessary to employ driving horns, which either are inserted into nicks in the periphery of the discs, or are supported from hubs keyed to the armature shaft at each end of the core. In ring armatures the centrifugal force presses the conductors at the inner circumference toward the armature core, and thus helps to drive, while the spider arms, by interlocking into the armature winding, serve as driving horns. If toothed discs are used, no better means of driving can be desired.

42. Armature Thrust.

If the field frame of a dynamo is not symmetrical, which is particularly the case in most of the bipolar types (see Figs. 77 to 85), unless special precautions are taken there will be a denser magnetic field at one side of the armature than at the other, and an attractive force will be exerted upon the armature, resulting in an armature thrust toward the side of the denser field.

The force with which the armature would be attracted, if only one-half of the field were acting, is:

$$f = 2\,\pi \times \frac{S_g}{2} \times \left(\frac{\mathcal{H}}{4\,\pi}\right)^2 = .0199 \times S_g \times \mathcal{H}^2 \text{ dynes,}$$

or, since 981,000 dynes = 1 kilogramme,

$$f = \frac{.0199}{981,000} \times S_g \times \mathcal{H}^2 = 2.03 \times 10^{-8} \times S_g \times \mathcal{H}^2 \text{ kilogrammes;}$$

S_g = gap area, in square centimetres;
$\mathcal{H}$ = field density, in lines of force per square centimetre.

Expressing the gap area by the dimensions of the armature, we obtain:

$$f = 2.03 \times 10^{-8} \times \frac{d_a \times \pi}{2} \times l_a \times \beta'_1 \times \mathcal{H}^2$$

$$= 32 \times 10^{-9} \times d_a \times l_a \times \beta'_1 \times \mathcal{H}^2 \text{ kilogrammes.} \quad ..\textbf{(101)}$$

If, now, both halves of the field are in action, but one half is stronger than the other, the armature will be acted upon by two forces:

$$f_1 = 32 \times 10^{-9} \times d_a \times l_a \times \beta'_1 \times \mathcal{H}_1^2 \text{ kilogrammes,}$$

and

$$f_2 = 32 \times 10^{-9} \times d_a \times l_a \times \beta'_1 \times \mathcal{H}_2^2 \text{ kilogrammes,}$$

and will be drawn toward the stronger side by the amount of the difference of their attractive forces. The armature thrust, therefore, is:

$$f_t = f_1 - f_2 = 32 \times 10^{-9} \times d_a \times l_a \times \beta'_1 \times (\mathcal{H}_1^2 - \mathcal{H}_2^2) \text{ kg.}; \quad ...\textbf{(102)}$$

f_t = thrusting force acting on armature, due to unsymmetrical field, in kilogrammes;
d_a = diameter of armature, in centimetres;
l_a = length of armature, in centimetres;
β'_1 = percentage of effective gap-circumference, see Table XXXVI.;
$\mathcal{H}_1$ = density of field, on stronger side, lines per square centimetre;
$\mathcal{H}_2$ = density of field, on weaker side, lines per square centimetre.

In English measure, 1 pound being = .4536 kilogrammes, and 1 square inch = 6.4515 square centimetres, the formula for the armature thrust becomes:

$$f_t = \frac{32 \times 10^{-9}}{6.4515 \times .4536} \times d_a \times l_a \times \beta'_1 \times (\mathcal{H}''^2_1 - \mathcal{H}''^2_2)$$

$$= 11 \times 10^{-9} \times d_a \times l_a \times \beta'_1 \times (\mathcal{H}''^2_1 - \mathcal{H}''^2_2) \text{ pounds,} \quad ....\textbf{(103)}$$

in which d_a and l_a are to be expressed in inches, and $\mathcal{H}''_1$ and $\mathcal{H}''_2$ in lines per square inch.

In such types, where the attractive force of the field manifests itself as a downward thrust, as in those shown in Figs.

78, 80, 82 and 85, the value obtained by (102) or (103), respectively, is to be added to the dead weight of the armature, in order to obtain the total down thrust upon the bearings. If, however, f_t is an upward thrust, as is indicated in Figs. 77,

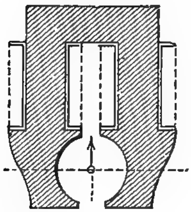


Fig. 77.

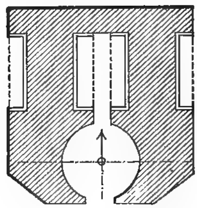


Fig. 78.

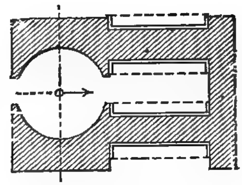


Fig. 79.

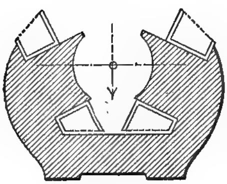


Fig. 80.

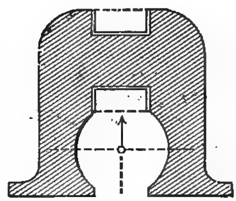


Fig 81.

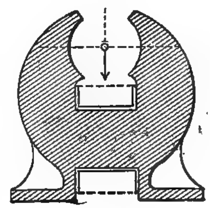


Fig. 82.

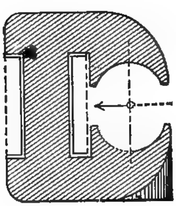


Fig. 83.

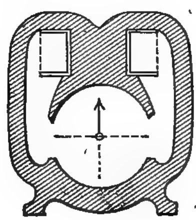


Fig. 84.

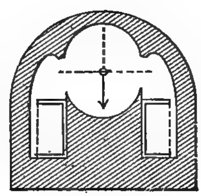


Fig. 85.

Figs. 77 to 85.—Unsymmetrical Bipolar Fields.

81 and 84, the down thrust upon the bearings is the weight of the armature, diminished by the amount of f_t. In the cases illustrated by Figs. 79 and 83 the action of the field causes a sideward thrust, which has to be taken care of by a proper design of the bearing pedestals, or of the journal brackets.

CHAPTER VIII.

ARMATURE WINDING OF DYNAMO-ELECTRIC MACHINES.

43. Types of Armature Winding.

a. Closed Coil Winding and Open Coil Winding.

If, in a continuous current dynamo, the reversal of the current would take place in all the conductors at once, considerable fluctuation of the E. M. F. would be the result. In order to obtain a steady current, the armature conductors are, therefore, to be so arranged relative to the poles, that a portion of them is in the strongest part of the field, while others are exposed to a weaker field, and some even are in the neutral position.

After having thus arranged the conductors, their connecting can be effected by one of the following two methods:

(I.) All conductors are connected among each other so as to form an endless winding, closed in itself, and consisting of two or more parallel branches, in each of which all the single E. M. Fs. induced have the same direction, and in which the reversal of the current occurs in such conductors only that at the time are in the neutral position. An armature with such connections is called a *closed coil armature* (Fig. 86).

(II.) The conductors are joined into groups, each group containing all such conductors in series which, relative to the field, have exactly the same position; and the current is taken off from such groups only which at the time have the maximum, or nearly the maximum, E. M. F., all other groups being at that time cut out altogether. An armature wound in this manner is styled an *open coil armature* (Fig. 87).

Although in a closed coil armature the sum of all the E. M. Fs. of the single coils is collected by the brushes (see § 9), while in an open coil armature the E. M. F. of one group of coils only is delivered to the external circuit, and although, therefore, the total E. M. F. output of an armature is smaller when connected up in the open coil fashion than it would be if

the same armature were run at the same speed but connected by the closed coil method, yet an open coil armature offers great advantages in case of high potential machines, as there is no difference of potential between adjoining commutator bars belonging to different groups of coils and only a small

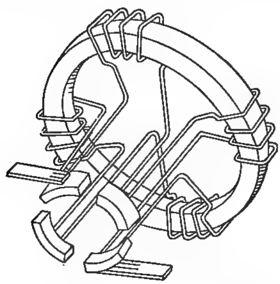


Fig. 86.—Closed Coil Winding.

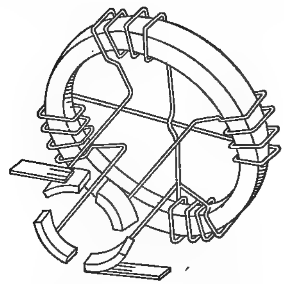


Fig. 87.—Open Coil Winding.

number of segments is required to bring the fluctuation of the E. M. F. within practical limits. Open coil armatures are therefore preferable to closed coil ones in case of machines for series arc lighting, where, if closed coil windings are employed, a great number of commutator segments is required on account of the high total potential around the commutator. (See Table XXI., § 25.)

b. Spiral Winding, Lap Winding, and Wave Winding.

According to the manner in which the connecting of the conductors by the above two methods is performed, the following types of armature windings can be distinguished:

(1) *Spiral Winding*, or *Ring Winding*, Figs. 88 and 89;

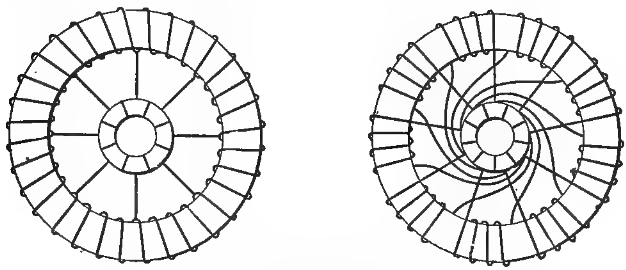
(2) *Lap Winding*, or *Loop Winding*, Figs. 90 and 91;

(3) *Wave Winding* or *Zigzag Winding*, Figs. 92 and 93.

In the *spiral winding*, Figs. 88 and 89, which can be applied in the case of ring armatures only, the connecting conductors are carried through the interior of the ring core, and the winding thus constitutes either one continuous spiral, Fig. 88, from which, at equal intervals, branch connections are led to the commutator, or a set of independent spirals, Fig. 89, which are separately connected to the commutator.

The *lap winding*, as well as the *wave winding*, is executed entirely exterior to the core, and can be applied to both drum and ring armatures.

In the *lap winding*, Figs. 90 and 91, the end of each coil,



Figs. 88 and 89.—Spiral Windings.

consisting of two or more conductors situated in fields of opposite polarity, is connected through a commutator segment to the beginning of a coil lying within the arc embraced by the former. With reference to the direction of connecting,

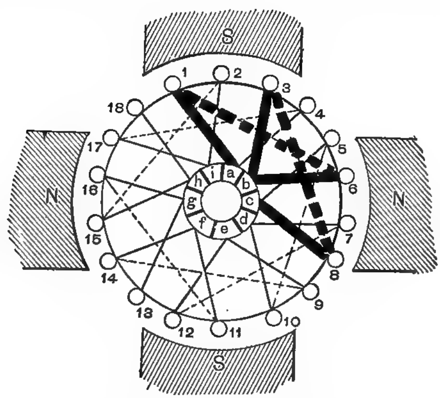


Fig. 90.—Lap Winding.

therefore, the beginning of every following coil lies back of the end of the foregoing, and the winding, consequently, forms a series of loops, which overlap each other. Fig. 90 represents such a lap winding for a four-pole drum armature, the development of which, Fig. 91, more clearly shows the forming of the loops and the manner of their overlapping.

In the *wave winding*, Figs. 92 and 93, the connecting continually advances in one direction, the end of each coil being connected to the beginning of the one having a corresponding position under the next magnet pole; and the winding, in con-

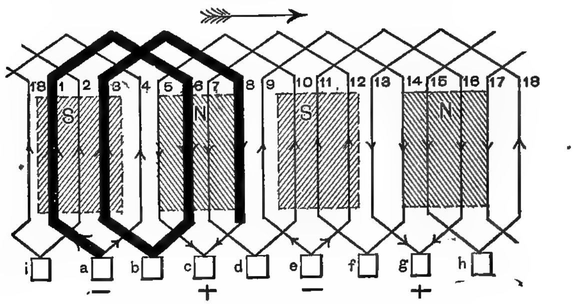


Fig. 91.—Development of Lap Winding.

sequence, represents itself in a zigzag, or wave shape. The wave winding is illustrated in Fig. 92, and for better comparison the same four-pole drum armature is chosen that in Fig.

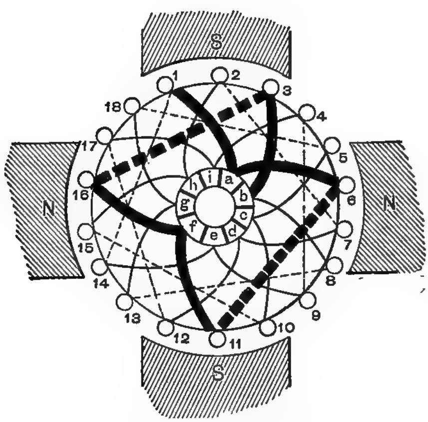


Fig. 92.—Wave Winding.

90 is shown with a lap winding. The development given in Fig. 93 distinctly shows the zigzag form of the wave winding.

In multipolar machines, the wave winding can be used for series as well as for parallel connection; the lap winding, however, for parallel grouping only.

While the lap winding necessitates as many sets of brushes as there are magnet poles, the wave winding for any number of poles invariably needs but two sets of brushes.

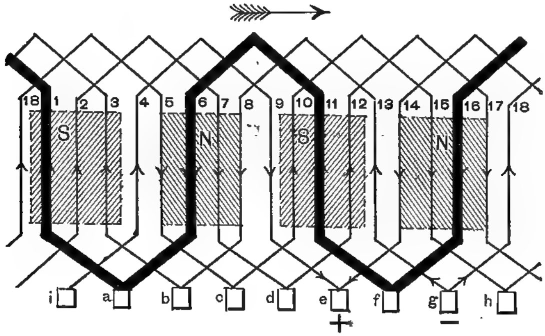


Fig. 93.—Development of Wave Winding.

For series-parallel connection, either wave winding may be used or lap and wave windings may be combined. Fig. 94 represents the development of such a "*mixed winding*," the

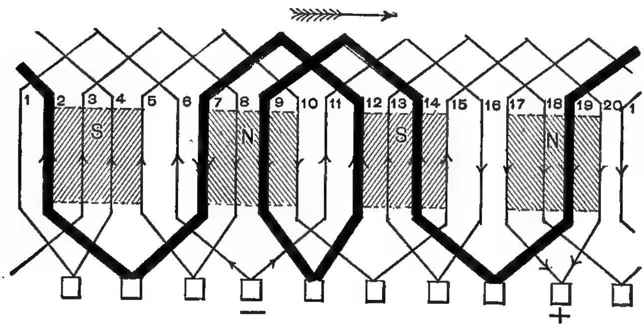


Fig. 94.—Development of " Mixed " Winding.

coils partly being connected in the lap and partly in the wave fashion. This winding, like the wave winding, has the peculiarity of requiring but two sets of brushes, independently of the number of magnet poles.

44. Grouping of Armature Coils.

When the conductors of a continuous current armature, as is usually the case, are to be so connected among each other

that the completed arrangement forms one continuous winding, or several separate continuous windings re-entrant on themselves, that is, when the armature is to have a closed coil winding, then, according to the voltage desired, the different conductors, or coils, may be grouped in two or more parallel circuits. In bipolar machines having the armature coils arranged in a single re-entrant winding, there can be but two such paths, the current bifurcating only once, dividing at the negative brush and reuniting at the positive brush as it leaves the armature. In multipolar dynamos, however, there may be more than one bifurcation in each re-entrant winding, and the current, therefore, split up into more than two electrically parallel paths. When there is but one bifurcation of the current, independent of the number of poles, the armature is said to have a *series grouping*, or a *two-circuit winding;* but when there are as many parallel branches in each winding as there are poles in the field-frame, the armature coils are said to be arranged in *parallel grouping*, or in *multiple circuit winding;* finally, when the number of the bifurcations of the current in each winding is greater than one and less than the number of poles, we have a combination of the above two methods, and the coils are arranged in what is called *series-parallel* or *mixed grouping.*

An armature with series grouping requires but two sets of brushes at neutral points of the commutator, while one with parallel grouping of the coils needs as many sets of brushes as there are poles. The number of brushes required with a mixed winding is greater than two and less than the number of poles, and is given by the number of parallel branches in each of the re-entrant windings.

While closed coil armatures usually form but one single re-entrant winding, in armatures for very large current output in which a difficulty in commutation is likely to arise if but one winding is employed, it is of advantage to have two or more distinct re-entrant windings, each connected to its own set of commutator bars, all the sets being interleaved in one commutator. The current from such armatures is collected by very thick brushes covering two or more consecutive commutator bars, or by sets of several thin brushes, connected in parallel with each other so as to virtually form thick brushes. If only

one commutator bar under each brush actually commutates the entire armature current, as shown in Fig. 95, the winding is called a *simplex*, or a *single*, or an *ordinary winding;* if, however, the coils are so grouped in a number of independent single windings that the current is commutated at several different parts of the contact surface of the brush, each independent volume of the current being a corresponding fraction of what it would be for a simplex winding, then we have a *mul-*

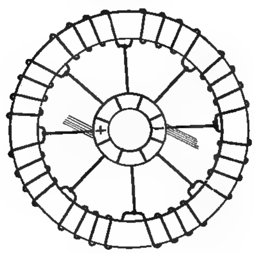


Fig. 95.—Diagram of Simplex Winding.

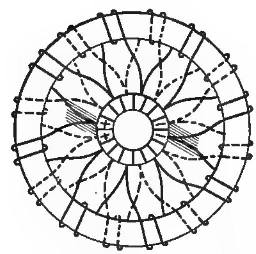


Fig. 96.—Diagram of Duplex Winding.

tiplex or a *multiple winding*. Fig. 96 gives the diagram of a *duplex* or *double winding*, having two commutating segments under each brush. If there are three points of commutation under each brush, corresponding to three independent re-entrant windings, we have a *triplex* or a *triple winding*, and so forth.

In a multiplex winding the successive commutator bars of one winding are not adjacent to each other, but alternate with the bars of the other windings, the result being that a section is very unlikely to be short-circuited by dirt or by an arc. The winding is very flexible owing to the readiness with which any number of independent parallel circuits can be arranged. The division of what would otherwise be a very heavy conductor into several smaller conductors, also has the effect of reducing the eddy current loss in the armature winding.

Considering a simplex winding, according to the grouping of the conductors or coils, the endless winding, when starting from any point within itself, has to be followed either once or more times around the armature in order to return to the

starting point. If the whole winding, following coil by coil in the order as actually connected, can be gone over in but one passage around the armature, Fig. 97, the winding is said to be *singly re-entrant*, if the armature has to be encircled twice to

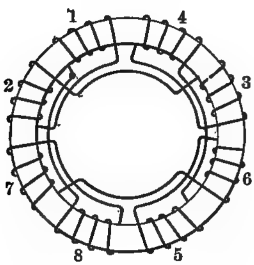


Fig. 97.—Singly Re-entrant Winding.

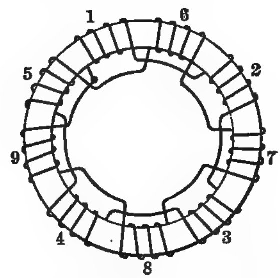


Fig. 98.—Doubly Re-entrant Winding.

return to the starting point, as in Fig. 98, we have a *doubly re-entrant winding*, and so on.

Using a circle ○ as the symbol for a single re-entrancy, a single loop (o) for a double re-entrancy, a double loop (oo) for a triple re-entrancy, and so forth, we can indicate the different kinds of windings as follows:

TABLE XL.—SYMBOLS FOR DIFFERENT KINDS OF ARMATURE WINDINGS.

KIND OF WINDING	SINGLY REENTRANT	DOUBLY REENTRANT	TRIPLY REENTRANT	QUADRUPLY REENTRANT
SIMPLEX WINDING	○	(o)	(oo)	(ooo)
DUPLEX WINDING	○○	(o)(o)	(oo)(oo)	(ooo)(ooo)
TRIPLEX WINDING	○○○	(o)(o)(o)	(oo)(oo)(oo)	(ooo)(ooo)(ooo)
QUADRUPLEX WINDING	○○○○	(o)(o)(o)(o)	(oo)(oo)(oo)(oo)	(ooo)(ooo)(ooo)(ooo)

According to the manner of grouping and to the kind of winding chosen, the voltage generated in the armature by a certain number of conductors (100), at a certain flux (1 mega-line = 1,000,000 lines of force, per pole) and at a certain cutting speed (100 revolutions per minute), varies between the following limits:

TABLE XLI.—E. M. F. GENERATED IN ARMATURE, AT VARIOUS GROUPING OF CONDUCTORS.

Number of Poles.	Volts Generated per 100 Conductors, per 100 Revolutions per min. and 1 Megaline Flux per pole. e_2						Average Volts Between Commutator Segments per Megaline and per 100 Revs. p. m. (Independent of Number of Conductors). e_3					
	Series Grouping.			Parallel Grouping.			Series Grouping.			Parallel Grouping.		
	Simplex Winding.	Duplex Winding.	Triplex Winding.	Simplex Winding.	Duplex Winding.	Triplex Winding.	Simplex Winding.	Duplex Winding.	Triplex Winding.	Simplex Winding.	Duplex Winding.	Triplex Winding.
2	1.667	.833	.556	1.667	.833	.556	.067	.033	.022	.067	.033	.022
4	3.333	1.667	1.111	1.667	.833	.556	.267	.133	.089	.133	.067	.044
6	5.000	2.500	1.667	1.667	.833	.556	.600	.300	.200	.200	.100	.067
8	6.667	3.333	2.222	1.667	.833	.556	1.067	.533	.356	.267	.133	.089
10	8.333	4.167	2.778	1.667	.833	.556	1.667	.833	.556	.333	.167	.111
12	10.000	5.000	3.333	1.667	.833	.556	2.400	1.200	.800	.400	.200	.133
14	11.667	5.833	3.889	1.667	.833	.556	3.267	1.633	1.089	.467	.233	.156
16	13.333	6.667	4.444	1.667	.833	.556	4.267	2.133	1.422	.533	.267	.178

Designating the voltages in columns 2 to 7 of this table by e_2, the number of conductors required for any particular case can be calculated from :

$$N_c = \frac{E \times 10^{10}}{e_2 \times N \times \Phi}, \quad \ldots\ldots\ldots\ldots (104)$$

in which N_c = number of armature conductors;

E = E. M. F. to be generated;

e_2 = volts per 100 conductors per 100 revolutions per minute and 1 megaline per pole, see Table XLI.;

N = speed, in revolutions per minute;

Φ = useful flux per pole, in webers.

The average voltage between adjoining commutator-bars can be found from

$$\left(\frac{E \times 2\, n'_p}{n_c}\right) = e_3 \times N \times \Phi \times 10^{-8}, \quad \ldots (105)$$

where e_s is the average voltage between segments at 100 revolutions per minute and at 1 megaline flux, as given in columns 8 to 13 of Table XLI.

45. Formula for Connecting Armature Coils.

a. Connecting Formula and its Application to the Different Methods of Grouping.

A general formula for connecting the conductors of a closed coil armature has been given by Arnold[1] as follows:

If N_c = number of conductors arranged around armature core;
n_a = number of conductors per commutator segment;
n'_p = number of bifurcations of current in armature;
$n'_p = 1$, single bifurcation, or 2 parallel circuits.
$n'_p = 2$, double bifurcation, or 4 parallel circuits, etc.
n_p = number of pairs of magnet poles;
y = "pitch," or "spacing" of armature winding; *i. e.*, the numerical step by which is to be advanced in connecting the armature conductors;

then the number of armature conductors can be expressed by

$$N_c = n_a \times (n_p \times y \pm n'_p),$$

from which follows the connecting formula for any armature:

$$y = \frac{1}{n_p} \times \left(\frac{N_c}{n_a} \mp n'_p \right). \quad \ldots\ldots\ldots \textbf{(106)}$$

The general rule, then, for connecting any armature is:

Connect the end (beginning) of any coil, x, of the armature to the beginning (end) of the $(x + y)^{\text{th}}$ *coil.*

For the various methods of grouping the armature coils, the above formula is applied as follows:

I. *Parallel Grouping.*—In this method of connecting there are as many parallel armature branches as there are poles, viz. $2\,n_p$ circuits, or n_p bifurcations. Spiral winding, lap winding, and wave winding may be applied:

(1) *Spiral Winding and Lap Winding.*—In this case the multipolar armature is considered as consisting of n_p bipolar

[1] E. Arnold, "Die Ankerwicklungen der Gleichstrom Dynamomaschinen." Berlin, 1891.

ones, and independently of the number of poles, $n_p = 1$ and $n'_p = 1$ is to be inserted in (106), and the formula applied to a set of conductors lying between two poles of the same polarity.

(2) *Wave Winding.*—Here the actual number of pairs of poles, n_p, and the actual number of bifurcations, $n'_p = n_p$, is to be introduced in (106), and the formula applied to the entire number of conductors.

II. *Series Grouping.*—This is characterized by having but two parallel armature circuits, or one bifurcation, no matter what the number of poles may be; for series connecting, therefore, we have $n'_p = 1$.

In the special case of $n_p = 1$, *bipolar* dynamos, the series connecting is identical with the parallel grouping, and the winding may be either a lap winding (spiral winding) or a wave winding; the latter holds good also for $n_p = 2$; *i. e.*, for *four-polar* machines. For dynamos with more than four poles, $n'_p > 2$, however, series grouping is only possible by means of wave winding.

III. *Series Parallel Grouping.*—In the mixed grouping the number of bifurcations is greater than 1, and must be less than n_p; hence, in the connecting formula we have $n'_p > 1$ and $n'_p < n_p$.

In this case there are either several circuits closed in itself, with separate neutral points on the commutator, or one single closed winding with n'_p parallel branches. The latter is the case if y and $\frac{N_c}{n_a}$ are prime to each other; the former if they have a common factor; this factor, then, indicates the number of independent circuits.

b. Application of Connecting Formula to the Various Practical Cases.

I. *Bipolar Armatures.*

(1) For any bipolar armature the number of pairs of poles, as well as the number of bifurcations is $= 1$; furthermore, the number of coils per commutator-bar is usually $= 1$; consequently $n_a = 1$, if in the connecting formula the number of conductors, N_c, is replaced by the number of coils, n_c. For ordinary bipolar armatures, therefore:

$$n_p = 1,\ n_a = 1,\ n'_p = 1; \ldots\ldots\ldots y = n_c \mp 1. \quad \ldots\ldots (107)$$

(2) If the number of commutator segments is half the number of armature coils, *i. e.*, two coils per commutator-bar, then

$$n_p = 1,\ n_a = 2,\ n'_p = 1; \ldots\ldots y = \frac{n_c}{2} \mp 1 \ldots\ldots \quad (108)$$

II. *Multipolar Armatures with Parallel Grouping.*

(1) By multiplying the bipolar method of connecting, we have:

$$n_p = 1,\ n_a = 1,\ n'_p = 1; \ldots\ldots y = n_c \mp 1. \quad \ldots (109)$$

This is a spiral winding; beginning and end of neighboring coils are connected with each other, and a commutator connection made between each two coils. The number of sets of brushes is $2\,n_p$.

For multipolar parallel connection and spiral winding with but two sets of brushes, either n_c divisions may be used in the commutator, and the bars, symmetrically situated with reference to the field, cross-connected into groups of n_p bars each, or only $\frac{n_c}{n_p}$ segments may be employed, and n_p coils of same relative position to the poles connected to each bar by means of n_p separate connection wires.

(2) In connecting after the wave fashion by joining coils of similar positions in different fields to the same commutator segments, the following formula is obtained:

$$n_p = n_p,\ n_a = 1,\ n'_p = n_p; \ldots\ldots$$

$$y = \frac{1}{n_p}(n_c \mp n_p) = \frac{n_c}{n_p} \mp 1. \quad \ldots\ldots (110)$$

If y and n_c have a common factor, this method of connecting furnishes several distinct circuits closed in itself, the common factor indicating their number.

(3) If n_p similarly situated coils are connected in series between each two consecutive commutator bars, only $\frac{n_c}{n_p}$ segments, but $2\,n_p$ sets of brushes are needed; the winding is of the wave type, and the connecting formula becomes:

mixed wire

$$n_p = n_p,\ n_a = n_p,\ n'_p = n_p; \ldots\ldots$$

$$y = \frac{1}{n_p}\left(\frac{n_c}{n_p} \mp n_p\right) = \frac{n_c}{n_p^2} \mp 1. \quad \ldots\ldots (111)$$

III. *Multipolar Armatures with Series Grouping.*

(1) If all symmetrically situated coils exposed to the same polarity, by joining the commutator segments into groups of n_p bars each, are connected to each other, they can be considered as one single coil, and we obtain:

$$n_p = n_p,\ n_a = 1,\ n'_p = 1;\ \dots y = \frac{1}{n_p}\ (n_c \mp 1). \quad \dots (112)$$

Each brush, in this case, short circuits n_p coils simultaneously.

The same formula holds good, if beginning and end of every coil are connected to a commutator-bar each. The latter can always be done if n_p is an uneven number; but if n_p is even, the number of coils, n_c, must be odd. In the case of n_p uneven, if n_c is even, the brushes embrace an angle of 180°; but if n_c is odd, an angle of only $\frac{180^\circ}{n_p}$ is inclosed by the brushes.

(2) Instead of cross-connecting the commutator, the winding itself can be so arranged that only $\frac{n_c}{n_p}$ bars are required. In this case the connections have to be made by the formula:

$$n_p = n_p,\ n_a = n_p,\ n'_p = 1;\dots y = \frac{1}{n_p}\left(\frac{n_c}{n_p} \mp 1\right). \quad (113)$$

NOTE.—In drum armatures the beginning and end of a coil being situated in different portions of the circumference, they should be numbered alike, and yet marked differently, in order to facilitate the application of the above connecting formulæ. By designating the beginnings of the coils by 1, 2, 3, ..., and the ends by 1', 2', 3', ..., this distinction is attained.

46. Armature Winding Data.

a. Series Windings for Multipolar Machines.

While a *parallel* winding for a multipolar armature is always possible if the number of coils is even, the possibility of a *series* winding depends upon the relation between the number of poles and the number of conductors per armature division, or the number of conductors per slot in case of a toothed or perforated armature, respectively. In the following Table XLII., which is compiled from data contained in Parshall and

Hobart's work,[1] the various kinds of series windings possible for different cases are given, the symbols shown in Table XL., § 44, being employed:

TABLE XLII.—KINDS OF SERIES WINDINGS POSSIBLE FOR MULTIPOLAR MACHINES.

Conductors per Armature Division (or per Slot)	Kind of Series Winding	SERIES WINDINGS possible for various numbers of Poles						
		4 Poles	6 Poles	8 Poles	10 Poles	12 Poles	14 Poles	16 Poles
1	Simplex	○	○	○	○	○	○	○
	Duplex	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@
	Triplex	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)
2	Simplex	○	○	○	○	○	○	○
	Duplex	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@
	Triplex	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)
4	Simplex		○		○		○	
	Duplex	°@ °@	@ @	°@ °@	@ @	°@ °@	@ @	°@ °@
	Triplex		°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)	
6	Simplex	○		○	○		○	○
	Duplex	°@ °@		°@ °@	°@ °@		°@ °@	°@ °@
	Triplex	(@@)(@@)(@@)	°(@@) °(@@) °(@@)	(@@)(@@)(@@)	(@@)(@@)(@@)	°(@@) °(@@) °(@@)	(@@)(@@)(@@)	(@@)(@@)(@@)
8	Simplex		○		○		○	
	Duplex	○○	@ @		@ @	○○	@ @	
	Triplex		°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)	
10	Simplex	○	○	○		○	○	○
	Duplex	°@ °@	°@ °@	°@ °@		°@ °@	°@ °@	°@ °@
	Triplex	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)
12	Simplex				○		○	
	Duplex	°@ °@		°@ °@	@ @		@ @	°@ °@
	Triplex		°(@@) °(@@) °(@@)		(@@)(@@)(@@)		(@@)(@@)(@@)	
14	Simplex	○	○	○	○	○		○
	Duplex	°@ °@	°@ °@	°@ °@	°@ °@	°@ °@		°@ °@
	Triplex	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)	°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)
16	Simplex		○		○		○	
	Duplex	○○	@ @		@ @	○○	@ @	
	Triplex		°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)		°(@@) °(@@) °(@@)	

○ = Singly reentrant Simplex Winding
°@ °@ = " " Duplex "
@ @ = Doubly " Duplex "
°(@@) °(@@) °(@@) = Triply " Triplex "
(@@)(@@)(@@) = " " Triplex "

[1] "Armature Windings for Electric Machines," H. F. Parshall and H. M. Hobart, New York, 1895.

b. Qualification of Number of Conductors for the Various Windings.

The approximate number of conductors for the generation of a certain E. M. F. being calculated from formula (104) and Table XXXIX., it is important to find the accurate number which is qualified to give correct connections for the desired kind of winding. In the following, practical rules and a number of tables are given for the various cases.

(1) *Simplex Series Windings.*—Simplex series windings may be arranged either so that coils in *adjacent fields*, or so that coils in *fields of same polarity* are connected to each other. In

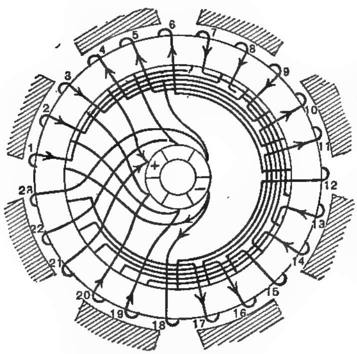


Fig. 99.—Short Connection Type Series Winding.

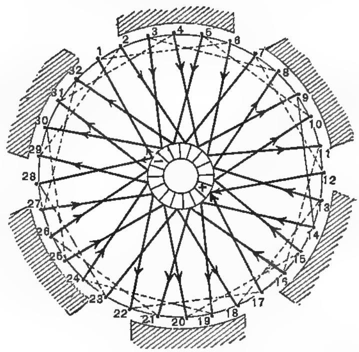


Fig. 100.—Long Connection Type Series Winding.

the former case, which is sometimes called the *short connection type* of series winding, each of the two armature circuits is influenced by all the poles; in the latter case, which is similarly styled the *long connection type* of series winding, each circuit is controlled by only half the number of poles. In the former, therefore, the E. M. Fs. of the two circuits are always equal, in the latter only then when the sum of all the lines of one polarity is equal to that of the other; a condition which, however, is fulfilled in all well designed machines.

In Fig. 99 a winding of the first kind, and in Fig. 100 one of the second kind is shown.

The formula controlling simple series windings is:

$$N_c = 2\ (n_p\, y \pm 1), \text{ for drum armatures,}$$

$$\text{and } n_c = n_p\, y \pm 1, \quad \text{for ring armatures;}$$

in which:

N_c = number of conductors;
n_c = number of coils;
n_p = number of pairs of poles;
y = average pitch.

While for the *short connection type* there are as many commutator segments as there are coils, in a ring armature, or half as many as there are conductors, in a drum armature, the number of commutator-bars for the *long connection type* of series winding is

$$\frac{n_c \pm 1}{n_p}.$$

It is preferable to have the pitch y the same at both ends, in order to have all end connections of same length, but the number of conductors is less restricted (when $n_p > 2$), if the front and back pitches differ by 2. Each pitch must be an *odd* number, so, in order that the winding passes through all conductors before returning upon itself, it must pass alternately through odd and even numbered conductors. Also when the bars, as is usually the case, occupy two layers, it is necessary to connect from a conductor of the upper to one of the lower layer, so as to obviate interference in the position of the spiral end connections.

The following Table XLIII., page 159, gives formulæ for the number of conductors for which simplex series windings are possible in various cases, and also gives the pitches for properly connecting the conductors among each other. The formulæ given refer to *drum* armatures, but can be used for *ring* armatures by replacing in every case half the number of conductors,

$$\frac{N_c}{2},$$

by the number of coils, n_c.

Example, showing use of Table XLIII.: A 6-pole simplex series-wound drum armature is to yield 1.25 volt of E. M. F. at 3,000 revolutions per minute, with a flux of 27,000 webers per pole. How many conductors are required, and how are they to be connected?

From (104) and Table XLI. we have

$$N_c = \frac{1.25 \times 10^{10}}{5 \times 3{,}000 \times 27{,}000} = 31,$$

and Table XLIII. shows that the number of conductors in this

TABLE XLIII.—NUMBER OF CONDUCTORS AND CONNECTING PITCHES FOR SIMPLEX SERIES DRUM WINDINGS.

Number of Poles. $2n_p$	Qualification of Number of Conductors, N_c. Equation for N_c *	Degree of Evenness.	Description.	Average Pitch.†	Front Pitch.‡	Back Pitch.‡
2	$N_c = 2x \pm 2$	N_c even	Any even number not divisible by 3.	$y = \frac{N}{2} - 1$ $y' = \frac{N_c}{2} + 1$	y y'	y y'
4	$N_c = 4x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number, *i. e.*, any odd multiple of 2.	$y = \frac{1}{2}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$
6	$N_c = 6x \pm 2$	N_c even	Any even number not a multiple of 3.	$y = \frac{1}{3}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$
8	$N_c = 8x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number.	$y = \frac{1}{4}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$
10	$N_c = 10x \pm 2$	N_c even	Any even number, having either 2 or 3 as remainder when divided by 5, *i. e.*, any number having a 2 or an 8 as the unit digit.	$y = \frac{1}{5}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$
12	$N_c = 12x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number not divisible by 3.	$y = \frac{1}{6}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$
14	$N_c = 14x \pm 2$	N_c even	Any even number having either 2 or 5 as remainder when divided by 7.	$y = \frac{1}{7}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$
16	$N_c = 16x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number having either 2 or 14 as remainder when divided by 16.	$y = \frac{1}{8}\left(\frac{N_c}{2} \pm 1\right)$	y $y'-1$	y $y'+1$

* General formula: $N_c = 2n_p x \pm 2$; $2n_p$ = number of poles, x = any integer.

† For ring armatures replace $\frac{N_c}{2}$ by n_c (number of coils).

‡ The front and back pitches must always be *odd* numbers. If the *average* pitch, y, is odd, both the front and back pitches are equal to y; but if y is *even*, then the front pitch is $y - 1$, and the back pitch $= y + 1$. If the average pitch is *either* odd (y) or even (y'), according to whether the + or − sign in the formula is used, then *two* connections are possible, one having the pitches y, y, and the other the pitches $y' - 1$, $y' + 1$.

case must fulfill the condition $N_c = 6x \pm 2$, which, for $x = 5$, and for the $+$ sign makes

$$N_c = 30 + 2 = 32 .$$

The same table gives the average pitch

$$y = \frac{1}{3}\left(\frac{32}{2} \pm 1\right) = 5 ,$$

from which follows that at both ends of the armature each conductor is to be connected to the sixth following (see Fig. 99, page 157).

(2) *Multiplex Series Windings.*—In case of multiplex series drum windings the number of conductors must be

$$N_c = 2\,(n_p\, y \pm n_m),$$

TABLE XLIV.—NUMBER OF CONDUCTORS AND CONNECTING PITCHES FOR DUPLEX SERIES DRUM WINDINGS.

Number of Poles $2n_p$	Kind of Series Winding.‖	Qualification of Number of Conductors, N_c: Equation for N_c.†	Degree of Evenness.	Description.	Average Pitch.‡	Front Pitch.§	Back Pitch.¶
2	○ ○	$N_c = 4x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number.	$y = \frac{N_c}{2} \pm 2$	y, y'	y, y'
	◎ ◎	$N_c = 4x \pm 4$	$\frac{N_c}{2}$ even	Any multiple of 4.	$y = \frac{N_c}{2} \pm 2$	$y-1$, $y'-1$	$y+1$, $y'+1$
4	○ ○	$N_c = 8x$	$\frac{N_c}{4}$ even	Any multiple of 8.	$y = \frac{1}{2}\left(\frac{N_c}{2}+2\right)$, $y' = \frac{1}{2}\left(\frac{N_c}{2}-2\right)$	y, y'	y, y'
	◎ ◎	$N_c = 8x \pm 4$	$\frac{N_c}{4}$ odd	Any quadruple of an odd integer.	$y = \frac{1}{2}\left(\frac{N_c}{3}+2\right)$, $y' = \frac{1}{2}\left(\frac{N_c}{2}-2\right)$	$y-1$, $y'-1$	$y+1$, $y'+1$
6	○ ○	$N_c = 12x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number, not divisible by 3.	$y = \frac{1}{3}\left(\frac{N_c}{2} \pm 2\right)$	y	y
	◎ ◎	$N_c = 12x \pm 4$	$\frac{N_c}{2}$ even	Any multiply even number (even multiple of 2) not divisible by 3.		$y-1$	$y+1$
8	○ ○, ◎ ◎	$N_c = 16x \pm 4$	$\frac{N_c}{4}$ odd	Any quadruple of an odd integer.	$y = \frac{1}{4}\left(\frac{N_c}{2}-2\right)$, $y' = \frac{1}{4}\left(\frac{N_c}{2}+2\right)$	y, $y'-1$	y, $y'-1$

TABLE XLIV.—NUMBER OF CONDUCTORS AND CONNECTING PITCHES FOR DUPLEX SERIES DRUM WINDINGS.—*Continued.*

Number of Poles. $2 n_p$.	Kind of Series Winding.*	Qualification of Number of Conductors, N_c. Equation for N_c.†	Degree of Evenness.	Description.	Average Pitch.‡	Front Pitch §	Back Pitch §
10	○ ○	$N_c = 20\,x \pm 6$	$\frac{N_c}{2}$ odd	Any singly even number having a 4 or a 6 as its unit digit.	$y = \frac{1}{5}\left(\frac{N_c}{2} \pm 2\right)$	y	y
	◎ ◎	$N_c = 20\,x \pm 4$	$\frac{N_c}{2}$ even	Any multiply even number having a 4 or a 6 as its unit digit.		$y' - 1$	$y' + 1$
12	○ ○	$N_c = 24\,x \pm 8$	$\frac{N_c}{4}$ even	Any multiple of 8, not divisible by 3.	$y = \frac{1}{6}\left(\frac{N_c}{2} \pm 2\right)$	y	y
	◎ ◎	$N_c = 24\,x \pm 4$	$\frac{N_c}{4}$ odd	Any quadruple of an odd integer, not divisible by 3.		$y' - 1$	$y' + 1$
14	○ ○	$N_c = 28\,x \pm 10$	$\frac{N_c}{2}$ odd	Any singly even number having 3 or 4 as remainder when divided by 7.	$y = \frac{1}{7}\left(\frac{N_c}{2} \pm 2\right)$	y	y
	◎ ◎	$N_c = 28\,x \pm 4$	$\frac{N_c}{2}$ even	Any multiply even number having 3 or 4 as remainder when divided by 7.		$y' - 1$	$y' + 1$
16	○ ○	$N_c = 32\,x \pm 12$	$\frac{N_c}{4}$ odd	Any quadruple of an odd integer of the form $8\,x \pm 3$.	$y = \frac{1}{8}\left(\frac{N_c}{2} \pm 2\right)$	y	y
	◎ ◎	$N_c = 32\,x \pm 4$	$\frac{N_c}{4}$ odd	Any quadruple of an odd integer of the form $8\,x \pm 1$.		$y' - 1$	$y' + 1$

* ○ ○ = singly re-entrant duplex winding; ◎ ◎ = doubly re-entrant duplex winding.

† General formula for ○ ○ : $N_c = 4\,n_p\,x \pm (2\,n_p - 4)$; $2\,n_p$ = number of poles.
General formula for ◎ ◎: $N_c = 4\,n_p\,x \pm 4$. x = any integer.

‡ In case of ring windings replace $\frac{N_c}{2}$ by n_c (number of coils).

§ If y is *odd*, both pitches are $= y$; if y is *even*, the pitches are $y - 1$ and $y + 1$; if the average pitch has two different *odd* values, y and y', the pitches may be either y, y, or y', y'; if the average pitch is either *odd* (y) or *even* (y'), the pitches may be either y, y, or $y' - 1, y' + 1$, respectively.

and for ring windings the number of coils

$$n_c = n_p\,y \pm n_m,$$

in which n_m is the number of multiplex windings. The greatest common factor of y and n_m indicates the number of re-entrancies. In Tables XLIV. and XLV., pages 160 to 163, data for duplex and triplex series windings, respectively, are given.

Example: The flux of a 10-pole dynamo is 8 megalines per pole. It is to give 145 volts at 125 revolutions per minute, with a triplex series drum winding. To find the number of conductors and the winding pitches.

The approximate number of conductors is, by (104):

$$N_c = \frac{145 \times 10^{10}}{2.778 \times 125 \times 8{,}000{,}000} = 522.$$

TABLE XLV.—NUMBER OF CONDUCTORS AND CONNECTING PITCHES FOR TRIPLEX SERIES DRUM WINDINGS.

NUMBER OF POLES. $2n_p$.	KIND OF SERIES WINDING.*	QUALIFICATION OF NUMBER OF CONDUCTORS, N_c. Equation for N_c.†	Degree of Evenness.	Description.	AVERAGE PITCH.‡	FRONT PITCH.§	BACK PITCH.§
2	○ ○ ○	$N_c = 6x \pm 2$	N_c even	Any even number, not a multiple of 3.	$y = \frac{N_c}{2} \pm 3$	y $y'-1$	y $y'+1$
	ꝏ ꝏ ꝏ	$N_c = 6x \pm 6$	N_c even	Any multiple of 6.		y $y'-1$	y $y'+1$
4	○ ○ ○	$N_c = 12x \pm 2$	$\frac{N_c}{2}$ odd	Any singly even number, not a multiple of 3.	$y = \frac{1}{2}\left(\frac{N_c}{2} - 3\right)$ $y' = \frac{1}{2}\left(\frac{N_c}{2} + 3\right)$	y $y'-1$	y $y'+1$
	ꝏ ꝏ ꝏ	$N_c = 12x \pm 6$	$\frac{N_c}{2}$ odd	Any odd multiple of 6.	$y = \frac{1}{2}\left(\frac{N_c}{2} + 3\right)$ $y' = \frac{1}{2}\left(\frac{N_c}{2} - 3\right)$	y $y'-1$	y $y'+1$
6	○ ○ ○	$N_c = 18x$	N_c even	Any multiple of 18.	$y = \frac{1}{3}\left(\frac{N_c}{2} + 3\right)$	y $y'-1$	y $y'+1$
	○ ○ ○ ꝏ ꝏ ꝏ	$N_c = 18x \pm 6$	N_c even	Any even multiple of 3, not divisible by 9.	$y' = \frac{1}{3}\left(\frac{N_c}{2} - 3\right)$	y $y'-1$	y $y'+1$
8	○ ○ ○	$N_c = 24x \begin{smallmatrix}\pm 2\\ \pm 10\end{smallmatrix}$	$\frac{N_c}{2}$ odd	Any singly even number, not a multiple of 3.	$y = \frac{1}{4}\left(\frac{N_c}{2} \pm 3\right)$	y $y'-1$	y $y'+1$
	ꝏ ꝏ ꝏ	$N_c = 24x \pm 6$	$\frac{N_c}{2}$ odd	Any odd multiple of 6.		y $y'-1$	y $y'+1$
10	○ ○ ○	$N_c = 30x \begin{smallmatrix}\pm 4\\ \pm 14\end{smallmatrix}$	N_c even	Any even number not divisible by 3, and having either a 4 or a 6 as unit digit.	$y = \frac{1}{5}\left(\frac{N_c}{2} \pm 3\right)$	y $y'-1$	y $y'+1$
	ꝏ ꝏ ꝏ	$N_c = 30x \pm 6$	N_c even	Any odd multiple of 6, having either a 4 or a 6 as unit digit.		y $y'-1$	y $y'+1$

TABLE XLV.—NUMBER OF CONDUCTORS AND CONNECTING PITCHES FOR TRIPLEX SERIES DRUM WINDINGS.—*Continued.*

Number of Poles. $2n_p$.	Kind of Series Winding.*	Qualification of Number of Conductors, N_c. Equation for N_c.†	Degree of Evenness.	Description.	Average Pitch.‡	Front Pitch.§	Back Pitch.§
12	○ ○ ○	$N_c = 36\,x \pm 18$	$\frac{N_c}{2}$ odd	Any odd multiple of 18.	$y = \frac{1}{6}\left(\frac{N_c}{2} + 3\right)$	y $y'-1$	y $y'+1$
	○ ○ ○ ◎ ◎ ◎	$N_c = 36\,x \pm 6$	$\frac{N_c}{2}$ odd	Any odd multiple of 6, not divisible by 9.	$y' = \frac{1}{6}\left(\frac{N_c}{2} - 3\right)$	y $y'-1$	y $y'+1$
14	○ ○ ○	$N_c = 42\,x \begin{smallmatrix}\pm 8\\ \pm 22\end{smallmatrix}$	N_c even	Any even number, not a multiple of 3, having either 1 or 6 as remainder when divided by 7.	$y = \frac{1}{7}\left(\frac{N_c}{2} \pm 3\right)$	y $y'-1$	y $y'+1$
	◎ ◎ ◎	$N_c = 42\,x \pm 6$	N_c even	Any multiple of 6, having either 1 or 6 as remainder when divided by 7.		y $y'-1$	y $y'+1$
16	○ ○ ○	$N_c = 48\,x \begin{smallmatrix}\pm 10\\ \pm 26\end{smallmatrix}$	$\frac{N_c}{2}$ odd	Any singly even number, not a multiple of 3, having either 6 or 10 as remainder when divided by 16.	$y = \frac{1}{8}\left(\frac{N_c}{2} \pm 3\right)$	y $y'-1$	y $y'+1$
	◎ ◎ ◎	$N_c = 48\,x \pm 6$	$\frac{N_c}{2}$ odd	Any odd multiple of 6, having either 6 or 10 as remainder when divided by 16.		y $y'-1$	y $y'+1$

* ○ ○ ○ = singly re-entrant triplex winding; ◎◎◎ = triply re-entrant triplex winding.

† General formula for ○ ○ ○ : $N_c = 6\,n_p\,x \pm (2\,n_p - 6)$, or $N_c = 6\,n_p\,x \pm (4\,n_p - 6)$;
General formula for ◎◎◎ : $N_c = 6\,n_p\,x \pm 6$.
$2\,n_p$ = number of poles. x = any integer.

‡ For ring windings replace $\frac{N_c}{2}$ by n_c (number of coils).

§ If y is *odd*, both pitches are $= y$; if y is *even*, the pitches are $y - 1$ and $y + 1$; if the average pitch is either *odd* (y), or *even* (y'), the pitches may be either y, y, or $y' - 1, y' + 1$, respectively.

By Table XLV., the number of conductors qualified for a singly re-entrant triplex series drum winding must be either

$$N_c = 30\,x \pm 4, \text{ or } N_c = 30\,x \pm 14\,,$$

the latter of which, for $x = 17$, when using the $+$ sign, furnishes the nearest number,

$$N_c = 30 \times 17 + 14 = 524\,,$$

for which a singly re-entrant triplex winding is possible. The average pitch,

$$y = \frac{1}{5}\left(\frac{524}{2} \pm 3\right) = 53,$$

being odd, the front and back pitches are equal, both being the same as the average pitch.

If a triply re-entrant triplex winding were desired the number of conductors would have to be determined from

$$N_c = 30\,x \pm 6\,;$$

and the two nearest numbers that fulfill this equation are

$$N_c = 30 \times 17 + 6 = 516,$$

and

$$N_c = 30 \times 18 - 6 = 534.$$

According to whether the former or the latter number of conductors is chosen, the average pitch will be either

$$y = \frac{1}{5}\left(\frac{516}{2} \pm 3\right) = 51,$$

or

$$y' = \frac{1}{5}\left(\frac{534}{2} \pm 3\right) = 54,$$

respectively. In the former case both pitches are $y = 51$; in the latter case, however, the front pitch has to be taken $y' - 1 = 53$, and the back pitch $y' + 1 = 55$.

(3) *Simplex Parallel Windings.*—For simplex parallel windings there may be any *even* number of conductors, except that in toothed and perforated armatures the number of conductors must also be a multiple of the number of conductors per slot. If it is desired to have exactly the same number of coils in each of the parallel branches, the number of coils must further be a multiple of the number of poles.

The pitches in parallel windings are alternately forward and backward, instead of being always forward, as in the series windings. The front and back pitches must both be *odd*, and should preferably differ by 2; therefore, the average pitch

$$y = \frac{1}{n_p}\left(\frac{N_c}{2} \pm 1\right)$$

should be *even*. The average pitch should not be very much different from the number of conductors per pole,

$$\frac{N_c}{n_p}.$$

For drum fashioned ring windings, or "chord" windings, the average pitch, y, should preferably be smaller than

$$\frac{N_c}{n_p},$$

and should differ from it by as great an amount as other conditions will permit.

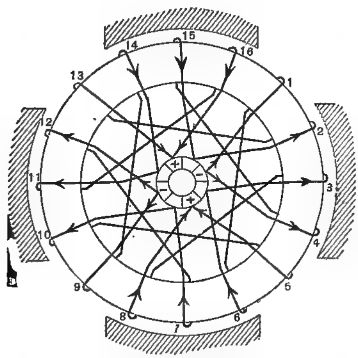


Fig. 101.—Simplex Parallel Ring Winding.

Fig. 101 shows a simplex parallel ring winding for 4 poles and 16 coils. The average pitch is

$$y = \frac{1}{2}\left(\frac{16}{2} \pm 1\right) = 4,$$

consequently the front pitch, $y - 1 = 3$, and the back pitch $y + 1 = 5$.

(4) *Multiplex Parallel Windings.*—In multiplex parallel windings the number of conductors, N_c, must be *even*. The connecting pitches must be *odd*. If the front pitch is $= y'$, then the back pitch is $= -(y' + 2\,n_m)$, where $n_m =$ number of multiple windings. The number of conductors (N_c), the average pitch (y) and the number of poles ($2\,n_p$) should be so chosen that $2\,n_p = y$ is somewhere nearly $= N_c$, preferably a little smaller than N_c.

The greatest common factor of

$$\frac{N_c}{2\,n_p} \text{ and } n_m$$

indicates the number of re-entrancies of the windings. If the number of conductors per pole,

$$\frac{N_c}{2\,n_p},$$

is not divisible by the number of multiple windings, n_m, there will be a singly re-entrant winding; and if it is divisible by

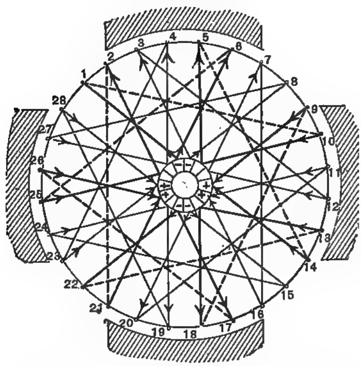


Fig. 102.—Duplex Parallel Drum Winding.

n_m, there will be a doubly re-entrant winding in case of $n_m = 2$ (duplex winding), and a triply re-entrant winding in case of $n_m = 3$ (triplex winding).

The winding pitches for multiple parallel windings are:

$$\text{Average pitch } y = \frac{1}{n_p} \times \left(\frac{N_c}{2} \pm n_m\right);$$

$$\text{Front pitch } \quad y_f = y - n_m,$$

$$\text{Back pitch } \quad y_b = y + n_m.$$

In case of a *duplex* parallel winding y should be chosen an *odd* number, so as to make $y - 2$ and $y + 2$ odd numbers also; and in case of a *triplex* parallel winding the average pitch should be taken *even*, in order to make the connecting pitches, $y - 3$ and $y + 3$, odd.

In Fig. 102 a singly re-entrant multiplex parallel winding is

given for $n_p = 2$, $n'_p = 2$, $n_m = 2$, and $N_c = 28$. The pitches in this case are

$$y = \frac{1}{2}\left(\frac{28}{2} \pm 2\right) = 7\,;$$

$$y - 2 = 5, \text{ and } y + 2 = 9\,.$$

There are two independent singly re-entrant windings, each having 4 parallel branches, making 8 paths altogether; 6 of these paths contain 4 conductors each, and the remaining 2 but 2 conductors each. In order to have an equal number of conductors in all branches, N_c must be a multiple of $2\,n'_p \times n_m$, or in the present example the number of conductors should be either 24 or 32; in the former case each of the 8 parallel branches would have 3, and in the latter case 4, conductors.

As further illustrations of the rules given above we take (1) $N_c = 486$, $n_p = 3$, $n'_p = 3$, $n_m = 2$; this is a 6-pole duplex parallel winding; since

$$\frac{N_c}{2\,n_p} = \frac{486}{6} = 81$$

is not divisible by $n_m = 2$, we have a singly re-entrant duplex winding (○○), for which the pitches are:

$$y = \frac{1}{3}\left(\frac{486}{2} \pm 2\right) = 81\;;$$

$$y - 2 = 79, \text{ and } y + 2 = 83\,.$$

(2) $N_c = 1{,}368$, $n_p = 6$, $n'_p = 6$, $n_m = 3$; in this case, which represents a triplex parallel winding for 12 poles,

$$\frac{N_c}{2\,n_p} = \frac{1{,}368}{12} = 144$$

is a multiple of $n_m = 3$, and therefore we have a triply re-entrant triplex winding (◎◎◎); the average pitch for this winding is

$$y = \frac{1}{6}\left(\frac{1{,}368}{2} \pm 3\right) = 114\,;$$

hence the front and back pitches are

$$y - 3 = 111, \text{ and } y + 3 = 117,$$

respectively.

CHAPTER IX.

DIMENSIONING OF COMMUTATORS, BRUSHES, AND CURRENT-CONVEYING PARTS OF DYNAMO.

47. Diameter and Length of Commutator Brush Surface.

In small and medium-sized machines the commutator is usually placed upon the shaft concentric with the armature, and has the collecting brushes sliding upon its peripheral surface. In large ring dynamos the armature winding is often performed by means of bare copper bars, and the current is then taken off directly from the winding; thus, in the Siemens Innerpole dynamo the brushes rest upon the external periphery of the armature, and in the Edison Radial Outerpole machine the two end surfaces of the armature are formed into commutators. If it is not convenient to use part of the armature winding itself as the commutator, in large diameter machines it is of advantage to provide a separate face-commutator, that is, a commutator with the brush surface perpendicular to the armature shaft; for in this case the otherwise unavailable space between the armature periphery and the shaft is made use of, and a saving in length of machine and in weight will be effected.

For the peripheral as well as for the face type commutator the same principles of construction hold good; the only difference is that in the latter case the outer diameter of the brush surface is fixed by the external diameter of the armature, and that therewith the top width of the bars is directly given by the number of commutator divisions, while in the former case the dimensions of the brush surface can be chosen between comparatively much wider limits.

In low potential machines with small number of divisions, the thickness of the substructure determines the diameter of the commutator; in high potential machines, however, especially those of multipolar type, where the number of commuta-

tor segments is very great, the width, at top, of the commutator bars, their number, and the thickness of the insulation between them fix the outside diameter.

The bars must be large enough in cross-section to carry the whole current generated in the armature without undue heating, and shall continue so after a reasonable amount of wear. They must be of sufficient length to allow a proper number of brushes to take off the current.

The same brush contact surface may be obtained by employing either a broad thin brush on a small diameter commutator, or a narrow thick one on a large diameter, the number of bars being the same in both cases, their width, consequently, larger in the latter case. With larger diameter and greater consequent peripheral velocity there will be more wear of both brushes and segments, and greater consumption of energy due to the increased friction of the brushes.

The segments are usually made of copper (cast, rolled, or forged), phosphor bronze, or gun metal, sometimes brass, and even iron being used; the materials for the substructure are phosphor bronze, brass, or cast iron.

From all this it will be obvious that a general formula for the diameter of the commutator cannot be established, and that, on the contrary, this dimension has to be properly chosen in every case with reference to the armature diameter to the design of the commutator, to the materials employed, to the strength of the substructure, or the thickness of the bar, respectively, and, finally, with reference to the wear of the segments.

The commutator diameter being decided upon, the size of the brushes can now be calculated, as shown in § 49, and, from this, the length of the commutator can be found.

In order to prevent annular grooves being cut around the commutator, the brushes ought to be so adjusted that the gaps between those in one set do not come opposite the gaps in the other set. Denoting, Fig. 103, the width of each brush by b_b, their number per set by n_b, and the gap between them by l'_b, we consequently obtain the total length of the commutator brush surface from:

$$l_c = \left(n_b + \frac{1}{2}\right) \times (b_b + l'_b) \quad \ldots\ldots\ldots (114)$$

This length of brush surface should be available even after the commutator has been turned down to its final diameter; the original diameter must therefore have a somewhat larger con-

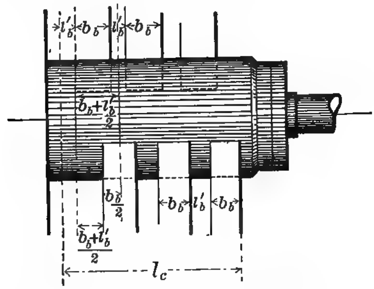


Fig. 103.—Arrangement of Commutator Brushes.

tact length. An addition to l_c of from ¼ to 1 inch, according to the depth of the bar, is thus necessitated.

As to the practical design of commutators, while the same general plan is followed in all, the details of construction are almost numberless. Structural cross-sections and descriptions of the commutators manufactured by the Electron Manufacturing Company, the Storey Motor and Tool Company, the Royal Electric Company, the Fort Wayne Electric Corporation, Paterson & Cooper, the Gülcher Company, the General Electric Company, the Triumph Electric Company, the Siemens & Halske Electric Company, the Walker Company, and others, are given in an article[1] in *American Electrician.*

48. Commutator Insulations.

In a commutator the insulation has to form a part of the general structure, and has to take strain in common with other material used; from its natural cleavage and hardness, therefore, *mica* is particularly suitable for commutator insulations, and is, in fact, almost exclusively used for this purpose, only asbestos and vulcanized fibre being employed in rare cases.

[1] "Modern Commutator Construction," *American Electrician*, vol. viii. p. 83 (July, 1896).

The thickness of the commutator insulation ought to be proportional to the voltage of the machine, and, for the various

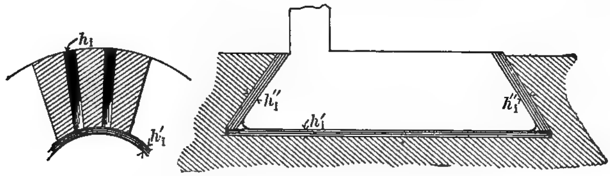


Fig. 104.—Commutator Insulations.

positions with reference to the bars, see h_i, h'_i, h''_i, Fig. 104, should be selected within the following limits:

TABLE XLVI.—COMMUTATOR INSULATIONS FOR VARIOUS VOLTAGES.

POSITION OF INSULATION.	THICKNESS OF INSULATION (MICA):					
	Up to 300 Volts.		400 to 700 Volts.		800 to 3,000 Volts.	
	inch.	mm.	inch.	mm.	inch.	mm.
Side Insulation (h_i)	.020 to .040	.5 to 1.0	.030 to .050	.75 to 1.25	.040 to .060	1 to 1.5
Bottom Insulation (h'_i)	$\frac{3}{64}$ " $\frac{3}{32}$	1.25 " 2.5	$\frac{1}{16}$ " $\frac{1}{8}$	1.5 " 3	$\frac{3}{32}$ " $\frac{3}{16}$	2.5 " 5
End Insulation (h''_i)	$\frac{1}{16}$ " $\frac{3}{32}$	1.5 " 2.5	$\frac{3}{32}$ " $\frac{1}{8}$	2.5 " 3	$\frac{1}{8}$ " $\frac{3}{16}$	3 " 5

49. Dynamo Brushes.[1]

a. Material and Kinds of Brushes.

For low potential machines having a large current output, it is the practice to employ thick *copper* brushes, made up either of copper wires, or copper strips, or copper wire gauze, in order to secure a large number of contact points, and to set them so as to make an angle of about 45° with the commutator surface, as shown in Fig. 105.

In small dynamos, often springy copper plates are used which are placed tangentially to the commutator periphery, as illustrated in Fig. 106.

For high potential machines, especially for railway generators and motors, *carbon* brushes are used in order to aid in the sparkless collection of the current at varying load. As each

[1] "Commutator Brushes for Dynamo-Electric Machines: their selection, their proper contact-area, and their best tension," by A. E. Wiener, *American Electrician*, vol. viii. p. 152 (September, 1896).

commutator segment enters under the brush, the area of contact is, at first, very small and, owing to the high specific resistance of carbon, a considerable resistance is offered to the passage of the current from the branch of the armature of which that segment at the time is the terminal, into the external circuit. This gives rise to a considerable local fall of potential, which diverts a comparatively large portion of the armature current through the neighboring coil into which it flows against the existing current, causing the latter to reverse quickly in opposition to the E. M. F. of self-induction, thereby

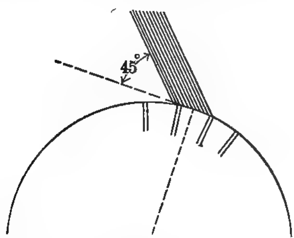


Fig. 105.—Sloping Copper Wire (or leaf) Brush.

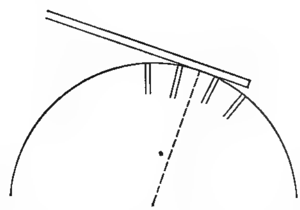


Fig. 106.—Tangential Copper Plate Brush.

preparing the short-circuited coil to join the successive armature circuit of opposite polarity without sparking. (Compare with sections on sparkless commutation of armature current, in § 13.) The resistance of the carbon brushes cannot be depended upon for the complete commutation of the entire current, but in most generators, especially in those with toothed and perforated armatures, fully half the armature current may be thus commutated. In railway generators it is usual to adjust the brushes so that at no load they are in the neighborhood of the forward pole-tips where the pole-fringe E. M. Fs. generated are sufficient to reverse one-half of the normal current, the remaining half being then taken care of by the brushes.

Carbon brushes are either set tangentially (Fig. 107), or radially (Fig. 108), with respect to the commutator circumference, the latter arrangement having the advantage of admitting of reversal of the rotation, without changing the brushes.

To use carbon brushes exclusively on machines of low voltage would be very bad practice, because carbon has so much

higher resistance than copper that the drop of potential would be excessive, and too great a percentage of the power of the machine would be used up for commutation. If, therefore, the resistance of an ordinary copper brush is not high enough

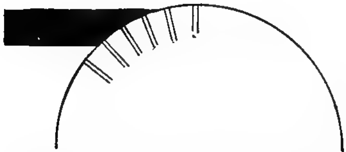


Fig. 107.—Tangential Carbon Brush.

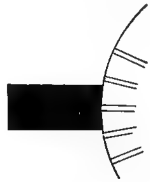
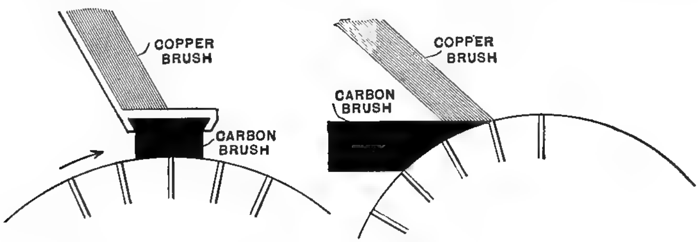


Fig. 108.—Radial Carbon Brush.

for sparkless collection, a copper gauze brush must be employed, which has a much higher resistance than a copper leaf brush, and while there are some mechanical advantages in using it, such as cooling effects and smoother wear of the commutator, yet the principal reason it stops sparking is that it has a higher resistance. In case the resistance is still too low, the next step is the application of a *brass* gauze brush having about twice the resistance of copper gauze. If that is not enough yet, some form of carbon brush which has its resistance reduced, must be resorted to. Carbon itself cannot have its resistivity changed, but by mixing copper filings with the carbon powder, or by molding layers of gauze in it, the conductivity of the brush can be increased. Instead of arti-



Figs. 109 and 110.—Combination Copper-Carbon Brushes.

ficially decreasing the resistance of carbon, *combination brushes* consisting either in copper brushes provided with carbon tips, Fig. 109, or in carbon brushes sliding upon the commutator and having, in turn, copper brushes resting against themselves, Fig. 110, are sometimes employed, and in case of very heavy

currents, the addition to each set of copper brushes, of a combination brush set somewhat ahead of the copper brushes as shown in Fig. 111, has been found to greatly improve the

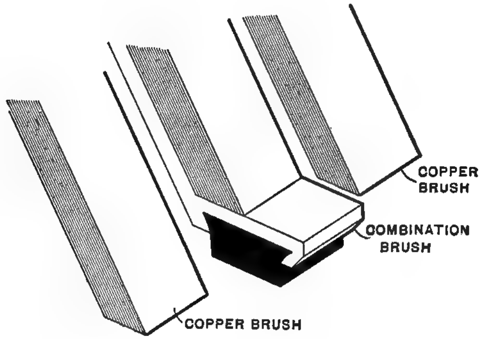
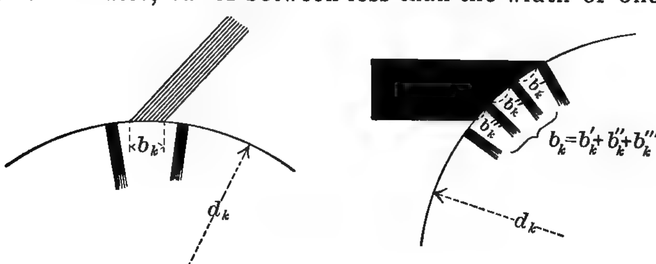


Fig. 111.—Arrangement of Copper and Combination Brushes for Collection of Large Currents.

sparkless running of the machines. With the latter arrangement, the tension on the combination brushes should exceed that on the copper brushes sufficiently to enable them to take their full share of the current as nearly as possible.

b. Area of Brush Contact.

The thickness of the brushes, according to the current capacity of the machine, to the grouping of the armature coils, to the material and kind of the brush and to the dimensions of the commutator, varies between less than the width of one to



Figs. 112 and 113.—Circumferential Breadth of Brush Contact.

that of three and even more commutator segments. In case the brush covers not more than the width of one bar, as in Fig. 112, only one armature coil is short-circuited at any time,

while in case of brushes thicker than the width of one bar plus two side insulations, Fig. 113, two or even more coils, at times, are simultaneously short-circuited under each set of brushes.

The breadth of the brush contact surface in the former case (Fig. 112) is equal to the thickness of the beveled end of the brush measured along the commutator circumference; in the latter case (Fig. 113) is the breadth of the brush bevel diminished by the sum of the thickness of the commutator insulations covered by the brush, and can be generally expressed by the formula

$$b_k = n_k \left(\frac{d_k \times \pi}{n_c} - h_i \right), \quad \ldots\ldots\ldots (115)$$

where b_k = circumferential breadth of brush contact, in inches;
n_k = number of commutator-bars covered by the thickness of one brush;
d_k = diameter of commutator, in inches;
n_c = number of commutator-divisions;
h_i = thickness of commutator side-insulation, in inches, see Table XLVI.

If the brush covers less than one bar, as in Fig. 112, n_k is a fraction; if the width of the brush is from one bar to one bar plus two side insulations, $n_k = 1$; when between two bars plus one insulation and two bars plus three insulations, $n_k = 2$, etc.; and if the brush covers from one bar plus two insulations to two bars plus one insulation, or from two bars plus three insulations to three bars plus two insulations, etc., the value of n_k is a mixed number, consisting of an integer and a fraction.

Having decided upon n_k and having calculated b_k from (115), the width of the contact area, and subsequently the width of the brushes, can be found for a given current output of the dynamo by providing contact area in proportion to the current intensity. In order to keep the brushes at a moderate temperature, and the *loss of commutation* within practical limits, the current density of the brush contact should not exceed 150 to 175 amperes per square inch in case of *copper* brushes (wire, leaf plate, and gauze), 100 to 125 amperes per square inch for *brass* gauze brushes, and 30 to 40 amperes per square inch in case of *carbon* brushes.

Taking the lower of the above limits of the current densities, the effective length of the brush contact can consequently be expressed by

$$l_k = \frac{I}{150 \times n''_p \times b_k} \quad \text{..........(116)}$$

for *copper* brushes, by

$$l_k = \frac{I}{100 \times n''_p \times b_k} \quad \text{..........(117)}$$

for *brass* brushes, and by

$$l_k = \frac{I}{30 \times n''_p \times b_k} \quad \text{..........(118)}$$

for *carbon* brushes, the symbols employed being

l_k = effective length of brush contact surface, in inches;
= $n_b \times b_b$ (n_b = number of brushes per set, b_b = width of brush);
I = total current output of dynamo, in amperes;
n''_p = number of pairs of brush sets (usually either $n''_p = 1$, or equal to the number of bifurcations of the armature current, $n''_p = n'_p$).

For the purpose of securing a good contact, the length l_k should be subdivided into a set of n_b individual brushes, of a width b_b each, not exceeding 1½ to 2 inches. In small machines, where one such brush would suffice, it is good practice to employ two narrow brushes, even down as low as 3/8 inch each, in order to facilitate their adjusting or exchanging while the machine is running.

c. Energy-Loss in Collecting Armature Current. Determination of Best Brush Tension.

The brushes give rise to two losses of energy: an *electrical* energy-loss due to overcoming contact resistance, and a *mechanical* loss caused by friction. Both of these losses depend upon the pressure with which the brushes are resting upon the commutator, the electrical loss decreasing and the mechanical loss increasing with increasing brush tension. There will, therefore, in every single case, be one certain pressure per unit area of brush contact, for which the sum of the brush losses will be a minimum. With the object of determining this criti-

cal pressure, E. V. Cox and H. W. Buck[1] have investigated the influence of the brush tension upon the contact resistance and upon the friction, for various kinds of brushes. They found (1) that the friction increases in direct proportion

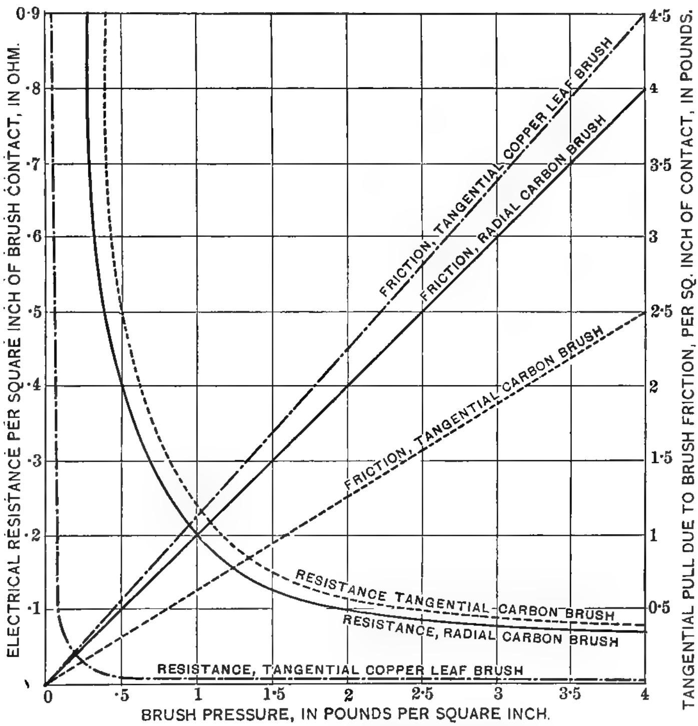


Fig. 114.—Contact Resistance and Friction per Square Inch of Brush Surface, on Copper Commutator (dry), at Peripheral Velocity of 1,000 Feet per Minute.

with the tension; (2) that the contact resistance decreases at first very rapidly, but that beyond a certain point a great increase in pressure produces only a slight diminution of resistance; (3) that slightly oiling the contact surface, while not perceptibly increasing the electrical resistance, greatly

[1] The Relation between Pressure, Electrical Resistance, and Friction in Brush Contact," Electrical Engineering Thesis, Columbia College, by E. V. Cox and H. W. Buck. *Electrical Engineer*, vol. xx. p. 125 (August 7, 1895); *Electrical World*, vol. xxvi. p. 217 (August 24, 1895).

diminishes the friction; (4) that for a copper brush the friction is greater and the contact resistance smaller than for a carbon brush of same area at the same pressure; (5) that the friction of a radial carbon brush is greater than that of a tangential carbon brush at the same pressure; (6) that for the same brush both the contact resistance and the friction are considerably less on a cast-iron cylinder than on a commutator; and

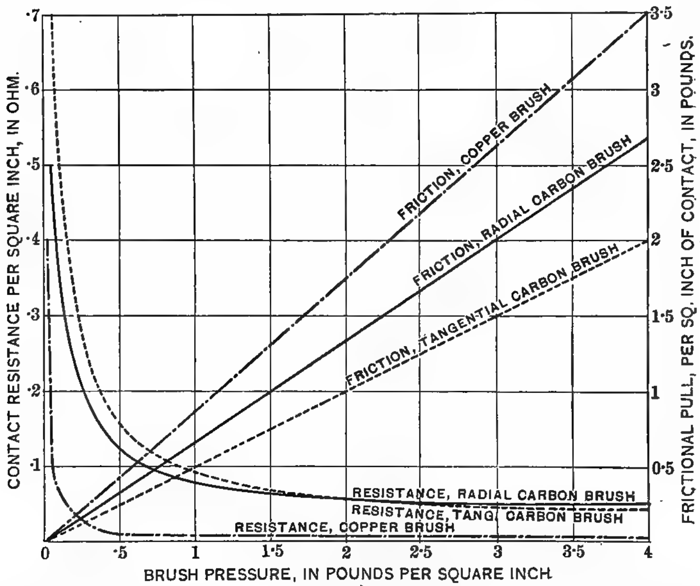


Fig. 115.—Contact Resistance and Friction per Square Inch of Brush Surface, on Cast-iron Cylinder.

(7) that for all kinds of brushes the friction is less at high than at low peripheral speeds, while the contact resistance is but slightly increased by raising the velocity.

In Figs. 114 and 115 the averages of their results are plotted, the former giving the curves of contact resistance and friction for an ordinary commutator, without lubrication, and the latter the corresponding curves for the case that the commutator is replaced by a cast-iron cylinder.

From Fig. 114 the following Table XLVII. is derived, which, in addition to the data obtained from the curves, also

contains the brush friction for the case the commutator is slightly oiled:

TABLE XLVII.—CONTACT RESISTANCE AND FRICTION FOR DIFFERENT BRUSH TENSIONS.

Brush Tension in Pounds per Square Inch.	Contact Resistance per Square Inch of Brush Surface, ρ_k, in Ohm.			Tangential Pull Due to Brush Friction per Square Inch of Contact at Peripheral Speed of 1,000 Feet per Minute. f_k, in Pounds.					
				Commutator Dry.			Commutator Oiled.		
	Tangential Copper Leaf Brush.	Tangential Carbon Brush.	Radial Carbon Brush.	Tangential Copper Brush.	Tangential Carbon Brush.	Radial Carbon Brush.	Tangential Copper Brush.	Tangential Carbon Brush.	Radial Carbon Brush.
.5	.010	.50	.40	.6	.3	.5	.16	.10	.15
1	.009	.24	.20	1.15	.63	1	.32	.20	.30
1.5	.008	.15	.13	1.7	.95	1.5	.48	.30	.45
2	.007	.12	.10	2.25	1.25	2	.64	.40	.60
2.5	.006	.10	.087	2.8	1.6	2.5	.80	.50	.75
3	.0055	.09	.08	3.4	1.9	3	.96	.60	.90
3.5	.0052	.083	.075	3.95	2.2	3.5	1.12	.70	1.05
4	.005	.08	.07	4.5	2.5	4	1.30	.80	1.20

The specific pull, f_k, due to brush friction, in columns 5 to 10 of the above table, is given for a peripheral velocity of 1,000 feet per minute; at 2,000 feet per minute it is 7/8, at 3,000 feet per minute 3/4, at 4,000 feet per minute 5/8, and at 5,000 feet per minute only 1/2 of what it is for that pressure at 1,000 feet per minute, and for any commutator velocity, v_k, can be found from the formula

$$f'_k = f_k \times \left(1 - \frac{v_k - 1,000}{8,000}\right). \quad \ldots\ldots(119)$$

From Table XLVII. the electrical brush loss is calculated by dividing the contact resistance given for the particular brush tension employed, by the contact area, and multiplying the

quotient by the number of sets of brushes and by the square of the current passing through each set, thus:

$$P_k = 2\, n''_p \times \frac{\rho_k}{l_k \times b_k} \times \left(\frac{I}{n''_p}\right)^2 = 2 \times \frac{\rho_k \times I^2}{l_k \times b_k \times n''_p} \text{ watts}$$

$$= .00268 \times \frac{\rho_k \times I^2}{l_k \times b_k \times n''_p} \text{ horse power,} \quad \ldots\ldots(\mathbf{120})$$

where P_k = energy absorbed by contact resistance of brushes;
ρ_k = resistivity of brush contact, ohm per square inch surface, from Table XLVII.;
$l_k \times b_k$ = contact area of one set of brushes, in square inches;
n''_p = number of pairs of brush sets;
I = current output of dynamo.

And the frictional loss is obtained in multiplying the tangential pull, given for the respective brush tension and corrected to the proper peripheral velocity according to formula (119), by the total brush contact area and by the peripheral velocity of the commutator, and dividing the product by 33,000, the equivalent of one horse power in foot-pounds per minute:

$$P_f = \frac{f'_k \times 2\, n''_p \times l_k \times b_k \times v_k}{33{,}000}$$

$$= 6 \times 10^{-5} \times f'_k \times l_k \times b_k \times n''_p \times v_k, \quad \ldots\ldots\ldots(\mathbf{121})$$

in which P_f = energy absorbed by brush friction, in HP;
f'_k = specific tangential pull due to friction, at velocity v_k, in pounds, see formula (119);
$2\, n''_p \times l_k \times b_k$ = total area of brush contact surfaces, in square inches;
v_k = peripheral velocity of commutator, in feet per minute,

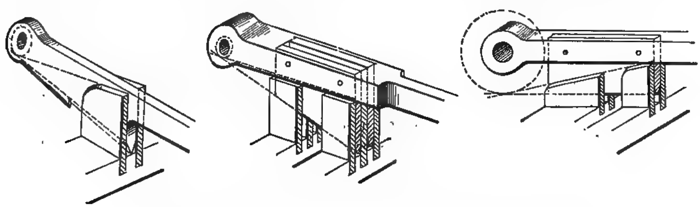
$$= \frac{d_k \times \pi \times N}{12}.$$

By calculating the amounts of P_k and P_f, from (120) and (121) respectively, for different brush tensions, the best tension giving a minimum value of the total brush-loss, $P_k + P_f$, can readily be found.

50. Current-Conveying Parts.

Care must also be exercised in the proportioning of those parts of a dynamo which serve to convey the current, collected by the brushes, to the external circuit. For, if material is wasted in these, the cost of the machine is unnecessarily increased; and if, on the contrary, too little material is used, an appreciable drop in the voltage and undue heating will be the result.

In the design of such current-conveying parts, among which may be classed brush holders, cables, conductor rods, cable lugs, binding posts, and switches, the attention should



Figs. 116 to 118.—Various Forms of Spring Contacts.

therefore be directed to the smallest cross-section through which the current has to pass, and to the surfaces of contact transferring the current from one part to another. The maximum permissible current density in the cross-section, while depending in a small degree upon the ratio of circumference to area of cross-section, is chiefly determined by the choice of the material; that in the area of contact between two parts, however, although the conductivity of the material employed is of some consequence, depends mainly upon the condition of the contact surfaces and upon the amount of pressure that can be applied to the joint.

The most usual forms of contact are those shown in Figs. 116 to 125. Figs. 116 to 118 represent *spring contacts* as used in switches; in Fig. 116 the switch blade is cast in one with the lever, while in Figs. 117 and 118 the levers are provided with separate copper blades. The former is a *single* switch making and breaking contact between the blade and the clips, the lever itself forming the terminal of one pole; the latter two are *double* switches, the connection being established between two sets of clips by way of the blade, when the switch is closed.

In order to prevent the forming of an arc in opening a switch, especially a double switch, each blade must leave all the clips with which it engages simultaneously over its entire length. For this purpose either the blade, or the clips, or both (Figs. 117, 118, and 116, respectively) have to be cut off at such an angle that, in the closed position of the switch, the enter-line of the blade and the line through the tops of the clips are both tangents to the same circle (shown in dotted lines in Figs. 116 to 118), described from the centre of the lever fulcrum. If all clips are then made of equal widths, as in Fig. 117, those

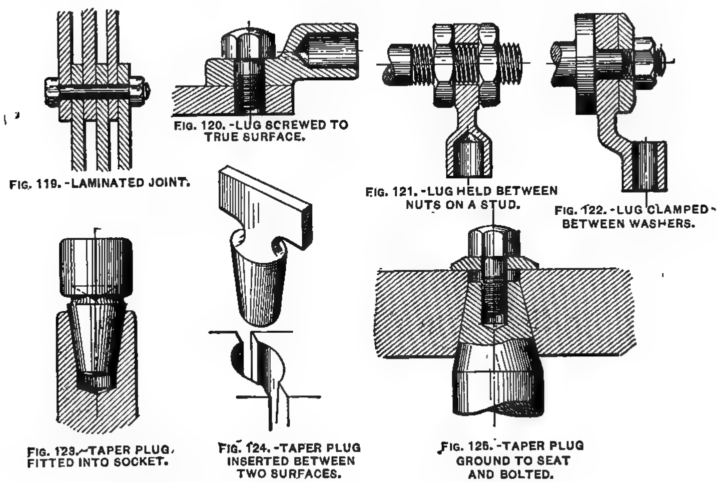


FIG. 119.-LAMINATED JOINT.
FIG. 120.-LUG SCREWED TO TRUE SURFACE.
FIG. 121.-LUG HELD BETWEEN NUTS ON A STUD.
FIG. 122.-LUG CLAMPED BETWEEN WASHERS.
FIG. 123.-TAPER PLUG FITTED INTO SOCKET.
FIG. 124.-TAPER PLUG INSERTED BETWEEN TWO SURFACES.
FIG. 125.-TAPER PLUG GROUND TO SEAT AND BOLTED.

Figs. 119 to 125.—Various Forms of Screwed, Clamped, and Fitted Contacts.

nearest to the fulcrum, in case of a double switch, have less contact area than the remote ones, and in designing such a switch this smaller contact area is to be made of sufficient size to carry half the armature current, if there is but one blade, and one-quarter of the total current when the lever has two blades. By making the clips near the fulcrum correspondingly wider than those at the other end of the blade, as in Fig. 118, all the contact surfaces can, however, be made of equal area.

Various forms of *screwed* or *bolted* contacts are shown in Figs. 119, 120, and 121; a *clamped* contact is illustrated in Fig. 122; two common forms of *fitted* contact in Figs. 123 and 124; and an excellent *fitted and screwed* contact in Fig. 125.

The permissible current densities for all these different kinds of contact as well as for the cross-section of different materials are compiled in the following Table XLVIII., which more in particular refers to the larger sizes of dynamos, since in small machines purely mechanical considerations lead to much heavier pieces than are required for electrical purposes:

TABLE XLVIII.—CURRENT DENSITIES FOR VARIOUS KINDS OF CONTACTS AND FOR CROSS-SECTION OF DIFFERENT MATERIALS.

KIND OF CONTACT.	MATERIAL.	CURRENT DENSITY.			
		ENGLISH MEASURE.		METRIC MEASURE.	
		Amps. per square inch.	Square mils per amp.	Amps. per cm.²	mm.² per ampere.
Sliding Contact (Brushes)	Copper Brush	150 to 175	5,700 to 6,700	23 to 28	3.5 to 4.5
	Brass Gauze Brush	100 to 125	8,000 to 10,000	15 to 20	5 to 7
	Carbon Brush	30 to 40	25,000 to 33,300	4.5 to 6	16 to 22
Spring Contact (Switch Blades)	Copper on Copper	60 to 80	12,500 to 16,700	9 to 12.5	8 to 11
	Composition on Copper	50 to 60	16,700 to 20,000	7.5 to 9.5	11.5 to 13.5
	Brass on Brass	40 to 50	20,000 to 25,000	6 to 8	12.5 to 16.5
Screwed Contact	Copper to Copper	150 to 200	5,000 to 6,700	23 to 31	3 to 4.5
	Composition to Copper	125 to 150	6,700 to 8,000	19 to 23	4.5 to 5.5
	Composition to Composition	100 to 125	8,000 to 10,000	15 to 20	5 to 6.5
Clamped Contact	Copper to Copper	100 to 125	8,000 to 10,000	15 to 20	5 to 6.5
	Composition to Copper	75 to 100	10,000 to 13,000	12 to 16	6 to 8.5
	Composition to Composition	70 to 90	11,000 to 14,000	11 to 14	7 to 9
Fitted Contact (Taper Plugs)	Copper to Copper	125 to 175	5,700 to 8,000	20 to 28	3.5 to 5
	Composition to Copper	100 to 125	8,000 to 10,000	15 to 20	5 to 7
	Composition to Composition	75 to 100	10,000 to 13,000	12 to 16	6 to 8.5
Fitted and Screwed Contact	Copper to Copper	200 to 250	4,000 to 5,000	30 to 40	2.5 to 3.5
	Composition to Copper	175 to 200	5,000 to 5,700	28 to 31	3 to 3.5
	Composition to Composition	150 to 175	5,700 to 6,700	23 to 28	3.5 to 4.5
Cross-section	Copper Wire	1,200 to 2,000	500 to 800	175 to 300	.35 to .55
	Copper Wire Cable	1,000 to 1,600	600 to 1,000	150 to 250	.4 to .65
	Copper Rod	800 to 1,200	800 to 1,200	125 to 175	.55 to .80
	Composition Casting	500 to 700	1,400 to 2,000	75 to 110	.90 to 1.35
	Brass Casting	300 to 400	2,500 to 3,300	45 to 60	1.60 to 2.25

CHAPTER X.

MECHANICAL CALCULATIONS ABOUT ARMATURE.

51. Armature Shaft.

The length of the armature shaft, varying considerably for the different arrangements of the field magnet frame, depends upon the type chosen, and, since the length of the commutator depends upon the current output of the machine, even varies in dynamos of equal capacity and of same design, but of different voltage, a general rule for the length of the shaft can therefore not be given.

Its diameter, however, directly depends only upon the output and the speed of the dynamo, and can be expressed as a function of these quantities, different functions, however, being employed for various portions of its length. For, while

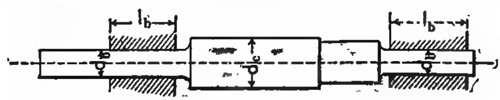


Fig. 126.—Dimensions of Armature Shaft.

in the bearing portions, d_b, Fig. 126, torsional strength only has to be taken into account, the center portion, d_c, between the bearings, which carries the armature core, is to be calculated to withstand the torsional force as well as the bending due to the weight.

For *steel* shafts the author has found the following empirical formulæ to give good results in practice:

For *bearing portions:*

$$d_b = k_8 \times \sqrt{P'} \times \sqrt[4]{N}, \quad \ldots\ldots(122)$$

where d_b = diameter of armature shaft, at bearings, in inches;
P' = capacity of dynamo, in watts;
N = speed, in revolutions per minute;
k_8 = constant, depending upon the kind of armature, see Table XLIX.

The value of k_8 varies between .0025 and .005, as follows:

TABLE XLIX.—VALUE OF CONSTANT IN FORMULA FOR JOURNAL-DIAMETER OF ARMATURE SHAFT.

KIND OF ARMATURE.	VALUE OF k_8.
High speed drum armature	.0025
High speed ring "	.003
Low speed drum "	.004
Low speed ring "	.005

For *core portion:*

$$d_c = k_9 \times \sqrt[4]{\frac{P'}{N}}, \qquad \text{..........(123)}$$

where d_c = diameter of core portion of armature shaft, in inches;

P' = capacity of machine, in watts;

N = speed, in revolutions per minute;

k_9 = constant depending upon output of machine, see Table L.

This constant indicates the dependence of the diameter of the shaft upon the length between its supports; and since the weight supported and also the length supporting it increase with the output, it is evident that k_9 has a greater value the larger the output of the machine. For capacities up to 2,000 kilowatts, k_9 numerically ranges between 1 and 1.8, thus:

TABLE L.—VALUE OF CONSTANT IN FORMULA FOR DIAMETER OF CORE PORTION OF ARMATURE SHAFT.

CAPACITY, IN WATTS. P'	VALUE OF k_9.
Up to 1,000 watts	1
" 5,000 "	1.1
" 10,000 "	1.2
" 50,000 "	1.3
" 100,000 "	1.4
" 200,000 "	1.5
" 500,000 "	1.6
" 1,000,000 "	1.7
" 2,000,000 "	1.8

Considering the speeds given in Tables X., XI., and XII., § 21, for various outputs, we obtain the following Tables LI., LII., and LIII., giving, respectively, the diameters of shafts for drum armatures, for high-speed ring armatures, and for low-speed ring armatures:

TABLE LI.—DIAMETERS OF SHAFTS FOR DRUM ARMATURES.

CAPACITY, IN WATTS. P'	SPEED, IN REVOLUTIONS PER MINUTE (FROM TABLE X.) N.	DIAMETER OF ARMATURE SHAFT, IN INCHES.		
		Bearing Portion.	Core Portion.	
		$d_b = .0025\ \sqrt{P'} \times \sqrt[4]{N}$	k_9	$d_c = k_9 \times \sqrt[4]{\frac{P'}{N}}$
100	3,000	$\frac{3}{16}$	.9	$\frac{3}{8}$
250	2,700	$\frac{5}{16}$	.95	$\frac{1}{2}$
500	2,400	$\frac{7}{16}$	1	$\frac{5}{8}$
1,000	2,200	$\frac{9}{16}$	1	$\frac{13}{16}$
2,000	2,000	$\frac{3}{4}$	1.05	1
3,000	1,900	$\frac{15}{16}$	1.1	$1\frac{1}{4}$
5,000	1,800	$1\frac{1}{8}$	1.15	$1\frac{1}{2}$
10,000	1,700	$1\frac{5}{8}$	1.2	2
15,000	1,600	$1\frac{7}{8}$	1.25	$2\frac{1}{4}$
20,000	1,500	$2\frac{1}{8}$	1.25	$2\frac{1}{2}$
25,000	1,350	$2\frac{3}{8}$	1.25	$2\frac{3}{4}$
30,000	1,200	$2\frac{1}{2}$	1.3	3
50,000	1,050	3	1.3	$3\frac{1}{2}$
75,000	900	$3\frac{5}{8}$	1.35	$4\frac{1}{8}$
100,000	750	$4\frac{1}{8}$	1.4	$4\frac{3}{4}$
150,000	600	$4\frac{3}{4}$	1.45	$5\frac{3}{4}$
200,000	500	$5\frac{1}{4}$	1.5	$6\frac{3}{4}$
300,000	400	6	1.55	8

For *wrought iron* shafts the diameters obtained by formulæ (122) and (123), or those taken from Tables LI., LII., and LIII., respectively, are to be multiplied by 1.25, that is, increased by 25 per cent.

52. Driving Spokes.

Ring armature cores usually are attached to the shaft either by means of spider frames or of skeleton pulleys. In both cases the driving of the conductors is effected by a number of spokes, which respectively form part of the spider itself, Fig. 127, or of a separate driving frame keyed to the skeleton pulley, Fig. 128, page 188.

TABLE LII.—DIAMETERS OF SHAFTS FOR HIGH-SPEED RING ARMATURES.

CAPACITY, IN WATTS. P'	SPEED, IN REVOLUTIONS PER MINUTE (FROM TABLE XI.) N.	DIAMETER OF ARMATURE SHAFT, IN INCHES.		
		Bearing Portion.	Core Portion.	
		$d_b = .003 \sqrt{P'} \times \sqrt[4]{N}$	k_9	$d_c = k_9 \times \sqrt[4]{\frac{P'}{N}}$
100	2,600	$\frac{1}{4}$	.9	$\frac{3}{8}$
250	2,400	$\frac{3}{8}$	.95	$\frac{1}{2}$
500	2,200	$\frac{1}{2}$	1	$\frac{11}{16}$
1,000	2,000	$\frac{5}{8}$	1	$\frac{7}{8}$
2,500	1,700	1	1.05	$1\frac{1}{4}$
5,000	1,500	$1\frac{3}{8}$	1.1	$1\frac{5}{8}$
10,000	1,250	$1\frac{3}{4}$	1.2	2
25,000	1,000	$2\frac{5}{8}$	1.25	$2\frac{7}{8}$
50,000	800	$3\frac{1}{2}$	1.3	$3\frac{3}{4}$
100,000	600	$4\frac{3}{4}$	1.4	5
200,000	500	$6\frac{1}{8}$	1.5	$6\frac{3}{4}$
300,000	450	$7\frac{1}{2}$	1.55	$7\frac{7}{8}$
400,000	400	$8\frac{1}{2}$	1.55	9
600,000	350	10	1.6	$10\frac{1}{2}$
800,000	300	11	1.65	12
1,000,000	250	12	1.7	$13\frac{1}{2}$
1,500,000	225	14	1.75	$15\frac{3}{4}$
2,000,000	200	16	1.8	18

TABLE LIII.—DIAMETERS OF SHAFTS FOR LOW-SPEED RING ARMATURES.

CAPACITY, IN WATTS. P'	SPEED, IN REVOLUTIONS PER MINUTE (FROM TABLE XII.) N.	DIAMETER OF ARMATURE SHAFT, IN INCHES.		
		Bearing Portion.	Core Portion.	
		$d_b = .005 \sqrt{P'} \times \sqrt[4]{N}$	k_9	$d_c = k_9 \times \sqrt[4]{\frac{P'}{N}}$
2,500	400	$1\frac{1}{8}$	1.05	$1\frac{5}{8}$
5,000	350	$1\frac{1}{2}$	1.1	$2\frac{1}{8}$
10,000	300	2	1.2	$2\frac{5}{8}$
25,000	250	$3\frac{1}{8}$	1.25	4
50,000	200	$4\frac{1}{4}$	1.3	$5\frac{1}{4}$
100,000	175	$5\frac{3}{4}$	1.4	$6\frac{7}{8}$
200,000	150	$7\frac{3}{4}$	1.5	9
300,000	125	$9\frac{1}{8}$	1.55	$10\frac{3}{4}$
400,000	100	10	1.55	$12\frac{1}{4}$
600,000	90	12	1.6	$14\frac{1}{2}$
800,000	80	$13\frac{1}{2}$	1.65	$16\frac{1}{2}$
1,000,000	75	15	1.7	$18\frac{1}{2}$
1,500,000	70	18	1.75	22
2,000,000	65	20	1.8	$24\frac{1}{2}$

In dimensioning these driving spokes, on the one hand sufficient mechanical strength for driving should be provided, while on the other hand, if spiral winding is to be used, not

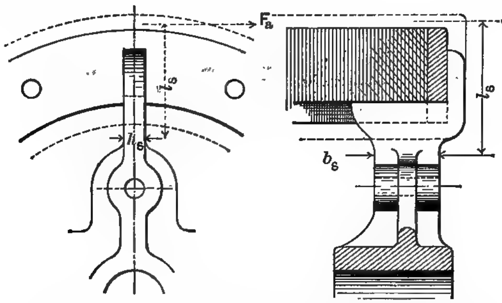


Fig. 127.—Ring Armature Driven by Spiders.

more than necessary of the inner winding circumference should be made unavailable.

For the latter reason the axial breadth of the spokes, b_s, Figs. 127 and 128, is to be made as large as the construction of the armature allows, and their thickness, h_s, calculated to give the requisite strength.

Multiplying equation (95) for the circumferential force per

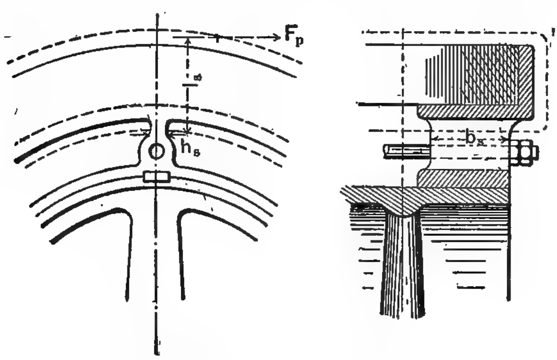


Fig. 128.—Ring Armature Driven by Pulley and End Rings.

armature conductor (§ 41) by the number of effective conductors, we obtain the total peripheral force of the armature:

$$F_a = f_a \times N_c \times \beta'_1 = .7375 \times \frac{E' \times I'}{v_c}, \quad \ldots .(124)$$

and, allowing a factor of safety of about 4, we get:

$$F'_a = 3 \times \frac{P'}{v_c}.$$

Dividing F'_a by the total number of spokes, we have the pull for each spoke, and this multiplied by the leverage gives the external momentum acting on each; the latter must be equal to the internal momentum, *i. e.*, the product of the modulus of the cross-section and the safe working stress of the material. In consequence, we have the equation:

$$\frac{3 \times \frac{P'}{v_c}}{n_s} \times l_s = \frac{b_s \times h_s^2}{6} \times p_s$$

or,

$$b_s \times h_s^2 = 18 \times \frac{P'}{v_c} \times \frac{l_s}{n_s \times p_s}, \quad \ldots\ldots(125)$$

in which b_s = smallest width of spoke (parallel to shaft), in inches;

h_s = smallest thickness of spoke (perpendicular to shaft), in inches;

l_s = leverage at smallest spoke section, *i. e.*, distance of smallest section from active armature conductors, in inches;

n_s = total number of spokes per armature;

P' = total capacity of dynamo, in watts;

v_c = conductor velocity of armature, in feet per second;

p_s = safe working load of material, in pounds per square inch;

for cast iron........p_s =	5,000	lbs. per	square inch.
" brass........... =	6,000	"	"
" phosphor-bronze =	7,000	"	"
" wrought iron.... =	10,000	"	"
" aluminum bronze =	12,000	"	"
" cast steel....... =	15,000	"	"

For spiral windings, now, b_s, as stated above, is given by making it as large as possible, and from (125) we therefore obtain:

$$h_s = 4.25 \times \sqrt{\frac{P'}{v_c} \times \frac{l_s}{n_s \times b_s \times p_s}}. \quad \ldots\ldots(126)$$

For windings external to the core, h_s may be fixed and then b_s calculated from:

$$b_s = 18 \times \frac{P'}{v_c} \times \frac{l_s}{n_s \times h^2_s \times p_s}. \quad \ldots .(127)$$

For very heavy duty dynamos a larger factor of safety should be taken, say from 6 to 8; this will change the numerical coefficient of formulæ (125) and (127) into 27 to 36, and that of equation (126) into 5.3 to 6, respectively.

53. Armature Bearings.

To determine the size of the armature bearings, ordinary engineering practice ought to be followed. In machine design, on account of the increased heat generation at higher velocities, it is the rule to provide a larger bearing surface the higher the speed of the revolving shaft. This rule may, for dynamo shafts, be expressed by the formula:

$$l_b = k_{10} \times d_b \times \sqrt{N}, \quad \ldots\ldots\ldots\ldots(128)$$

where l_b = length of bearing, in inches;
d_b = diameter of bearing, in inches, from formula (122);
N = speed of shaft in revolutions per minute;
k_{10} = constant depending upon kind of armature and on capacity of dynamo. (See Table LIV.)

The numerical values of k_{10} range between .1 and .225 for high-speed armatures, and from .15 to .3 for low-speed armatures, as follows:

TABLE LIV.—VALUE OF CONSTANT IN FORMULA FOR LENGTH OF ARMATURE BEARINGS.

CAPACITY, IN KILOWATTS.	VALUE OF CONSTANT k_{10}.	
	High-Speed Armatures.	Low-Speed Armatures.
Up to 5	.1	.15
" 10	.1	.175
" 50	.125	.2
" 100	.15	.225
" 500	.175	.25
" 1,000	.2	.275
" 2,000	.225	.3

Applying these values to formula (128), and using the journal diameters previously determined, the following Tables LV., LVI., and LVII. are obtained, giving the sizes of bearings for drum armatures, high-speed ring armatures, and low-speed ring armatures, respectively:

TABLE LV.—BEARINGS FOR DRUM ARMATURES.

CAPACITY, IN KILO-WATTS.	VALUE OF CONSTANT k_{10}.	SPEED IN REVS. PER MIN. (FROM TABLE X.) N.	SIZE OF BEARING.		
			Diameter (from Table LI.) d_b	Length. $l_b = k_{10} \times d_b \times \sqrt{N}$.	Ratio. $l_b : d_b$
.1	.1	3,000	$\frac{3}{16}$	1	5.3
.25	.1	2,700	$\frac{5}{16}$	$1\frac{5}{8}$	5.2
.5	.1	2,400	$\frac{7}{16}$	$2\frac{1}{8}$	4.9
1	.1	2,200	$\frac{9}{16}$	$2\frac{5}{8}$	4.7
2	.1	2,000	$\frac{3}{4}$	$3\frac{3}{8}$	4.5
3	.1	1,900	$\frac{15}{16}$	4	4.3
5	.1	1,800	$1\frac{1}{8}$	$4\frac{3}{4}$	4.2
10	.1	1,700	$1\frac{5}{8}$	$6\frac{3}{4}$	4.2
15	.105	1,600	$1\frac{7}{8}$	$7\frac{7}{8}$	4.2
20	.11	1,500	$2\frac{1}{8}$	9	4.2
25	.115	1,350	$2\frac{3}{8}$	10	4.2
30	.12	1,200	$2\frac{1}{2}$	$10\frac{1}{2}$	4.2
50	.125	1,050	3	12	4.0
75	.13	900	$3\frac{5}{8}$	14	3.9
100	.14	750	$4\frac{1}{8}$	$15\frac{3}{4}$	3.8
150	.15	600	$4\frac{3}{4}$	$17\frac{1}{2}$	3.7
200	.16	500	$5\frac{1}{4}$	$18\frac{3}{4}$	3.6
300	.175	400	6	21	3.5

54. Pulley and Belt.

The pulley diameter is determined by the speed of the dynamo and the linear belt velocity:

$$D_p = \frac{12 \times v_B}{\pi \times N} = 3.7 \frac{v_B}{N}, \quad \ldots\ldots\ldots\ldots (129)$$

where D_p = diameter of pulley, in inches;

v_B = belt speed, in feet per minute, see Table LVIII.;

N = speed of dynamo, in revolutions per minute.

The belt speed in modern dynamos ranges between 2,000

TABLE LVI.—BEARINGS FOR HIGH-SPEED RING ARMATURES.

CAPACITY, IN KILO-WATTS.	VALUE OF CONSTANT k_{10}.	SPEED IN REVS. PER MIN. (FROM TABLE XI.) N.	SIZE OF BEARING.		
			Diameter (from Table LII.) d_b	Length. $l_b = k_{10} \times d_b \times \sqrt{N}$.	Ratio. $l_b : d_b$
.1	.1	2,600	$\frac{1}{4}$	$1\frac{1}{4}$	5.0
.25	.1	2,400	$\frac{3}{8}$	$1\frac{7}{8}$	5.0
.5	.1	2,200	$\frac{1}{2}$	$2\frac{3}{8}$	4.75
1	.1	2,000	$\frac{5}{8}$	$2\frac{3}{4}$	4.4
2.5	.1	1,700	1	$4\frac{1}{8}$	4.1
5	.1	1,500	$1\frac{3}{8}$	$5\frac{3}{8}$	3.9
10	.11	1,250	$1\frac{3}{4}$	$6\frac{3}{4}$	3.85
25	.12	1,000	$2\frac{5}{8}$	10	3.8
50	.13	800	$3\frac{1}{2}$	13	3.7
100	.15	600	$4\frac{3}{4}$	$17\frac{1}{2}$	3.7
200	.16	500	$6\frac{3}{8}$	23	3.6
300	.17	450	$7\frac{1}{2}$	27	3.6
400	.175	400	$8\frac{1}{2}$	30	3.5
600	.18	350	10	$33\frac{1}{2}$	3.35
800	.19	300	11	36	3.3
1,000	.2	250	12	38	3.2
1,500	.21	225	14	45	3.2
2,000	.225	200	16	51	3.2

TABLE LVII.—BEARINGS FOR LOW-SPEED RING ARMATURES.

CAPACITY, IN KILO-WATTS.	VALUE OF CONSTANT k_{10}.	SPEED IN REVS. PER MIN. (FROM TABLE XII.) N.	SIZE OF BEARING.		
			Diameter (from Table LIII.) d_b	Length. $l_b = k_{10} \times d_b \times \sqrt{N}$.	Ratio. $l_b : d_b$
2.5	.15	400	$1\frac{1}{8}$	$3\frac{3}{8}$	3.0
5	.16	350	$1\frac{1}{2}$	$4\frac{1}{2}$	3.0
10	.17	300	2	$5\frac{7}{8}$	2.9
25	.18	250	$3\frac{1}{4}$	$8\frac{3}{4}$	2.8
50	.19	200	$4\frac{1}{4}$	$11\frac{1}{2}$	2.7
100	.20	175	$5\frac{3}{4}$	$15\frac{1}{4}$	2.65
200	.21	150	$7\frac{3}{4}$	20	2.6
300	.23	125	$9\frac{1}{8}$	$23\frac{3}{4}$	2.6
400	.25	100	10	25	2.5
600	.265	90	12	30	2.5
800	.27	80	$13\frac{1}{2}$	$32\frac{1}{2}$	2.4
1,000	.28	75	15	36	2.4
1,500	.29	70	18	$43\frac{1}{2}$	2.4
2,000	.30	65	20	48	2.4

and 6,000 feet per minute (= 600 and 1,800 metres per minute), as follows:

TABLE LVIII.—BELT VELOCITIES FOR HIGH-SPEED DYNAMOS OF VARIOUS CAPACITIES.

CAPACITY, IN KILOWATTS.	BELT SPEED, v_B	
	Feet per Minute.	Metres per Minute.
Up to 5	2,000 to 3,000	600 to 900
2.5 " 25	3,000 " 4,000	900 " 1,200
10 " 100	4,000 " 5,000	1,200 " 1,500
50 " 500	5,000 " 6,000	1,500 " 1,800

The pull at the pulley circumference, in pounds, is:

$$F_p = \frac{33{,}000 \times \mathrm{HP}}{\frac{D_p \times \pi}{12} \times N} = \frac{33{,}000 \times \mathrm{HP}}{v_B}$$

$$= \frac{33{,}000 \times \frac{\mathrm{Watts}}{746}}{v_B} = 44.2 \times \frac{P'}{v_B}.$$

For an arc of belt contact of 180°, which can safely be assumed for dynamo pulleys, the pull F_p, is to be multiplied by 1.4 in order to obtain the tension on the tight side of the belt; hence the greatest strain upon the belt:

$$F_B = 1.4 \times F_p = 62 \times \frac{P'}{v_B}.$$

Allowing 300 pounds per square inch as the safe working strain of leather, the necessary sectional area of the belt can be found from

$$b_B \times h_B \doteq \frac{F_B}{300} = .2 \times \frac{P'}{v_B}; \quad \ldots\ldots (130)$$

b_B = width of belt, in inches;
h_B = thickness of belt, in inches;
F_B = greatest strain in belt, *i. e.*, tension on its tight side, in pounds;
P' = capacity of dynamo, in watts;
v_B = belt speed, in feet per minute, Table LVIII.

The approximate thicknesses for the various kinds of belts are:

Single belt................ $h_B = \frac{3}{16}$ inch
Light double belt.......... " $= \frac{9}{32}$ "
Heavy double belt.......... " $= \frac{11}{32}$ "
Three-ply belt............. " $= \frac{7}{16}$ "

Inserting these figures into (130), the width of the belt is obtained:

$$\text{Single belt..........} b_B = \frac{.2}{\frac{3}{16}} \times \frac{P'}{v_B} = 1.1 \times \frac{P'}{v_B} \qquad \textbf{(131)}$$

$$\text{Light double belt....} b_B = \frac{.2}{\frac{9}{32}} \times \frac{P'}{v_B} = .7 \times \frac{P'}{v_B} \qquad \textbf{(132)}$$

TABLE LIX.—SIZES OF BELTS FOR DYNAMOS.

OUTPUT OF DYNAMO IN KILO-WATTS.	THICKNESS OF BELT, INCH.	WIDTH OF BELT, IN INCHES. Belt Speed, in Feet per Minute:								
		2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000
1	Single Belt $\frac{3}{16}$	.8	.7	.6	..	..	..	..	..	..
2	$\frac{3}{16}$	1.6	1.3	1.1	..	..	..	..	..	..
3	$\frac{3}{16}$	2.3	1.8	1.5	1.3	1.2	..	..	..	..
5	$\frac{3}{16}$	3.1	3.0	2.5	2.1	1.9	..	..	..	..
7.5	$\frac{3}{16}$	5.4	4.4	3.6	3.1	2.7	..	..	..	..
10	$\frac{3}{16}$	7.1	5.6	4.7	4.0	3.5	3.1	2.8	..	..
15	$\frac{3}{16}$	10.3	8.3	6.9	5.9	5.2	4.6	4.1	..	..
20	$\frac{3}{16}$	13.4	10.7	9.0	7.7	6.7	6.0	5.4	..	..
25	$\frac{3}{16}$	..	..	10.9	9.4	8.2	7.3	6.6	..	..
30	$\frac{3}{16}$	..	..	13	11.1	9.7	8.6	7.8	..	..
40	$\frac{3}{16}$	..	..	17	14.6	12.8	11.4	10.2	..	..
50	$\frac{3}{16}$	..	..	21	18.0	15.8	14	12.7	11.5	10.5
60	$\frac{3}{16}$	..	..	25	21.4	18.8	16.7	15	13.7	12.5
75	$\frac{3}{16}$	..	..	31	26.5	23.2	20	18.5	17	15.5
100	$\frac{3}{16}$	..	..	..	..	30	27	24.4	22.2	20.4
150	Double Belt. $\frac{9}{32}$	..	..	..	..	29	25.7	23	21	19.3
200	$\frac{11}{32}$	..	..	..	..	33	29.2	26.3	24	22
250	$\frac{11}{32}$	..	..	..	..	40.7	36.2	32.6	29.6	27.2
300	$\frac{11}{32}$	..	..	..	..	51.3	45.5	41	37.3	34.2
400	3-ply $\frac{7}{16}$	..	..	..	..	48.5	43	38.7	35.2	32.2
500	$\frac{7}{16}$	..	..	..	..	60	53.5	48	44	40

$$\text{Heavy double belt} \ldots b_B = \frac{.2}{\frac{11}{32}} \times \frac{P'}{v_B} = .6 \times \frac{P'}{v_B} \quad \textbf{(133)}$$

$$\text{Three-ply belt} \ldots\ldots b_B = \frac{.2}{\frac{7}{16}} \times \frac{P'}{v_B} = .45 \times \frac{P'}{v_B} \quad \textbf{(134)}$$

Single belts are used for all the smaller sizes, up to 100 KW output, *light double* belts up to 200 KW, *heavy doubles* up to 400 KW, and *three-ply* belts for capacities from 400 KW up.

Based upon the above formulæ the author has prepared the preceding Table LIX., from which the belt dimensions for various outputs and for different belt speeds can readily be taken.

The width of the belt being thus determined, the breadth of the pulley-rim is found by adding from ½ inch to 2 inches according to the width of the belt.

PART III.

CALCULATION OF MAGNETIC FLUX.

CHAPTER XI.

USEFUL AND TOTAL MAGNETIC FLUX.

55. Magnetic Field. Lines of Magnetic Force. Magnetic Flux. Field-Density.

The surrounding of a magnetic body, as far as the magnetic effects of the latter extend, is called its *Magnetic Field.*

According to the modern theory of magnetism, magnetic attractions and repulsions are assumed to take place along certain lines, called *Lines of Magnetic Force;* the magnetic field of a magnet, therefore, is the region traversed by the magnetic lines of force emanating from its poles.

The lines of magnetic force are assumed to pass out from the north pole and back again into the magnet at its south pole; their *direction*, therefore, indicates the *polarity* of the magnetic field.

The total *number* of lines of magnetic force in any magnetic field is termed its *Magnetic Flow*, or *Magnetic Flux*, and is a measure of the *amount*, or *quantity* of its magnetism.

The density of the magnetism at any point within the region of magnetic influence of a magnet, or the *Field Density* of a magnet, is expressed by the number of these magnetic lines of force per unit of field area at that point, measured perpendicularly to their direction.

The *Unit of Field Density*—that is, the field density of a unit pole—is 1 line of magnetic force per square centimetre of field area, and is called 1 *gauss.*

A *Single Line of Force*, or the *Unit of Magnetic Flux*, is that amount of magnetism that passes through every square centimetre of cross-section of a magnetic field whose density is unity. To this unit, which was formerly called 1 *weber*, the name of 1 *maxwell* was given at the Paris Electrical Congress, in 1900.

A *Magnet Pole of Unit Strength* is that which exerts unit force upon a second unit pole, placed at unit distance from the former. The lines of force of a single pole, concentrated in one point, are straight lines emanating from this point to all

directions; *i. e.*, radii of a sphere. The surface of a sphere of 1 centimetre radius is 4π square centimetres; a pole of unit strength, therefore, has a magnetic flux of 4π absolute or C. G. S. lines of magnetic force, or of 4π maxwells.

The number of C. G. S. lines of force, or the number of webers expressing the strength of a certain magnetic field, must consequently be divided by 4π, or by 12.5664, in order to give that same field strength in absolute units of magnetism, *i. e.*, in unit-poles.

A magnetic field of unit intensity also exists at the center of curvature of an arc of a circle whose radius is 1 centimetre and whose length is 1 centimetre, when a current of 1 absolute electromagnetic unit of intensity, or of 10 practical electromagnetic units, that is, of 10 amperes, flows through this arc. Therefore, the unit of magnetic flux, *i. e.*, 1 C. G. S. line of force, or 1 maxwell, is equal to

$$\frac{10}{4\pi}$$

practical electromagnetic units, or one practical electromagnetic unit

$$= \frac{4\pi}{10} \text{ maxwells.}$$

56. Useful Flux of Dynamo.

The total number of lines of force cutting the armature conductors is called the *Useful Flux* of the dynamo.

According to the definition given in § 15, we have:

$$\text{Volts} = \frac{\text{Number of C. G. S. Lines cut per second}}{10^8}. \quad \textbf{(135)}$$

Let now Φ = total number of useful lines, or useful flux, in maxwells;

N_c = number of conductors all around pole-facing circumference of armature;

$N_c = n_c \times n_a$, for *ring* armatures;

$N_c = 2 \times n_c \times n_a$, for *drum* armatures and for *drum-wound* ring armatures;

(where n_c = number of commutator-divisions,
n_a = number of turns per commutator-division,
$n_c \times n_a$ = total number of convolutions of armature, see § 25);
N = speed, in revolutions per minute; and
n'_p = number of bifurcations of current in armature, *i. e.*, number of pairs of armature portions connected in parallel, see § 45;

then,—

1 conductor in 1 revolution cuts 2Φ lines of force,

for, the Φ lines emanating from all the north poles, after passing the armature core, return to the south poles, hence pass twice across the air-gaps, and, in consequence, are cut twice in each revolution by every armature conductor.

The armature makes

$$\frac{N}{60}$$

revolutions in 1 second, hence,

$$1 \textit{ conductor in } 1 \textit{ second cuts } 2\Phi \times \frac{N}{60} \textit{ lines.}$$

Each one of the $2\,n'_p$ parallel armature portions contains

$$\frac{N_c}{2\,n'_p}$$

conductors connected in series; in each of these $2\,n'_p$ armature circuits, therefore,

$$\frac{N_c}{2\,n'_p} \textit{ conductors in } 1 \textit{ second cut } 2\,\Phi \times \frac{N}{60} \times \frac{N_c}{2\,n'_p} \textit{ lines.}$$

But, according to the law of the divided circuit, the E. M. F. generated in one of the parallel branches is the output voltage of the machine; the E. M. F. generated by any armature, consequently, by virtue of (135), is

$$E' = \frac{\Phi \times N_c \times N}{n'_p \times 60 \times 10^8} \text{ volts,} \qquad \ldots\ldots\ldots \textbf{(136)}$$

and from this we obtain the number of useful lines required to produce the E. M. F. of E' volts, thus:

$$\Phi = \frac{6 \times n'_p \times E' \times 10^9}{N_c \times N}. \qquad \ldots\ldots\ldots\ldots \textbf{(137)}$$

For dynamos with but one pair of parallel circuits in the armature, *i. e.*, for bipolar machines and for multipolar dynamos with series connections, we have $n'_p = 1$, see (112) and (113), and the useful flux for this special case is:

$$\Phi = \frac{6 \times E' \times 10^9}{N_c \times N}. \qquad \ldots\ldots\ldots\ldots (138)$$

This formula also gives the useful flux *per pole* in *multipolar* dynamos, with *parallel* grouping, and therefore in text-books is usually given instead of (137) as the general formula for the useful flux of a dynamo, which, however, is not strictly correct, and, in consequence, misleading.

57. Actual Field Density of Dynamo.

According to the definition given in § 55, the actual field density of a dynamo is the total useful flux cutting the armature conductors, divided by the area of the actual magnetic field, thus:

$$\mathcal{H}'' = \frac{\Phi}{S_f}, \qquad \ldots\ldots\ldots\ldots\ldots \quad (139)$$

where $\mathcal{H}''$ = field density, in lines of force per square inch;
Φ = useful flux, in maxwells, from formula (137) or (138), respectively; and
S_f = actual field-area, in square inches, *i. e.*, area occupied by the effective armature conductors.

The same formula also holds good for the metric system, the density, $\mathcal{H}$, in gausses, being obtained, if the area, S_f, is expressed in square centimetres.

The actual field density, calculated from (139), is, in general, slightly different from the original field density, selected from Table VI., § 18, and used for the determination of the length of armature conductor, for the reason that, in practice, the length of the polar arc is not fixed with relation to exactly obtaining the assumed field density, but is dimensioned according to a construction rule having reference to the ratio of the distance between pole-corners to the length of gap-spaces (see § 58).

It would be an easy matter to obtain the length of the polar arc and the percentage of its embrace from the assumed field

density, for, supposing that, in a machine with smooth armature, the length of the polepieces is equal to that of the armature core, we would simply have to make the sum of the lengths of the polar arcs of half the number of poles equal to

$$\frac{\Phi}{\mathcal{H}'' \times l_a},$$

or the percentage of the polar arc:

$$\beta_1 = \frac{\Phi}{\mathcal{H}'' \times l_a \times d_f \times \frac{\pi}{2}}, \quad \dots\dots\dots\dots (140)$$

in which β_1 = percentage of polar arc, or quotient of sum of all polar arcs by circumference of mean field-circle;

Φ = useful flux, in maxwells, from (137) or (138);

$\mathcal{H}''$ = assumed field density, in lines per square inch, from Table VI., § 18;

l_a = length of armature core, in inches, formula (40); and

d_f = mean diameter of magnetic field, in inches, which is given by the core diameter of the armature, by the height of its winding space, and by the clearance between the armature winding and the polepieces: $d_f = d_a + h_a + h_c$; see § 58.

But since the polar embrace so determined may not be within the limits of practical design in accordance with the construction rule referred to, it is advisable not to follow the process indicated by formula (140), but to fix the distance between the pole-corners, and thereby the percentage β_1, by that rule, and to calculate the actual field density corresponding to the same by formula (142) or (146), respectively.

This latter method is in no way objectionable, as the new, actual value of $\mathcal{H}''$ only enters the calculation of the magnetomotive force, and the change does not affect any of the previous calculations concerning the dimensions and the winding data of the armature. For, according to formula (136), the same E. M. F. will be generated by a certain number of conductors moving at a constant speed, as long as the total useful

flux remains the same; the E. M. F. generated by a certain armature, therefore, remains constant as long as the product field density and field area is kept at the same value, and it matters not whether this product is made up of the original field density and an area corresponding to the polar embrace found from formula (140), or of a larger actual density and a

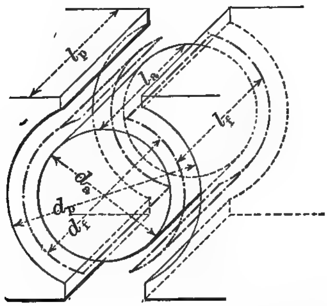


Fig. 129.—Field Area of Bipolar Dynamo.

corresponding reduced field area, or of a smaller density spread over a larger area.

a. Smooth Armatures.

In *smooth*-core armatures, Fig. 129, the area, S_f, occupied by the effective conductors, is obtained from:

$$S_f = d_f \times \frac{\pi}{2} \times \beta'_1 \times l_f; \quad \ldots\ldots\ldots (141)$$

the actual field density, therefore, by inserting (141) into (139) can be found:

$$\mathcal{H}'' = \frac{\Phi}{d_f \times \frac{\pi}{2} \times \beta'_1 \times l_f}, \quad \ldots\ldots\ldots (142)$$

where $\mathcal{H}''$ = actual field density of dynamo, in lines of force per square inch;

Φ = total useful flux of machine, in maxwells, from formula (137) or (138);

d_f = mean diameter of magnetic field, in inches;

$= \frac{1}{2}(d_a + d_p)$;

d_a = diameter of armature core, in inches;

d_p = diameter of bore of polepieces, in inches;

β'_1 = ratio of effective field circumference, obtained from the percentage of polar embrace, β, by means of Table XXXVIII., § 38;

l_f = mean length of magnetic field, in inches,

$= \frac{1}{2}(l_a + l_p)$;

l_a = length of armature core, in inches;

l_p = length of polepiece, in inches;

If d_f and l_f are expressed in centimetres, the field density in gausses, $\mathcal{H}$, is obtained from the same formula (142).

b. Toothed and Perforated Armatures.

For toothed and perforated armatures, the area S_f occupied in the magnetic field by the effective armature conductors, cannot be directly calculated from the dimensions of the armature core, since the path area of the actually useful flux cutting the conductors depends upon too many conditions to be formulated satisfactorily, and it is, therefore, advisable to compute the actual field density, $\mathcal{H}''$, directly from the electrical data of the armature.

According to § 15, the E. M. F. generated per foot of effective armature wire moving at a velocity of 1 foot per second in a field of the density of 1 line per square inch, is

$$\frac{72 \times 10^{-8}}{n'_p} \text{ volt,}$$

if there are n'_p bifurcations of the current in the armature.

For the total effective length of L_e feet of conductor moving at the speed of v_c feet per second in a field of density $\mathcal{H}''$, therefore, the E. M. F. generated is

$$E' = \frac{72 \times 10^{-8}}{n'_p} \times L_e \times v_c \times \mathcal{H}'', \quad \ldots.(143)$$

from which follows the field density:

$$\mathcal{H}'' = \frac{n'_p \times E' \times 10^8}{72 \times L_e \times v_c}. \quad \ldots\ldots\ldots\ldots(144)$$

In this, L_e depends upon the polar embrace, which, in turn, is determined by the ratio of the distance between pole-cor-

ners to the length of the air spaces, and can be expressed in terms of the total active length of wire, by

$$L_e = L_a \times \beta_1 \quad \ldots\ldots\ldots\ldots (145)$$

Inserting (145) into (144), we obtain the actual field density:

$$\mathcal{H}'' = \frac{n'_p \times E' \times 10^8}{72 \times \beta_1 \times L_a \times v_c}, \quad \ldots\ldots\ldots (146)$$

where $\mathcal{H}''$ = actual field density of dynamo, in lines of force per square inch;
n'_p = number of bifurcations of current in armature;
E' = total E. M. F. to be generated in armature, in volts;
β_1 = percentage of polar arc, see § 58;
L_a = length of active armature conductor, in feet, formula (26) or (148);
v_c = conductor speed, in feet per second.

The field density in metric units is obtained from

$$\mathcal{H} = 20{,}000 \times \frac{E \times n'_p}{\beta_1 \times L_a \times v_c}, \quad \ldots\ldots\ldots (147)$$

if L_a is expressed in metres and v_c in metres per second.

Since, in a newly designed armature, on account of rounding off the number of conductors to a readily divisible number and the length of the armature to a round dimension, the actual length, L_a, of the armature conductor, in general, is somewhat different from that found by formula (26), (as a rule, a little greater a value is taken), it is preferable to deduce the accurate value of L_a from the data of the finished armature:

$$L_a = N_c \times \frac{l_a}{12} = \frac{n_w \times n_l}{n_\delta} \times \frac{l_a}{12}, \quad \ldots\ldots (148)$$

where N_c = total number of conductors on armature;
l_a = length of armature core, in inches;
n_w = number of wires per layer;
n_l = number of layers of armature wire;
n_δ = number of wires stranded in parallel.
(see § 23 for n_w, n_l, n_δ.)

Formula (146) for the actual field density of toothed and perforated armatures, can also be used for smooth cores, and may be applied to check the result obtained from (142).

For the application to smooth armatures, however, the polar embrace, β_1, in formula (146) and (147), is to be replaced by the corresponding value of the effective field circumference, β_1, obtained from the former by means of Table XXXVIII., § 38.

If it is desired to know the real field area in toothed and perforated armatures, an expression for S_f can be obtained by combining formulæ (139) and (146), thus:

$$S_f = \frac{\Phi}{\mathcal{H}''} = \frac{72 \times \beta_1 \times L_a \times v_c \times \Phi}{n'_p \times E' \times 10^8}. \quad ..(149)$$

This formula, which gives the mean effective area actually traversed by the useful lines cutting the armature conductors, is very useful for the investigation of the magnetic field of toothed and perforated armatures.

58. Percentage of Polar Arc.

The ratio of polar embrace, to which frequent reference has been had in § 57, is determined by the distance between the pole-corners and by the bore of the polepieces.

a. Distance Between Pole-corners.

The mean distance between the pole-corners, l_p, Fig. 130, depends upon the length of the gap-space between the arma-

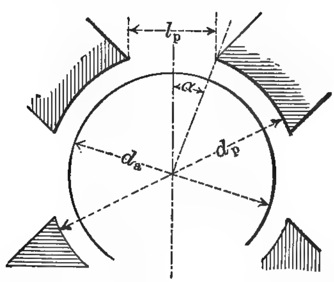


Fig. 130.—Distance Between Pole-corners, and Pole Space Angle.

ture core and the pole face, and is determined by the rule of making that distance from 1.25 to 8 times the length of the two gap-spaces, according to the kind and size of the armature and to the number of poles, see Table LX.

Denoting this ratio of the distance between the pole-corners

to the length of the gaps by k_{11}, this rule can be expressed by the formula:

$$l_p = k_{11} \times (d_p - d_a), \quad \ldots\ldots\ldots\ldots (150)$$

where l_p = mean distance between pole-corners;
d_p = diameter of polepieces;
d_a = diameter of armature core; for toothed and perforated armatures, d_a is the diameter at the bottom of the slots.

The value of k_{11} for various cases may be chosen within the following limits:

TABLE LX.—RATIO OF DISTANCE BETWEEN POLE-CORNERS TO LENGTH OF GAP-SPACES, FOR VARIOUS KINDS AND SIZES OF ARMATURES.

CAPACITY IN KILO-WATTS.	VALUE OF RATIO k_{11}.					
	Smooth Armature.				Toothed or Perforated Armature.	
	Bipolar.		Multipolar.			
	Drum.	Ring.	Drum.	Ring.	Bipolar.	Multipolar.
.1	1.5	2.5	1.5	2.5	1.25	1.25
.25	1.75	3	1.75	2.75	1.5	1.3
.5	2	3.5	2	3	1.75	1.4
1	2.25	4	2.25	3.25	2	1.5
2.5	2.5	4.5	2.75	3.5	2.25	1.6
5	3	5	3	3.75	2.5	1.7
10	3.5	5.5	3.25	4	2.75	1.8
25	4	6	3.5	4.25	3	1.9
50	4.5	6.5	3.75	4.5	3.25	2
100	5	7	4	4.75	3.5	2.1
200	5.5	7.5	4.25	5	3.75	2.2
300	6	8	4.5	5.25	4	2.3
400	....	....	4.75	5.5	....	2.4
600	....	....	5	5.75	....	2.5
800	....	....	....	6	....	2.6
1,000	....	....	....	6.5	....	2.7
1,200	....	....	....	7	....	2.8
1,500	....	....	....	7.5	....	2.9
2,000	....	....	....	8	....	3

Whenever k_{11} can be made larger than given in the above table without reducing the percentage of the polar embrace below its practical limit, it is advisable to do so, and in fact this ratio in some modern machines has values as high as $k_{11} = 12$.

b. Bore of Polepieces.

The diameter of the polepieces, or the bore of the field, d_p, is given by the diameter of the armature core, the height of the armature winding, and the clearance between the armature winding and the polepieces:

$$d_p = d_a + 2 \times (h_a + h_c), \quad \ldots\ldots (151)$$

d_a = diameter of armature core, in inches;
h_a = height of winding space, including insulations and binding wires, in inches;
h_c = radial height of clearance between external surface of finished armature and polepieces, in inches; see Table LXI.

TABLE LXI.—RADIAL CLEARANCE FOR VARIOUS KINDS AND SIZES OF ARMATURES.

Diameter of Armature.		Radial Clearance, h_c.											
		Smooth Armature.										Toothed or Perforated Armature.	
		Disc or Ribbon Core.						Wire Core.					
		Wire Wound.				Copper Bars.		Wire Wound.		Copper Bars.			
		Drum.		Ring.									
inches.	cm.	inch.	mm.	inch.	mm.	inch.	mm.	inch.	mm.	inch.	mm.	inch.	mm.
2	5	$\frac{3}{32}$	2.4	..	...	..	...	$\frac{5}{32}$	4.0	..	...	$\frac{1}{16}$	1.6
4	10	$\frac{1}{8}$	3.2	$\frac{3}{32}$	2.4	..	...	$\frac{3}{16}$	4.8	..	...	$\frac{5}{64}$	2.0
8	15	$\frac{5}{32}$	4.0	$\frac{1}{8}$	3.2	..	...	$\frac{7}{32}$	5.6	..	...	$\frac{3}{32}$	2.4
12	30	$\frac{3}{16}$	4.8	$\frac{5}{32}$	4.0	$\frac{5}{32}$	4.0	$\frac{1}{4}$	6.4	$\frac{7}{32}$	5.6	$\frac{1}{8}$	3.2
18	45	$\frac{7}{32}$	5.6	$\frac{3}{16}$	4.8	$\frac{11}{64}$	4.4	$\frac{9}{32}$	7.2	$\frac{1}{4}$	6.4	$\frac{5}{32}$	4.0
24	60	$\frac{1}{4}$	6.4	$\frac{7}{32}$	5.6	$\frac{3}{16}$	4.8	$\frac{5}{16}$	8.0	$\frac{9}{32}$	7.2	$\frac{3}{16}$	4.8
30	75	$\frac{9}{32}$	7.2	$\frac{1}{4}$	6.4	$\frac{7}{32}$	5.6	$\frac{11}{32}$	8.8	$\frac{5}{16}$	8.0	$\frac{7}{32}$	5.6
40	100	..	...	$\frac{9}{32}$	7.2	$\frac{1}{4}$	6.4	$\frac{3}{8}$	9.6	$\frac{11}{32}$	8.8	$\frac{1}{4}$	6.4
50	125	..	...	$\frac{5}{16}$	8.0	$\frac{9}{32}$	7.2	$\frac{13}{32}$	10.4	$\frac{3}{8}$	9.6	$\frac{9}{32}$	7.2
75	200	..	...	$\frac{3}{8}$	9.6	$\frac{5}{16}$	8.0	$\frac{7}{16}$	11.2	$\frac{13}{32}$	10.4	$\frac{5}{16}$	8.0
100	250	..	...	$\frac{7}{16}$	11.2	$\frac{11}{32}$	8.8	$\frac{1}{2}$	12.8	$\frac{7}{16}$	11.2	$\frac{11}{32}$	8.8
125	300	..	...	$\frac{1}{2}$	12.8	$\frac{3}{8}$	9.6	..	...	..	...	$\frac{3}{8}$	9.6
150	400	..	...	$\frac{9}{16}$	14.4	$\frac{7}{16}$	11.2	..	...	..	...	$\frac{7}{16}$	11.2
200	500	..	...	$\frac{5}{8}$	16.0	$\frac{1}{2}$	12.8	..	...	..	...	$\frac{1}{2}$	12.0

The radial clearance, which is to be taken as small as possible, in order to keep the air-gap reluctance at a minimum, ranges between 1/32 and 7/16 inch, according to the kind of the armature and its size. The preceding Table LXI. may serve as a guide in fixing its limits for any particular case.

The above table shows that with toothed and perforated armatures the smallest clearance can be used, a fact which is explained by the consideration that the exteriors of these armatures offer a solid body, and may be turned off true to the field-bore. For a similar reason wire-core armatures need a larger clearance than disc-core armatures, since the former cannot be tooled in the lathe, and have to be used in the more or less oval form in which they come from the press. Since copper bars can be put upon the body with greater precision than wires, a somewhat larger clearance is to be allowed in the latter case. Finally, a drum armature, in general, has a higher winding space than a ring armature of same size; the unevenness in winding will, consequently, be more prominent in the former case, and therefore a drum armature should be provided with a somewhat larger clearance than a ring of equal diameter.

The figures given in Table LXI. may be considered as average values, and, in specially favorable cases, may be reduced, while under certain unfavorable conditions an increase of the clearance may be desirable.

c. Polar Embrace.

The dimensions of the magnetic field having thus been determined, half the pole-space angle, α, Fig. 130, can be found from the trigonometrical equation:

$$\sin \alpha = \frac{l_p}{d_p}; \quad \ldots\ldots\ldots\ldots\ldots (\mathbf{152})$$

l_p = pole distance, from formula (150);
d_p = diameter of polepieces, from formula (151).

The ratio of polar embrace, or the percentage of polar arc, then, is:

$$\beta_1 = \frac{90^\circ - \alpha \times n_p}{90^\circ}, \quad \ldots\ldots\ldots (\mathbf{153})$$

in which α = half pole-space angle, from (152);
n_p = number of pairs of magnet poles.

From (153) follows, by transposition:

$$\alpha = \frac{90 \times (1 - \beta_1)}{n_p}, \quad \ldots\ldots\ldots\ldots(154)$$

from which the pole-space angle, α, can be calculated in the case that the ratio of embrace, β_1, of the polepieces is given.

59. Relative Efficiency of Magnetic Field.

The useful flux of the dynamo being found from formula (137), the number of lines of force per watt of output, at unit conductor-velocity, will be a measure for the magnetic qualities of the machine, and may be regarded as the *relative efficiency of the magnetic field.*

The field efficiency for any dynamo can accordingly be obtained from the equation:

$$\Phi'_P = \frac{\Phi}{E' \times I'} \times v_c = \frac{\Phi}{P'} \times v_c, \quad \ldots(155)$$

where Φ'_P = relative efficiency of magnetic field, in maxwells per watt of output at a conductor velocity of 1 foot per second.

Φ = useful flux of dynamo, from formula (137) or (138);

E' = total E. M. F. to be generated in machine, in volts;

I' = total current to be generated in machine, in amperes;

P' = $E' \times I'$ = total capacity of machine, in watts;

v_c = conductor velocity, in feet per second.

The numerical value of this constant, Φ'_P, varies between 4,000 and 40,000 lines of force per watt at 1 foot per second, according to the size of the machine, the lower figure corresponding to the highest field efficiency; and for outputs from 1/4 KW to 2,000 KW, for bipolar and for multipolar fields, respectively, ranges as per the following Table LXII., which is averaged from a great number of modern dynamos of all types of field-magnets:

TABLE LXII.—FIELD EFFICIENCY FOR VARIOUS SIZES OF DYNAMOS.

CAPACITY, IN KILOWATTS.	VALUE OF Φ'_p IN MAXWELLS PER WATT, AT UNIT CONDUCTOR VELOCITY. Bipolar Fields.	Multipolar Fields.
Up to .25	15,000 to 40,000	....
.25 to 1	10,000 to 20,000	....
1 to 10	8,000 to 15,000	10,000 to 20,000
10 to 50	7,000 to 12,000	8,000 to 15,000
50 to 100	6,000 to 10,000	7,000 to 12,000
100 to 500	5,000 to 7,500	6,000 to 10,000
500 to 1,000	....	5,000 to 7,500
1,000 to 2,000	....	4,000 to 6,000

For a newly designed machine, the value Φ'_p, obtained by means of formula (155), will be within the limits given in this

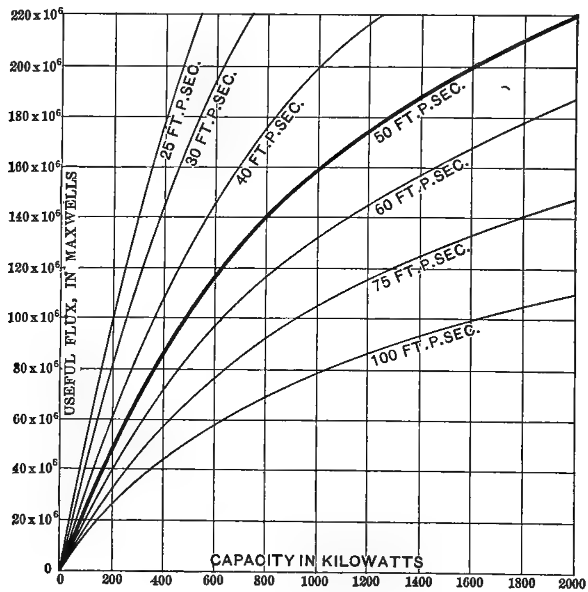


Fig. 131.—Average Useful Magnetic Flux at Different Conductor Velocities for Various Outputs.

table, provided the armature has been calculated in accordance with the rules and tables furnished in the respective Chapters of Part II.

As from Table LXII. follows the self-evident fact that the magnetic fields of large dynamos are more efficient than those of small ones, a curve was plotted in order to examine the rate of this increase. For this purpose the useful fluxes of all the dynamos considered were reduced to the basis of a conductor velocity of 50 feet per second, when the heavy curve, Fig. 131, was obtained by averaging the values of the flux thus found.

From this curve a law can be deduced for the increase of the field efficiency with increasing size. In the following Table LXIII., from the average useful flux for 50 feet conductor velocity, as plotted in Fig. 131, the specific flux per kilowatt has been calculated, showing the rate of increase of the field efficiency:

TABLE LXIII.—VARIATION OF FIELD EFFICIENCY WITH OUTPUT OF DYNAMO.

CAPACITY IN KILOWATTS.	TOTAL AVERAGE USEFUL FLUX AT VELOCITY OF 50 FEET PER SECOND.	SPECIFIC FLUX, IN MAXWELLS PER KILOWATT, AT 50 FEET PER SECOND.
.1	100,000	1,000,000
.25	200,000	800,000
.5	350,000	700,000
1	600,000	600,000
2.5	1,300,000	520,000
5	2,300,000	460,000
10	4,800,000	400,000
25	8,500,000	340,000
50	15,500,000	310,000
75	22,000,000	294,000
100	28,000,000	280,000
200	50,000,000	250,000
300	70,000,000	233,000
400	88,000,000	220,000
500	104,000,000	208,000
600	118,000,000	197,000
700	130,000,000	186,000
800	141,000,000	176,000
900	151,000,000	168,000
1,000	160,000,000	160,000
1,200	175,000,000	146,000
1,500	195,000,000	130,000
2,000	220,000,000	110,000

By the law of inverse proportionality between useful flux and conductor velocity, the remaining curves for 25, 30, 40, 60,

75, and 100 feet per second, respectively, were then drawn in Fig. 131.

Tabulating all the values thus received, we obtain the following Table LXIV., giving average values of the useful flux for various conductor velocities:

TABLE LXIV.—USEFUL FLUX FOR VARIOUS SIZES OF DYNAMOS AT DIFFERENT CONDUCTOR VELOCITIES.

CAPACITY IN KILOWATTS.	AVERAGE USEFUL FLUX, IN MAXWELLS, AT CONDUCTOR VELOCITY, PER SECOND, OF:						
	25 feet (= 7.5 m.)	30 feet (= 9 m.)	40 feet (= 12 m.)	50 feet (= 15 m.)	60 feet (= 18 m.)	75 feet (= 22.5 m.)	100 feet (= 30 m.)
.1	200,000	167,000	125,000	100,000	83,000	....	....
.25	400,000	333,000	250,000	200,000	167,000	....	....
.5	700,000	583,000	438,000	350,000	292,000	....	....
1	1,200,000	1,000,000	750,000	600,000	500,000	400,000	....
2.5	2,600,000	2,200,000	1,600,000	1,300,000	1,100,000	870,000	....
5	4,600,000	3,800,000	2,900,000	2,300,000	1,900,000	1,500,000	....
10	8,000,000	6,700,000	5,000,000	4,000,000	3,300,000	2,700,000	....
25	17,000,000	14,200,000	10,600,000	8,500,000	7,100,000	5,700,000	....
50	31,000,000	25,800,000	19,400,000	15,500,000	12,900,000	10,300,000	....
75	44,000,000	36,700,000	27,500,000	22,000,000	18,300,000	14,700,000	11,000,000
100	56,000,000	46,700,000	35,000,000	28,000,000	23,300,000	18,700,000	14,000,000
200	100,000,000	83,300,000	62,500,000	50,000,000	41,700,000	33,300,000	25,000,000
300	140,000,000	117,000,000	87,500,000	70,000,000	58,300,000	46,700,000	35 000,000
400	....	147,000,000	110,000,000	88,000,000	73,300,000	58,700,000	44,000,000
500	....	173,000,000	130,000,000	104,000,000	86,700,000	68,300,000	52,000,000
600	....	197,000,000	148,000,000	118,000,000	98,300,000	78,700,000	59,000,000
700	....	216,000,000	163,000,000	130,000,000	108,000,000	86,700,000	65,000,000
800	....	235,000,000	176,000,000	141,000,000	117,000,000	94,000,000	70,500,000
900	....	....	189,000,000	151,000,000	126,000,000	101,000,000	75,500,000
1,000	....	....	200,000,000	160,000,000	133,000,000	107,000,000	80,000,000
1,200	....	....	219,000,000	175,000,000	146,000,000	117,000,000	87,500,000
1,500	....	....	244,000,000	195,000,000	163,000,000	130,000,000	97,500,000
2,000	....	....	275,000,000	220,000,000	183,000,000	147,000,000	110,000,000

60. Total Flux to be Generated in Machine.

The *total flux* to be generated in any dynamo is the product obtained in multiplying its *useful flux* by the factor of its *magnetic leakage :*

$$\Phi' = \lambda \times \Phi = \lambda \times \frac{6 \times n'_p \times E' \times 10^9}{N_c \times N}; \quad ..(156)$$

Φ' = total flux to be generated in machine, in lines of force;

Φ = useful flux necessary to produce the required E. M. F. under the given conditions, from formula (137);

λ = factor of magnetic leakage (see Chapters XII. and XIII).

The value of the total magnetic flux in a dynamo directly determines the sectional areas of the various portions of the magnetic circuit in the frame (see Chapter XVI.), and—since the magnetomotive force required depends upon the total magnetic flux to be effected—λ has a direct influence also upon the magnet winding. In calculating a dynamo-electric machine, therefore, it is of great importance to compute the actual value of the total flux, and, consequently, to predetermine with sufficient accuracy the amount of the magnetic leakage.

But, since the dimensions of the magnetic circuit depend upon the total flux to be generated, and since the accurate value of the latter is given by the coefficient of magnetic leakage which in turn for a newly designed machine must be calculated from the dimensions of the magnet frame, it is necessary to proceed as follows:

An approximate value of λ for the type and size of dynamo in question is taken from Table LXVIII., § 70, and the corresponding approximate total flux calculated from formula (156). With the value of Φ' thus obtained the principal dimensions of the magnet frame are determined according to the rules given in Chapter XVI. The dimensions now being known, the probable leakage factor, λ, can be figured from formula (157) or (158), respectively, § 61, the single terms of which are found from the respective formulæ given in Chapter XII.

From formula (156), finally, the accurate value of the total flux is obtained. Should the latter prove so much different from the assumed approximate value of Φ', as to necessitate a change in the dimensions of the frame, then the calculation of λ will have to be partly or wholly repeated.

That such a calculation of the probable leakage factor is necessary in every single case, is evident from the fact that not only the leakage in two machines of same general design, and even of approximately the same size, which are merely differently proportioned in their essential parts, may widely differ from each other, but that in one and the same dynamo the amount of the leakage can be considerably varied by using armatures of different core-diameters in its magnetic field.

From the same reason it can also be concluded that the method of assuming a value of λ from previous experience

with a certain type, or even with an individual machine, is an entirely unreliable one, and that the calculation of the magnetomotive force based upon such an assumption cannot be depended upon.

The author's method of predetermining, from the dimensions of a machine, the *probable* factor of its magnetic leakage is given in the following Chapter XII., while a practical method used by the author for computing the *real* leakage coefficient, from the test of an actual machine, is treated in Chapter XIII.

Professor Forbes' logarithmic formulæ,[1] which are usually given in text-books[2] for the predetermination of magnetic leakage, in the first place are too cumbersome for the practical electrical engineer, and besides leave room for doubt as to their application in special cases; Professor Thompson's formula[3] for the case of leakage between parallel cylinders has been shown[4] to be incorrect; and the empirical formulæ given by Kapp[5] for the leakage resistance of upright and inverted horseshoe types, although being extremely simple, have not much practical value, as they merely have reference to the size of the machine and are independent of the dimensions and the design of the field frame, and will therefore give correct results only in case of dynamos having exactly the same relative proportions as those experimented upon by Kapp.

It is therefore believed that the establishment of the geometrical formulæ presented in Chapter XII., which are simple in form, concise in application, and accurate in result, has removed the principal difficulties heretofore experienced with leakage calculations.

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[1] George Forbes, *Journal Society Telegraph Engineers*, vol. xv. p. 531, 1886.

[2] S. P. Thompson, "Dynamo-Electric Machinery," fifth edition, p. 156.

[3] S. P. Thompson, "Lectures on the Electro-Magnet," authorized American edition, p. 147.

[4] A. E. Wiener, "Magnetic Leakage in Dynamo-Electric Machinery," *Electrical Engineer*, vol. xviii. p. 164 (August 29, 1894).

[5] Gisbert Kapp, "Electric Transmission of Energy," third edition, p. 122.

CHAPTER XII.

CALCULATION OF LEAKAGE FACTOR FROM DIMENSIONS OF MACHINE.

A. Formula for Probable Leakage Factor.

61. Coefficient of Magnetic Leakage for Dynamos with Smooth and with Toothed or Perforated Armatures.

Since air is a conductor of magnetism, the conditions of the magnetic circuit of a dynamo-electric machine resemble those of a closed metallic electric circuit immersed in a conducting fluid. In the latter case, the main current will flow through the metallic conductors, but a portion will pass through the fluid. Similarly, in the dynamo, the main path for the lines of force being the magnetic circuit consisting of the iron field frame, the air gaps, and the armature core, a portion of the magnetic flux will take its way through the surrounding air. The amount of electric current passing through the surrounding medium, the fluid, depends upon the ratio between the conductances of the main to the shunt paths. In order to calculate the amount of magnetic leakage in a dynamo, therefore, it is, analogically, only necessary to determine the ratio between the permeances of the useful and the stray paths.

a. Smooth Armature.

The leakage factor in any dynamo having a smooth armature can accordingly be expressed as the quotient of the total joint permeance of the system by the permeance of the useful path. But since the reluctance of the iron portion of the main path is very small compared with that of the air gaps, the sum of their reciprocals, that is, the total permeance of the useful path, is practically equal to the permeance of the gaps; hence the permeance of the gaps can be taken as a substitute of the permeance of the whole magnetic circuit within

the machine, and we obtain the following formula for the probable leakage factor of any dynamo having a *smooth* armature :

$$\lambda = \frac{\text{Joint permeance of useful and stray paths}}{\text{Permance of useful path}},$$

or,

$$\lambda = \frac{\mathfrak{P}_1 + \mathfrak{P}_2 + \mathfrak{P}_3 + \mathfrak{P}_4}{\mathfrak{P}_1}, \quad \ldots\ldots\ldots(157)$$

where $\mathfrak{P}_1$ = relative permeance of the air gaps (useful path);
$\mathfrak{P}_2$ = relative average permeance across magnet cores (stray path);
$\mathfrak{P}_3$ = relative permeance across polepieces (stray path);
$\mathfrak{P}_4$ = relative permeance between polepieces and yoke (stray path).

The *relative* permeances—by which are understood the absolute permeances divided by the magnetic potential, and which, therefore, include a constant factor, on account of the units chosen—are taken for convenience, for, in each individual case the maximum magnetic potential is the same for all permeances, and a constant numerical factor, if absolute permeances were used, would be common to all terms in (157), and consequently would cancel.

b. Toothed and Perforated Armature.

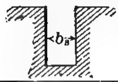
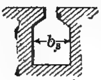
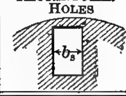
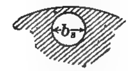
In *toothed* and *perforated* armatures a portion of the magnetic lines of the main path enters the iron projections of the core and passes through the armature without cutting the conductors. This portion, therefore, cannot be considered as useful, and has to be taken into account in computing the total leakage coefficient of the machine. Introducing this leakage into the calculation in the form of a factor, the *factor of armature leakage*, we obtain the probable leakage factor of any dynamo having a *toothed* or *perforated* armature:

$$\lambda' = \lambda_1 \times \lambda = \lambda_1 \times \frac{\mathfrak{P}_1 + \mathfrak{P}_2 + \mathfrak{P}_3 + \mathfrak{P}_4}{\mathfrak{P}_1}. \quad (158)$$

The factor, λ_1, of this core-leakage, that is, the ratio of the total flux of the useful path passing the air gaps to the actual useful flux cutting the armature conductors, or to the total flux

through the gaps minus that portion leaking through the teeth, depends upon the relative sizes of the slots to the teeth, and for armatures otherwise properly dimensioned, has been found to average within the following limits:

TABLE LXV.—CORE LEAKAGE IN TOOTHED AND PERFORATED ARMATURES.

RATIO OF WIDTH OF SLOTS TO THEIR PITCH ON OUTER CIRCUMFERENCE $b_s : \frac{d''_a \pi}{n'_c}$	FACTOR OF ARMATURE LEAKAGE, λ_1			
	TOOTHED ARMATURES		PERFORATED ARMATURES	
	STRAIGHT TEETH 	PROJECTING TEETH 	RECTANGULAR HOLES 	ROUND HOLES 
0.35	1.06 to 1.04			
.4	1.05 " 1.03	1.10 to 1.04		
.45	1.04 " 1.02	1.07 " 1.03		
.5	1.03 " 1.01	1.05 " 1.02	1.07 to 1 04	1.10 to 1.06
.55	1.02 " 1.005	1.03 " 1.01	1.06 " 1.03	1.08 " 1.05
.6	1.01 " 1.0025	1.02 " 1.005	1.05 " 1.02	1.06 " 1.04
.65			1.04 " 1.01	1.05 " 1.03
.7			1.03 " 1.01	1.04 " 1.02

B. GENERAL FORMULÆ FOR RELATIVE PERMEANCES.

62. Fundamental Permeance Formula and Practical Derivations.

In order to obtain the values of the permeances of the various paths, we start from the general law of conductance:

$$\text{Conductance} = \left\{ \begin{matrix} \text{Conductivity} \\ \text{of medium} \end{matrix} \right\} \times \frac{\text{Area of medium}}{\text{Distance in medium}},$$

or, in our case of *magnetic* conductance:

$$\text{Permeance} = \text{Permeability} \times \frac{\text{Area}}{\text{Length}}.$$

Since the permeability of air = 1, the relative leakage permeance between two surfaces can be expressed by the general formula:

$$\mathfrak{P} = \frac{\text{Mean area of surfaces exposed}}{\text{Mean length of path between them}}. \quad \textbf{(159)}$$

From this, formulæ for the various cases occurring in practice can be derived.

a. Two plane surfaces, inclined to each other.

In order to express, algebraically, the relative permeance of the air space between two inclined plane surfaces, Fig. 132, the mean path is assumed to consist of two circular arcs joined by a straight line tangent to both circles, said arcs to be described from the edges of the planes nearest to each other, as

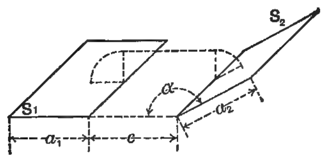


Fig. 132.—Two Plane Surfaces Inclined to Each Other.

centres, with radii equal to the distances of the respective centres of gravity from those edges. Hence:

$$\mathfrak{P} = \frac{\frac{1}{2}(S_1 + S_2)}{c + \frac{a_1 + a_2}{2} \times \frac{\pi}{2} \times \frac{\alpha}{180^\circ}}, \qquad \ldots\ldots\ldots(160)$$

where S_1, S_2 = areas of magnetic surfaces;
c = least distance between them;
a_1, a_2 = widths of surfaces S_1 and S_2, respectively;
α = angle between surfaces S_1 and S_2.

b. Two parallel plane surfaces facing each other.

If the two surfaces S_1 and S_2 are parallel to one another, Fig. 133, the angle inclosed is $\alpha = 0^\circ$, and the formula for

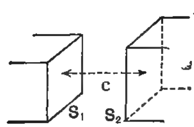


Fig. 133.—Two Parallel Plane Surfaces Facing Each Other.

the relative permeance, as a special case of (160), becomes:

$$\mathfrak{P} = \frac{\frac{1}{2}(S_1 + S_2)}{c}. \qquad \ldots\ldots\ldots\ldots(161)$$

c. Two equal rectangular surfaces lying in one plane.

In case the two surfaces lie in the same plane, Fig. 134, they inclose an angle of $\alpha = 180°$, and the permeance of

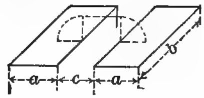


Fig. 134.—Two Equal Rectangular Surfaces Lying in One Plane.

the air between them, by formula (160), is:

$$\mathfrak{P} = \frac{a \times b}{c + a \times \frac{\pi}{2}}, \quad \ldots\ldots\ldots\ldots\ldots (162)$$

a = width of rectangular surface;
b = length of rectangular surface;
c = least distance between surfaces.

d. Two equal rectangles at right angles to each other.

If the two surfaces are rectangular to each other, Fig. 135,

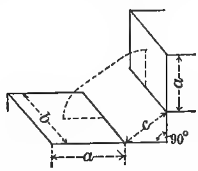


Fig. 135.—Two Equal Rectangles at Right Angles to Each Other.

the angle $\alpha = 90°$, formula (160), consequently, reduces to

$$\mathfrak{P} = \frac{a \times b}{c + a \times \frac{\pi}{4}}. \quad \ldots\ldots\ldots\ldots\ldots (163)$$

e. Two parallel cylinders.

In case the two surfaces are cylinders of diameter, d, and length, l, Fig. 136, the areas of their surfaces are $d \times \pi \times l$; and if they are placed parallel to each other, at a distance, c, apart, the mean length of the magnetic path is $c + \frac{3}{4} d$; hence the permeance of the air between them:

$$\mathfrak{P} = \frac{d \times \pi \times l}{c + \frac{3}{4} d}. \quad \ldots\ldots\ldots\ldots\ldots (164)$$

In this formula the expression for the mean length of the path is deduced from Fig. 137, in which it is assumed that the

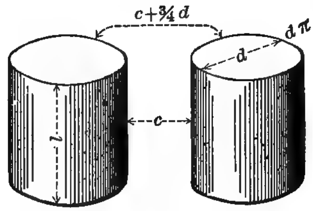


Fig. 136.—Two Parallel Cylinders.

mean path consists of two quadrants joined by a straight line of length c, and extends between two points of the cylinder-peripheries situated at angles of 60° from the centre line.

Since in an equilateral triangle the perpendicular, dropped from any one corner upon the opposite side, bisects that side,

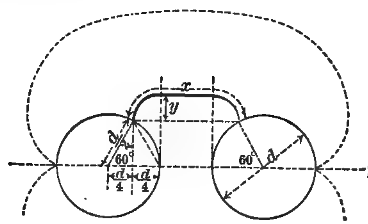


Fig. 137.—Leakage Path Between Parallel Cylinders.

the perpendicular, from either of the endpoints of the mean path upon the centre line, bisects the radius of the corresponding cylinder-circle, and the radius of the leakage-path quadrant is

$$y = \frac{d}{4};$$

hence the length of the mean path:

$$x = c + y \times \pi = c + d \times \frac{\pi}{4},$$

or, approximately:

$$x = c + \tfrac{3}{4} d.$$

This approximation even better meets the practical truth, as most of the leakage takes place directly across the cylinders.

and the mean path, therefore, in reality is situated at an angle of somewhat less than 60°, which was taken for convenience in the geometrical consideration.

f. Two parallel cylinder-halves.

If two cylinder-halves face each other with their curved surfaces, Fig. 138, the mean length of the magnetic path is $c + .3\,d$, where c is the least distance apart of the curved sur-

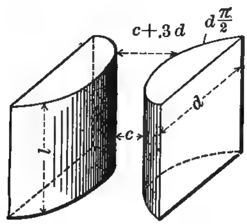


Fig. 138.—Two Parallel Cylinder-Halves.

faces, and d the diameter of the cylinders, and we have for the permeance:

$$\mathfrak{P} = \frac{d \times \frac{\pi}{2} \times l}{c + .3\,d} = \frac{d \times \pi \times l}{2\,c + .6\,d}. \quad \ldots (165)$$

The mean length of the path is geometrically found from Fig. 139, as follows:

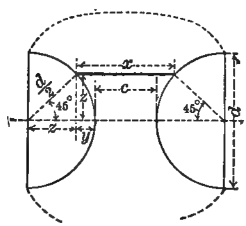


Fig. 139.—Leakage Path Between Parallel Cylinder-Halves.

$$2\,z^2 = \left(\frac{d}{2}\right)^2; \quad z = \frac{d}{2}\sqrt{\frac{1}{2}}$$

$$\therefore y = \frac{d}{2} - z = \frac{d}{2}\left(1 - \sqrt{\frac{1}{2}}\right) = .15\,d$$

$$x = c + 2\,y = c + .3\,d.$$

For, in this case, the extent of the leakage field is much smaller than in that of full cylinders, and the mean path can be assumed a straight line meeting the two semicircles at an angle of 45° from the centre line.

C. RELATIVE PERMEANCES IN DYNAMO-ELECTRIC MACHINES.

63. Principle of Magnetic Potential.

In taking the magnetic potential between two polepieces of opposite polarity as unity for calculating the relative permeances in dynamo-electric machines, the potentials between various points of the magnetic circuit depend upon the number of magnet-cores magnetically in series between two consecutive poles of opposite polarity. If, as is the case in the majority of types, there are two magnets between any north-pole and the next south-pole of the machine, then the magnetic potential between two points of the magnetic circuit separated by but one magnet, is $= \frac{1}{2}$; and two points not separated by a magnet core, have no difference of magnetic potential, their potential $= 0$. If the circuit consists of but one magnet, or of several magnets magnetically in parallel, then the magnetic potential between any two leakage surfaces of opposite polarity is $= 1$, *i. e.*, the difference of magnetic potential between the polepieces.

The observance of this general principle enables us to bring all the relative permeances into proper relation to each other, and we can now apply formulæ (160) to (165) to the cases of a dynamo.

64. Relative Permeance of the Air Gaps ($\mathfrak{P}_1$).

a. Smooth Armature.

In dynamos with smooth-core armatures the relative permeance of the air gaps simply is the quotient of the mean field area by the mean length of the lines in the gap-space. The mean area of the gap-space for any armature facing poles opposite its outer periphery is given by formula (141), § 57, while the mean length of the path, in both gaps, for smooth armature cores is:

$$l''_g = k_{12} \times (d_p - d_a), \quad \ldots\ldots\ldots (166)$$

where k_{12} is a constant depending upon the degree of deflection of the lines of force. It is well known that in a dynamo-electric machine, when the armature is in motion, the lines do not cross the air gaps at right angles, but are deflected into an oblique position, Fig. 140, owing to the shifting of the neutral line.

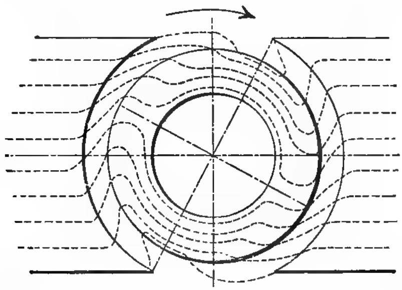


Fig. 140.—Deflection of Lines of Force in Gap-Space.

The amount of this deflection naturally depends upon the speed of the revolving armature and upon the density of the lines, and in machines with smooth-surface armatures increases solely with the product of these two quantities:

TABLE LXVI.—FACTOR OF FIELD DEFLECTION IN DYNAMOS WITH SMOOTH-SURFACE ARMATURES.

PRODUCT OF CONDUCTOR VELOCITY AND FIELD DENSITY, $v_c \times \mathcal{H}$.		FACTOR OF FIELD DEFLECTION, k_{12}.
English Measure. Velocity, in feet per second. Density, in lines per sq. inch.	Metric Measure. Velocity, in metres per second. Density, in lines per cm.²	Smooth-Core and Perforated Armatures.
Below 400,000	Below 20,000	1.10 to 1.15
400,000 to 500,000	20,000 to 25,000	1.11 " 1.16
500,000 " 600,000	25,000 " 30,000	1.12 " 1.18
600,000 " 700,000	30,000 " 35,000	1.13 " 1.20
700,000 " 800,000	35,000 " 40,000	1.14 " 1.22
800,000 " 900,000	40,000 " 45,000	1.15 " 1.25
900,000 " 1,000,000	45,000 " 50,000	1.16 " 1.27
1,000,000 " 1,100,000	50,000 " 55,000	1.17 " 1.30
1,100,000 " 1,250,000	55,000 " 62,500	1.18 " 1.32
1,250,000 " 1,500,000	62,500 " 75,000	1.19 " 1.35
1,500,000 " 1,750,000	75,000 " 87,500	1.20 " 1.40
1,750,000 " 2,000,000	87,500 " 100,000	1.22 " 1.45
Over 2,000,000	Over 100,000	1.25 " 1.50

The preceding Table LXVI. is averaged from a great number of dynamos under various conditions, and gives values of k_{12} for smooth as well as perforated armatures.

Combining formulæ (141) and (166), we obtain for the case of cylindrical smooth-core armatures with *external* poles:

$$\mathfrak{P}_1 = \frac{S_g}{l''_g} = \frac{d_f \times \frac{\pi}{2} \times \beta'_1 \times l_f}{k_{12} \times (d_p - d_a)}, \quad \ldots\ldots (167)$$

in which $\mathfrak{P}_1$ = relative permeance of air gaps;

S_g = mean area of gap-space;

l''_g = mean length of path in both gaps;

d_f = mean diameter of magnetic field, $= \frac{1}{2}(d_a + d_p)$;

d_a = diameter of armature core;

d_p = diameter of bore of polepieces;

l_f = mean length of magnetic field, $= \frac{1}{2}(l_a + l_p)$;

l_a = length of armature core;

l_p = length of polepieces;

(See Fig. 129, page 204.)

β'_1 = percentage of effective gap circumference, see Table XXXVIII., § 38;

k_{12} = factor of field deflection, Table LXVI.

For armature revolving *outside* of a magnetic field, as in the innerpole types, in the denominator of formula (167), the order of the diameters d_a and d_p is to be reversed, as in this case d_a, the *internal* diameter of the armature core, is larger than the diameter of the pole-bore.

If poles are situated *interior* as well as *exterior* to the armature, the mean of the outer and inner gap areas has to be taken by applying formula (141) to the inner diameter as well as to the outer diameter of the core; and instead of $(d_p - d_a)$ the sum of the outer and inner gaps is to be substituted. Finally, in case of armatures facing the poles in the *axial* direction, as in the flat ring armature type, the gap area, if polepieces are used, is the mean of half the pole area and the ring area of the armature core; and if no separate polepieces

are employed, is practically equal to half the sectional area of the magnet cores. The mean length of the path is the difference between the axial pole distance and the axial breadth of the armature core, multiplied by the factor of field deflection.

b. Toothed and perforated armatures.

In machines with *toothed* and *perforated* armatures the air gaps are composed of the clearance spaces between the tops of the iron projections and the pole surfaces, and of the spaces between the tops of the projections and the bottoms of the

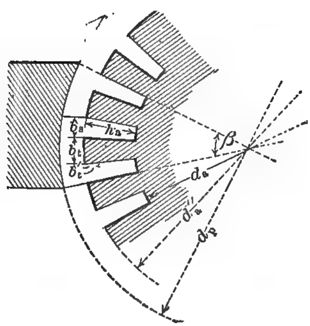


Fig. 141.—Gap-Space of Toothed Armature.

slots; and the relative permeance of the gaps, consequently, is the sum of the relative permeance of the clearance spaces ($\mathfrak{P}'$) plus the relative joint permeance of the teeth ($\mathfrak{P}''$) and of the slots ($\mathfrak{P}'''$), or, in symbols:

$$\mathfrak{P}_1 = \frac{1}{\frac{1}{\mathfrak{P}'} + \frac{1}{\mathfrak{P}'' + \mathfrak{P}'''}} = \frac{\mathfrak{P}' \times (\mathfrak{P}'' + \mathfrak{P}''')}{\mathfrak{P}' + \mathfrak{P}'' + \mathfrak{P}'''} \quad \ldots (168)$$

The permeances of the clearance spaces, of the teeth, and of the slots, respectively, can be expressed, with reference to Fig. 141, as follows:

$$\mathfrak{P}' = \frac{\frac{1}{4}[d_p \, \pi \times \beta_1 + (b_t + b'_t) \times n'_c \times \beta'_1] \times l_f}{k_{12} \times (d_p - d''_a)}; \quad (169)$$

for *straight* teeth: .... $b'_t = \frac{b_s}{2}$.

" *projecting* teeth: .... b'_t = radial depth of tooth projection.

" *perforated* armatures: ... $(b_t + b'_t) \times n'_c = d''_a \times \pi$.

$$\mathfrak{T}'' = \frac{d_a \pi - b_s \times n'_c}{h_a} \times l_a \times k_2 \times \beta_1 \times \mu ; \quad (170)$$

$$\mathfrak{T}''' = \frac{b_s \times l_a}{h_a} \times n'_c \times \beta_1 . \quad \ldots\ldots\ldots\ldots (171)$$

The symbols used in these formulæ are:

$\mathfrak{T}'$ = relative permeance of clearance spaces;
$\mathfrak{T}''$ = relative permeance of teeth;
$\mathfrak{T}'''$ = relative permeance of slots;
d_a = diameter at bottom of slots;
d''_a = diameter at top of teeth;
d_p = diameter of bore of polefaces;
b_s = breadth of armature slots;
b_t = top width of armature teeth;
b'_t = radial spread of magnetic lines along teeth;
l_a = length of armature core;
l_f = length of magnetic field;
n'_c = number of armature slots;
β_1 = percentage of polar arc,
$= \frac{n_p \times \beta}{180}$;
n_p = number of pairs of poles,
β = pole angle;
β'_1 = percentage of effective gap circumference, see Table XXXVIII., § 38;
k_2 = ratio of magnetic to total length of armature core, Table XXIII., § 26;
k_{12} = factor of field deflection, see Table LXVII., below;
μ = permeability of iron in armature teeth, at density employed, see Table LXXV., § 81.

Formulæ (170) and (171) apply directly only to *straight-tooth* armatures. For *projecting teeth* the same formulæ, however, can be used if the dimensions of the projecting tooth are

replaced by those of a straight tooth of equal volume, as indicated by Fig. 142, the reduced width of the slot, b_{s_1}, taking the place of the actual width, b_s. For *perforated* armatures with *rectangular* holes (Fig. 143) the slot permeance is directly expressed by formula (171), while the permeance of the iron projections is equal to that of straight teeth having equal volume. In formula (170), consequently, the reduced width, b_{s_1}, and in (171) the actual width, b_s, of the holes is to be used. For *round* and *oval* perforations, Figs. 144 and 145, respectively, the iron projections being transformed into straight

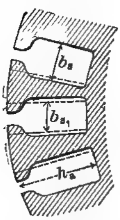


Fig. 142.

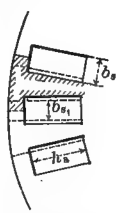


Fig. 143.

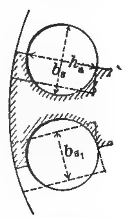


Fig. 144.

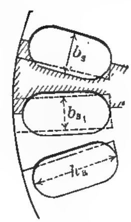


Fig. 145.

Figs. 142 to 145.—Geometrical Substitution of Projecting Teeth and Hole-Projections by Straight Teeth of Equal Volume.

teeth of equal volume, the reduced width, b_{s_1}, of the perforation is to be used in both (170) and (171).

The permeance of the teeth, $\mathfrak{P}''$, on account of the high value of the permeability, μ, at even comparatively high saturation of the teeth, is very large compared with the permeance of the slots, $\mathfrak{P}'''$, so that for all practical purposes $\mathfrak{P}''^{r}$ in (168) may be neglected, and we have:

$$\mathfrak{P}_1 = \frac{\mathfrak{P}' \times \mathfrak{P}''}{\mathfrak{P}' + \mathfrak{P}''} \quad \ldots\ldots\ldots\ldots\ldots(172)$$

The permeance of the clearance space, $\mathfrak{P}'$, furthermore, is so small compared with $\mathfrak{P}''$, that their sum $\mathfrak{P}' + \mathfrak{P}''$ is practically equal to $\mathfrak{P}''$, and by canceling we obtain the approximate formula:

$$\mathfrak{P}_1 = \mathfrak{P}', \quad \ldots\ldots\ldots\ldots\ldots\ldots(173)$$

which can be used with sufficient accuracy in all cases where the magnetization in the teeth is not driven beyond 100,000 lines per square inch (= 15,500 gausses).

Inserting the values from (169) into (173) we obtain for *straight-tooth* armatures:

$$\mathfrak{T}_1 = \frac{\frac{1}{4}\left[d_p \times \pi \times \beta_1 + \left(b_t + \frac{b_s}{2}\right) \times n'_c \times \beta'_1\right] \times l_f}{k_{12} \times (d_p - d''_a)}; \quad (174)$$

for *projecting-tooth* armatures:

$$\mathfrak{T}_1 = \frac{\frac{1}{4}\left[d_p \times \pi \times \beta_1 + (b_t + b'_t) \times n'_c \times \beta'_1\right] \times l_f}{k_{12} \times (d_p - d''_a)}; \quad (175)$$

and for *perforated* armatures:

$$\mathfrak{T}_1 = \frac{\frac{\pi}{4}(d_p \times \beta_1 + d''_a \times \beta'_1) \times l_f}{k_{12} \times (d_p - d''_a)}. \quad \ldots (176)$$

TABLE LXVII.—FACTOR OF FIELD DEFLECTION IN DYNAMOS WITH TOOTHED ARMATURES.

RATIO OF RADIAL CLEARANCE TO PITCH OF SLOTS ON OUTER CIRCUMFERENCE.	FACTOR OF FIELD DEFLECTION, k_{12}, FOR TOOTHED ARMATURES. Product of Conductor Velocity and Field Density, in English Measure.				
	500,000	1,000,000	1,500,000	2,000,000	2,500,000
0.1	1.90	2.00	2.10	2.20	2.30
.15	1.80	1.90	2.00	2.10	2.20
.2	1.70	1.80	1.90	2.00	2.10
.25	1.60	1.70	1.80	1.90	2.00
.3	1.50	1.60	1.70	1.80	1.90
.35	1.40	1.50	1.60	1.70	1.80
.4	1.35	1.40	1.50	1.60	1.70
.45	1.30	1.35	1.40	1.50	1.60
.5	1.25	1.30	1.35	1.40	1.50
.55	1.20	1.25	1.30	1.35	1.40
.6	1.15	1.20	1.25	1.30	1.35
.65	1.12	1.15	1.20	1.25	1.30
.7	1.10	1.12	1.15	1.20	1.25

The amount of the field deflection in machines with toothed armatures is primarily governed by the ratio of the clearance space to the pitch of the slots, and only secondarily depends upon the product of conductor velocity and field density.

The values for use with formulæ (174) and (175) are compiled in the above Table LXVII., while those for use with formula (176) are contained in the previous Table LXVI. Table LXVII. refers to *straight* teeth only; in case of armatures with *projecting* teeth, the average of the values from Table LXVII. and from LXVI. for a corresponding perforated armature must be taken.

65. Relative Average Permeance between the Magnet Cores ($\mathfrak{P}_2$).

Since in dynamo-electric machines the magnet cores, with their ends averted from the armature, are magnetically joined by special "yokes" or by the frame of the machine, forming the magnetic return circuit, the magnetic potential between these joined ends is practically = 0, while the full magnetic potential is operating between the free ends toward the armature. The average magnetic potential over the whole length of the magnet cores, therefore, is one-half of the maximum potential, and the average relative permeance, consequently, one-half of that which would exist between the cores, if they had the same magnetic potential all over their length.

For the various forms of magnet cores, by virtue of formulæ (160) to (165), respectively, we therefore obtain the following relative average permeances:

a. Rectangular Cores.

The permeance between two rectangular magnet cores, Fig. 146, is the sum of the permeances between the inner surfaces

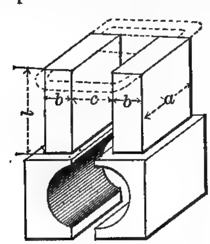


Fig. 146.—Rectangular Magnet Cores.

which face each other, formula (161), and between the end surfaces which lie in the same plane, formula (162); and therefore the average permeance:

$$\mathfrak{P}_2 = \frac{a \times l}{2\,c} + \frac{b \times l}{c + b \times \frac{\pi}{2}}, \quad \ldots\ldots(177)$$

where a, b, c, and l are the dimensions of the cores in inches, see Fig. 146.

b. Round Cores.

According to formula 164, we have in this case, see Fig. 147:

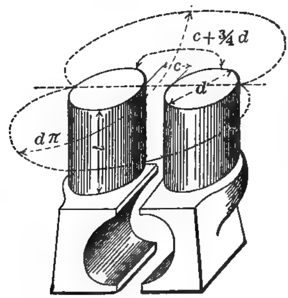


Fig 147.—Round Magnet Cores.

$$\mathfrak{P}_2 = \frac{1}{2} \times \frac{d\,\pi \times l}{c + \frac{3}{4}\,d} = \frac{d\,\pi \times l}{2\,c + 1.5\,d}, \quad \ldots(178)$$

c. Oval Cores.

For oval cores, Fig. 148, the permeance path consists of two parts, a straight portion between the inner surfaces, and a

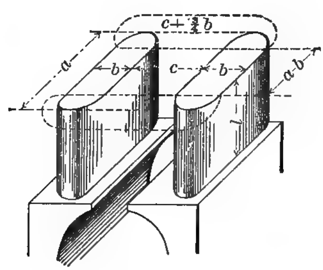


Fig. 148.—Oval Magnet Cores.

curved portion between the round end surfaces. Combining, therefore, formulæ (161) and (164), we obtain:

$$\mathfrak{P}_2 = \frac{(a - b) \times l}{2\,c} + \frac{b\,\pi \times l}{2\,c + 1.5\,b}, \quad \ldots\ldots(179)$$

d. Inclined Cores.

If the cores, instead of being parallel to each other, are set at an angle, Fig. 149, the distance, c, in formulæ (177), (178),

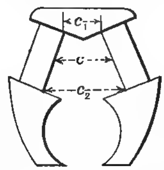


Fig. 149.—Inclined Magnet Cores.

and (179), respectively, has to be averaged from the least and greatest distance of the cores:

$$c = \frac{c_1 + c_2}{2}, \quad \ldots\ldots\ldots\ldots\ldots(180)$$

e. Multipolar Types.

In case of multipolar dynamos of n_p pairs of poles, the total permeance across the magnet cores is $2\,n_p$ times that between each pair of cores. In calculating the latter, it has to be considered that, while the permeance across two opposite side surfaces of the cores does not change by increasing their number, the leakage across two end surfaces is reduced, half of the lines leaking to the neighboring core at one side, and half to that on the other side.

For *rectangular* cores, therefore, we have, with reference to Fig 150:

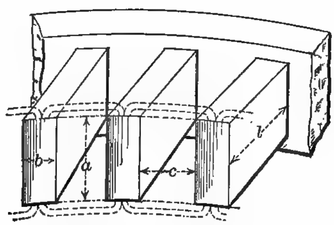


Fig. 150—Multipolar Frame with Rectangular Cores.

$$\mathfrak{P}_2 = 2n_p \times \left(\frac{a \times l}{2\,c} + \frac{\frac{1}{2}\,b \times l}{c + b\,\frac{\pi}{4}}\right)$$

$$= n_p \times \left(\frac{a \times l}{c} + \frac{b \times l}{c + b\,\frac{\pi}{4}}\right); \quad \ldots\ldots(181)$$

for *round* cores, according to formula (165):

$$\mathfrak{T}_2 = 2\, n_p \times \frac{d\,\pi \times l}{2(2\,c + .6\,d)} = n_p \times \frac{d\,\pi \times l}{2\,c + .6\,d}; \quad (182)$$

and for *oval* cores:

$$\mathfrak{T}_2 = n_p \times \left(\frac{(a-b) \times l}{2\,c} + \frac{b \times \pi \times l}{2\,c + .6\,b}\right). \quad (183)$$

In multipolar machines, for c, the smaller of either the mean distance between two magnets, Fig. 151, or twice the

Figs. 151 and 152.—Mean Length of Leakage Path between Magnet Cores in Multipolar Dynamos.

mean distance between magnet core and yoke, Fig. 152, is to be taken.

f. Iron-clad Types.

In certain types of dynamos, known as "*Iron-clad*" forms because of their yokes constituting a complete encompassment around the machine, if there are two magnet cores, they are not side by side of each other, but lie in line and are faced by the yokes connecting the same (Figs. 153 and 155).

1. *Bipolar Iron-clad Type.*

Considering that in the ordinary bipolar iron-clad type, Fig. 153, the magnetic potential between the pole ends of the

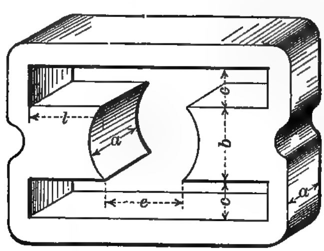


Fig. 153.—Bipolar Iron-clad Type.

cores is unity, between the yoke ends, however, is zero, and at intermediate points, consequently, is given by the ratio of

the distance from the yoke to the entire length of the core, and further that only half the magnetic potential exists between the poles and the yokes, and that, therefore, the average potential between cores and yokes at any point is but one-quarter the maximum potential of the core at that point, we obtain the following expression for the total average permeance between the cores:

$$\mathfrak{P}_2 = \frac{b \times l}{e + l \times \frac{\pi}{2}} + \left(1 + \frac{l - c}{l}\right) \times \frac{a \times c}{e + c \times \frac{\pi}{2}}$$

$$+ \frac{l - c}{l} \times \frac{a \times \left(l - \frac{c}{2}\right)}{1.285\, c}. \qquad \ldots\ldots(184)$$

In this formula it is assumed that leakage takes place in three directions: (1) from magnet core to magnet core along the entire length of their end surfaces (parallel to the arma-

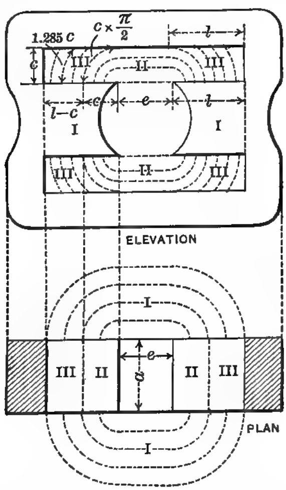


Fig. 154.—Leakage Paths in Bipolar Iron-clad Type.

ture heads), paths *I*, *I*, Fig. 154, having an average potential half that between the poles; (2) from core to core across the surfaces facing the yoke portions of the frame, along a distance, from the pole corners, equal to the distance c between

cores and yokes, paths *II*, *II*; and (3) from the cores to the yokes along the remainder of the length of the core-surfaces opposite the yokes, paths *III*, *III*.

From Fig. 154 the mean area of these paths, *III*, *III*, is obtained:

$$\frac{1}{2}\,[a \times (l - c) + a \times l] = a \times \left(l - \frac{c}{2}\right),$$

and the mean length:

$$\frac{1}{2}\left(c + c \times \frac{\pi}{2}\right) = \frac{1}{2}\left(1 + \frac{\pi}{2}\right) \times c = 1.285\,c.$$

The magnetic potential at the leakage division point of the core is

$$\frac{l - c}{l},$$

the mean potentials of the pole and yoke portions of the core consequently are

$$\frac{1}{2}\left(1 + \frac{l - c}{l}\right) \text{ and } \frac{1}{2}\left(\frac{l - c}{l}\right),$$

respectively, and the average potentials of the paths *II*, *II*, between the pole portions of the cores, and of the paths *III*, *III*, between the yoke portions of the cores and the yokes, are

$$\frac{1}{2}\left(1 + \frac{l - c}{l}\right) \text{ and } \frac{1}{4}\left(\frac{l - c}{l}\right),$$

respectively.

2. *Fourpolar Iron-clad Type.*

If the magnets are so wound that consequent poles are produced in the yokes, Fig. 155, then the full magnetic potential

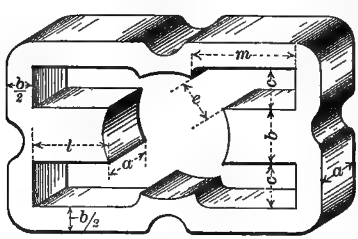


Fig. 155.—Fourpolar Iron-clad Type.

prevails between the cores and the yokes, and the average potential along their length is one-half the potential between

the poles. For the fourpolar iron-clad type, Fig. 155, having two salient and two consequent poles, the permeance between the cores consequently is:

$$\mathfrak{P}_2 = \frac{a \times (l + m)}{c} + \frac{b \times (l + m)}{c + b \times \frac{\pi}{4}}. \quad \ldots\ldots(185)$$

Here all the leakage is assumed to take place between the cores and the adjacent yokes, for in this type the distance, e, between the pole-corners is generally not much smaller, if any, than the distance, c, between cores and yokes, and consequently no additional leakage needs to be considered at this point, unless separate polepieces are used. See formula (196).

There being no further stray paths in the types considered, formulæ (184) and (185) give the total stray permeances of the bipolar and fourpolar iron-clad types, respectively.

3. *Single Magnet Iron-clad Type.*

There being but one magnet core in this type, Fig. 156, the stray paths from that core to the polepiece of opposite polar-

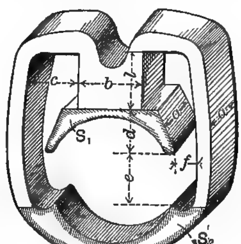


Fig. 156.—Single Magnet Iron-clad Type.

ity and to the adjoining yokes constitute the entire waste permeance which can be formulated as follows:

$$\mathfrak{P}_2 = \frac{S_1 + S_2}{e + d \times \frac{\pi}{2}} + \frac{2\, a \times (l + d)}{c + f}. \quad \ldots\ldots(186)$$

g. Horizontal Double Magnet Type.

This type, Fig. 157, of which the bipolar iron-clad type can be considered a special case, concerning the magnetic poten-

tials of the leakage paths, has features similar to the iron-clad form, and the permeance across the magnet cores of the hori-

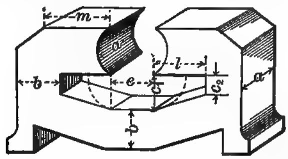


Fig. 157.—Horizontal Double Magnet Type.

zontal double magnet type, which, at the same time is its total waste permeance, accordingly can be expressed by the formula:

$$\mathfrak{T}_2 = \frac{b \times l}{e + b \times \frac{\pi}{2}} + \frac{1}{2} \left\{ \frac{a \times m}{e + m \times \frac{\pi}{2}} + \left(1 + \frac{l - c_1}{l}\right) \times \frac{a \times c_1}{e + c_1 \times \frac{\pi}{2}} + \frac{\frac{l - c_1}{l} \times a \times \left(l - \frac{c_1}{2}\right)}{\frac{1}{2}\,(c_1 + c_2)} \right\}, \quad (187)$$

ı which

$$\frac{l - c_1}{l}$$

; the magnetic potential at the leakage division points of the ıagnet cores.

6. Relative Permeance across Polepieces ($\mathfrak{T}_3$).

The amount of leakage across the end and side surfaces of ıe polepieces, that is, across all their surfaces not facing the rmature core, depends upon the shape of the polepieces and pon the design of the machine with reference to an external ·on surface (bedplate) near the polepieces.

For the most usual shapes the following formulæ can be deved for the relative permeance across the polepieces:

a. Polepieces Having an External Iron Surface Opposite Them.

Upright Horseshoe Type.

In the upright horseshoe type, Fig. 158, the entire direct :akage across the polepieces can be assumed to pass through ıe iron bedplate, hence:

$$\mathfrak{L}_3 = \frac{\frac{1}{2}\left[f \times \left(g + \frac{h}{2}\right) + S\right]}{2\,z}. \qquad \ldots .(188)$$

S = half area of iron surface facing polepieces (centre portion of bedplate), in square inches;

z = distance from polepiece to iron surface (height of zinc base), in inches;

f, g, h are dimensions in inches, see Fig. 158.

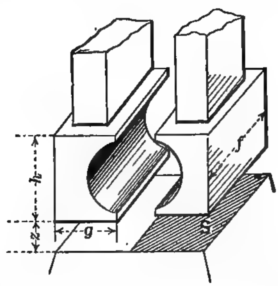


Fig. 158.—Polepieces of Upright Horseshoe Type.

2. *Horizontal Horseshoe Type.*

In this type, Fig. 159, the lines from the lower halves of the polepieces leak to the bedplate, while from the upper halves, and from the end surfaces, they pass across the pole gaps:

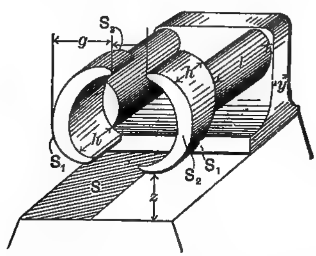


Fig. 159.—Horizontal Horseshoe Type.

$$\mathfrak{L}_3 = \frac{\frac{1}{2}\,(S + S_1)}{2\,z} + \frac{S + S_2}{e + g \times \frac{\pi}{4}}. \qquad \ldots ..(189)$$

S_1 = surface of polepiece opposite bedplate (= half of external surface);

S_2 = end surface of polepiece;

S = half area of iron surface facing polepieces (or area of portion opposite one polepiece).

3. *Fourpolar Double Magnet Type.*

In machines of this type, Fig. 160, there are two leakage paths across the polepieces, the lines from the lower pair of

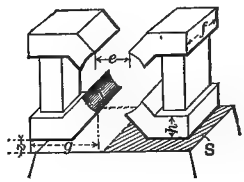


Fig. 160.—Fourpolar Double Magnet Type.

polepieces passing across the bedplate, those from the upper pair across the pole gap:

$$\mathfrak{P}_3 = \frac{\frac{1}{2}[f \times (g + 2h) + S]}{2z} + \frac{g \times (f + 2h)}{e + g \times \frac{\pi}{4}}. \quad (190)$$

Since there are no further essential leakages in this type, formula (190) gives the entire relative permeance of the waste-paths for the fourpolar double magnet type.

b. Polepieces Having No External Iron Surface Opposite Them.

1. *Inverted Horseshoe Type with Rectangular Polepieces.*

For rectangular polepieces, Fig. 161, the mean length of all

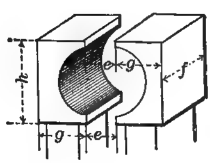


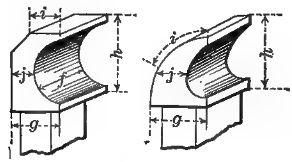
Fig. 161.—Inverted Horseshoe Type with Rectangular Polepieces.

leakage paths are equal, and the relative permeance between the polepieces may consequently be expressed by:

$$\mathfrak{P}_3 = \frac{g \times (f + 2\,h)}{e + g \times \frac{\pi}{2}}. \quad \ldots\ldots\ldots(\mathbf{191})$$

2. *Inverted Horseshoe Type with Beveled or Rounded Polepieces.*

In beveled and rounded polepieces, Figs. 162 and 163, respectively, the length of the path across the upper surfaces is



Figs. 162 and 163.—Inverted Horseshoe Type with Beveled and Rounded Polepieces.

somewhat smaller than that of the side surfaces, and the permeance formula consists of two terms:

$$\mathfrak{P}_3 = \frac{i \times f}{e + i \times \frac{\pi}{2}} + \frac{2\,h \times j}{e + g \times \frac{\pi}{2}}. \quad \ldots(\mathbf{192})$$

3. *Single Magnet Type.*

Here there are four distinct paths for the leakage lines from polepiece to polepiece, viz., across the end surfaces of the

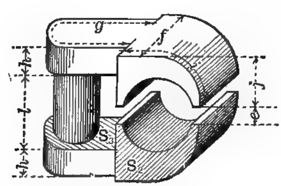


Fig. 164.—Single Magnet Type.

yoke portions, the end surfaces of the pole portions, the facing surfaces of the pole portions, and the inside projections of the pole portions; hence we obtain, with reference to Fig. 164:

$$\mathfrak{P}_s = \frac{g \times h}{l + h\frac{\pi}{2}} + \frac{2\,S_2 + f \times i}{e + j} + \frac{S_3}{l} + \frac{f \times (j-h)}{e + (j-h)}. \quad (193)$$

All leakage paths of the single magnet type being considered in this formula, (193) gives the total relative permeance of the waste field of that type.

4. *Double Magnet Type.*

There is no leakage between the magnet cores nor between polepieces and yoke in this type, the total stray permeance of

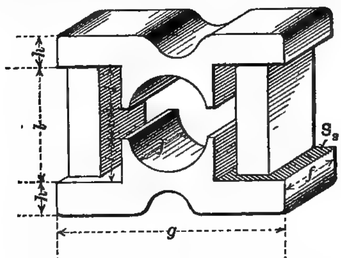


Fig. 165.—Double Magnet Type.

the double magnet type, Fig. 165, therefore, is given by the formula:

$$\mathfrak{P}_s = 2\left[\frac{(g + f) \times h + S_3}{l} + \frac{f \times i}{e + i}\right]. \quad (194)$$

5. *Double Horseshoe Type.*

In the double horseshoe type, Fig. 166, the only leakage across the polepieces takes place at the end surfaces and at

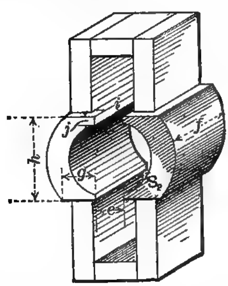


Fig. 166.—Double Horseshoe Type.

the pole corners, hence we have for this, and for similar symmetrical bipolar types:

$$\mathfrak{P}_s = 2 \times \left(\frac{S_2}{e + g \times \frac{\pi}{2}} + \frac{j \times f}{e} + \frac{i \times f}{e + i \times \frac{\pi}{2}} \right). \quad \textbf{(195)}$$

6. *Iron-clad Type.*

In iron-clad types, Fig. 167, the leakage from the end surfaces and the back surface of the polepieces takes place to the

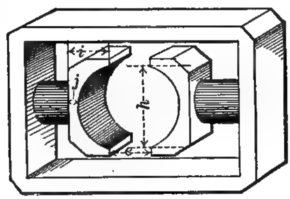


Fig. 167.—Iron-clad Type.

yoke, see formula (204); for the permeance across the polepieces, only the side surfaces are to be considered, and we obtain:

$$\mathfrak{P}_s = \frac{h \times (i + j)}{e + i \times \frac{\pi}{2}}. \quad \ldots\ldots\ldots\ldots\textbf{(196)}$$

7. *Radial Multipolar Type.*

In radial multipolar dynamos, Fig. 168, lines pass from the end surfaces of the polepieces across the pole gaps:

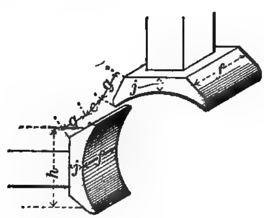


Fig. 168.—Radial Multipolar Type.

$$\mathfrak{P}_s = 2\, n_p \left(\frac{g \times f}{e + g \times \frac{\pi}{2}} + \frac{h \times j}{e + \frac{h}{2} \times \frac{\pi}{2}} \right) \quad ..\textbf{(197)}$$

n_p = number of pairs of magnet poles.

8. *Tangential Multipolar Type.*

The leakage between adjacent polepieces in tangential multipolar machines, Fig. 169, takes place across the length of the magnet cores:

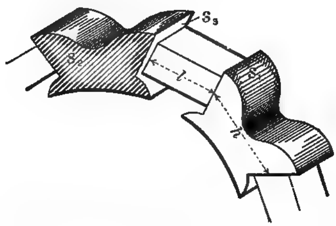


Fig. 169.—Tangential Multipolar Type.

$$\mathfrak{P}_3 = 2\, n_p \times \left(\frac{S_3}{l} + \frac{S_1 + S_2}{l + h \times \frac{\pi}{4}}\right). \quad \ldots(198)$$

S_1 = half area of external surface of polepiece;
S_2 = area of side surface of polepiece;
S_3 = area of projecting portion of end surface, = end surface — area of magnet core.

67. Relative Permeance between Polepieces and Yoke ($\mathfrak{P}_4$).

According to the general principle of calculating relative permeances, the magnetic potential between polepieces and yoke is to be taken $= \frac{1}{2}$, with reference to the potential between two polepieces of opposite polarity. For, the yokes serve to join two magnet cores in series, magnetically, and are therefore separated from the polepieces by but one magnet core. If the yokes join the magnets in parallel, then they usually serve as polepieces also, and must be considered as such in leakage calculations.

Since the amounts of the leakages in the various paths are proportional to their permeances, in dynamos having an external iron surface near the polepieces, most of the leakage takes place between the polepieces through that external surface; and in such machines the leakage from the polepieces to the yoke is comparatively small.

a. Polepieces Having an External Iron Surface Opposite Them.

1. *Upright Horseshoe Type.*

From the polepiece area facing the yoke, S_3, Fig. 170, the leakage takes place in a straight line equal in length to that of

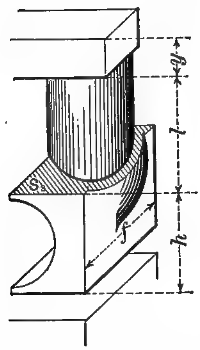


Fig. 170.—Upright Horseshoe Type.

the magnet cores, while from the end surfaces the leakage paths are quadrants joined by straight lines:

$$\mathfrak{L}_4 = \frac{S_3}{l} + \frac{1}{2}\left(\frac{f \times h}{l + (h + y)\frac{\pi}{2}}\right) = \frac{S_3}{l} + \frac{f \times h}{2\,l + (h + y)\frac{\pi}{2}}. \quad (199)$$

S_3 = projecting area of polepiece, = top area of polepiece minus area of magnet core.

2. *Horizontal Horseshoe Type.*

The leakage from the polepieces to the yoke partly passes directly across the cores, and partly takes its path through the iron bed; hence, with reference to Fig. 159, page 239, we have approximately:

$$\mathfrak{L}_4 = \frac{S_1 + S_2}{\frac{1}{2}(l \times z)}. \quad \ldots\ldots\ldots\ldots (200)$$

S_1 = half area of external surface of polepiece;
S_2 = projecting area of polepiece, = area of yoke-end of polepiece minus area of magnet core;
l = length of magnet core;
z = distance of polepiece from iron bedplate.

b. Polepieces Having No External Iron Surface Opposite Them.

1. *Inverted Horseshoe Type with Rectangular Polepieces.*

In this case the leakage from the side surfaces of the pole-

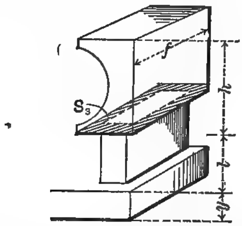


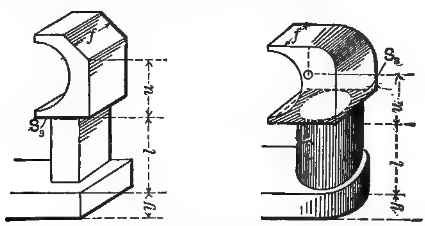
Fig. 171.—Inverted Horseshoe Type with Rectangular Polepieces.

pieces to the yoke, Fig. 171, is twice that of the upright type:

$$\mathfrak{T}_4 = \frac{S_3}{l} + \frac{f \times h}{l + (h + y)\frac{\pi}{4}}. \quad \ldots\ldots\ldots(201)$$

2. *Inverted Horseshoe Type with Beveled or Rounded Polepieces.*

Similar to the former case we have for these forms of the polepieces, Figs. 172 and 173, respectively:



Figs. 172 and 173.—Inverted Horseshoe Type with Beveled and Rounded Polepieces.

$$\mathfrak{T}_4 = \frac{S_3}{l} + \frac{f \times u}{l + (u + y)\frac{\pi}{4}}. \quad \ldots\ldots\ldots(202)$$

3. *Horizontal Double Magnet Type.*

If in this type special polepieces are applied, Fig. 174, lines

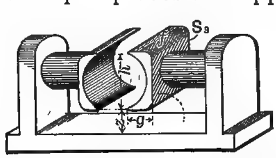


Fig. 174.—Horizontal Double Magnet Type.

pass from the lower surfaces of the same to the yoke:

$$\mathfrak{P}_4 = \frac{f \times g}{z} + \frac{S_3}{2\left(z + h \times \frac{\pi}{4}\right)}. \quad \ldots . (203)$$

Here it is supposed that the path from the projecting back surfaces of the polepieces to the yoke below them is shorter than the length of magnet cores; if the latter is not the case, the term

$$\left(z + h \times \frac{\pi}{4}\right)$$

in the denominator of the second portion of formula (203) is to be replaced by l, the length of the cores.

4. *Iron-clad Types.*

In the *bipolar* iron-clad type, with separate poleshoes, Fig. 175, lines leak to the yoke from the back surfaces of the pole-

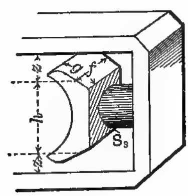


Fig. 175.—Bipolar Iron-clad Type with Poleshoes.

pieces; hence the relative permeance, half of the total magnetic potential existing between polepieces and yoke:

$$\mathfrak{P}_4 = \frac{2(g \times f)}{z} + \frac{S_3}{z + h \times \frac{\pi}{4}}. \quad \ldots (204)$$

As to the denominator of the second term, see remark to formula (203).

This amount, formula (204), as well as the relative permeance across the side surfaces of the polepieces, formula (196), is to be added to the relative permeance found by formula (184), iron-clad type without polepieces, in order to obtain the total relative permeance of this type.

In the *fourpolar* iron-clad type, since the total magnetizing force of each circuit is supplied by one magnet only, there is

full magnetic potential between polepieces and frame, and both terms of formula (204) must consequently be *multiplied by 2.*

5. *Radial Multipolar Type.*

In this type leakage lines pass from the projecting portions, S_3, Fig. 176, of the back surfaces of the polepieces to those of

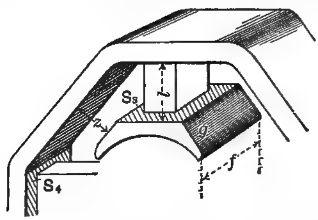


Fig. 176.—Radial Multipolar Type.

the yoke, S_4; and if the yoke is relatively near to the pole gap, leakage also takes place from the end surfaces of the polepieces to the yoke:

$$\mathfrak{P}_4 = n_p \times \left(\frac{\frac{1}{2}(S_3 + S_4)}{l} + \frac{2f \times g}{z} \right). \quad \ldots (205)$$

According to the design of the frame, then, either formula (205) is to be used together with the latter portion of formula (197), or the entire formula (197) is to be combined with the first portion of formula (205), in order to obtain the total joint permeance across the polepieces and from polepieces to yoke of the radial multipolar type.

By the proper combination of formulæ (167) to (205) the probable leakage factor of any dynamo can be calculated from the dimensions of the machine.

D. Comparison of Various Types of Dynamos.

68. Application of Leakage Formulæ for Comparison of Various Types of Dynamos.

In order to illustrate the application of the above formulæ, and at the same time to afford the means of comparing the relative leakages in various well-known types of dynamos, in the following, frames of various types are designed for the same armature, and the leakage factor for each machine thus obtained is calculated.

In order to accommodate all the types to be considered here, the armature has been chosen of a square cross-section, viz., 16 inches core diameter, and 16 inches long. This armature, if wound to a height of about $\frac{5}{8}$ inch, and driven at a speed of 800 revolutions per minute, will yield an output of 50 KW.

The polepieces for this armature must have a bore of $17\frac{1}{2}$ inches, and must be 16 inches long; the pole angle, for all bipolar types, is chosen $\beta = 136°$, and the distance between the pole corners, therefore, is $17\frac{1}{2} \times \sin \frac{1}{2} (180° - 136°) = 6\frac{1}{2}$ inches.

Figs. 177 to 186 give the dimensions of various types of frames for this armature, viz., (1) Upright Horseshoe Type;

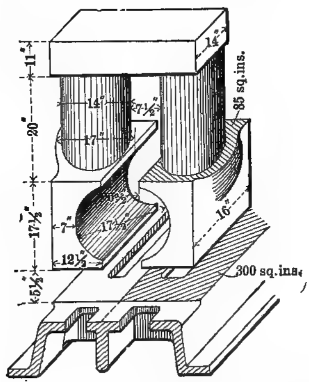


Fig. 177.—Upright Horseshoe Type.

(2) Inverted Horseshoe Type; (3) Horizontal Horseshoe Type; (4) Single Magnet Type; (5) Vertical Double Magnet Type; (6) Vertical Double Horseshoe Type; (7) Horizontal Double Horseshoe Type; (8) Horizontal Double Magnet Type; (9) Bipolar Iron-clad Type; and (10) Fourpolar Iron-clad Type, respectively.

The probable leakage factors of these machines figure out as follows:

1. *Upright Horseshoe Type, Fig. 177.*

By (167):

$$\mathfrak{P}_1 = \frac{\frac{1}{4}\left\{16\,\pi + 17\frac{1}{2}\,\pi \times \frac{136}{180}\right\} \times 16}{1.30 \times (17\frac{1}{2} - 16)} = \frac{375}{1.95} = 192.$$

By (178):

$$\mathfrak{P}_2 = \frac{14 \times \pi \times 20}{2 \times 7\frac{1}{2} + 1.5 \times 14} = \frac{43.98 \times 20}{15 + 21} = 24.5.$$

By (188):

$$\mathfrak{P}_3 = \frac{\frac{1}{2}\left[16 \times (12\frac{1}{2} + 8\frac{3}{4}) + 300\right]}{2 \times 5\frac{1}{2}} = 29.1.$$

By (199):

$$\mathfrak{P}_4 = \frac{85}{20} + \frac{16 \times 17\frac{1}{2}}{2 \times 20 + (17\frac{1}{2} + 11)\frac{\pi}{2}} = 4.3 + 3.3 = 7.6.$$

By (157):

$$\lambda = \frac{192 + 24.5 + 29.1 + 7.6}{192} = \frac{253.2}{192} = \mathbf{1.32.}$$

2. *Inverted Horseshoe Type, Fig. 178.*

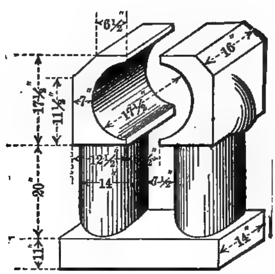


Fig. 178.—Inverted Horseshoe Type.

$\mathfrak{P}_1 = 192.$

$\mathfrak{P}_2 = 24.5.$

By (192):

$$\mathfrak{P}_3 = \frac{6\frac{1}{2} \times 16}{6\frac{1}{2} + 6\frac{1}{2} \times \frac{\pi}{2}} + \frac{2 \times 17\frac{1}{2} \times 7}{6\frac{1}{2} + 12\frac{1}{2} \times \frac{\pi}{2}} = 6.2 + 9.4 = 15.6.$$

By (202):

$$\mathfrak{P}_4 = \frac{85}{20} + \frac{16 \times 11\frac{1}{2}}{20 + (11 + 11\frac{1}{2} \times \frac{\pi}{4}} = 4.3 + 4.9 = 9.2.$$

By (157):

$$\lambda = \frac{192 + 24.5 + 15.6 + 9.2}{192} = \frac{241.3}{192} = \mathbf{1.255}.$$

3. *Horizontal Horseshoe Type, Fig. 179.*

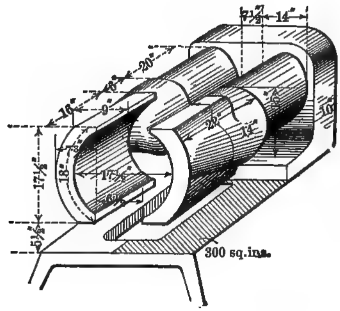


Fig. 179.—Horizontal Horseshoe Type.

$\mathfrak{P}_1 = 192.$

$\mathfrak{P}_2 = 24.5.$

By (189):

$$\mathfrak{P}_3 = \frac{\frac{1}{2}(14 \times 22 + 300)}{2 \times 5\frac{1}{2}} + \frac{14 \times 22 + 2\frac{1}{4} \times 18}{6\frac{1}{2} + 9 \times \frac{\pi}{4}} = 27.6 + 25.7 = 53.3.$$

By (200):

$$\mathfrak{P}_4 = \frac{(14 \times 22) + \left\{15^2\frac{\pi}{4} - 14^2\frac{\pi}{4}\right\}}{\frac{1}{2}(20 + 5\frac{1}{2})} = 27.6.$$

$$\lambda = \frac{192 + 24.5 + 53.3 + 27.6}{192} = \frac{297.4}{192} = \mathbf{1.55}.$$

4. *Single Magnet Type, Fig. 180.*

$\mathfrak{P}_1 = 192.$

By (193):

$$\mathfrak{P}_3 = \frac{\left\{2 \times 11 + 16 \times \frac{\pi}{2}\right\} \times 14}{24 + 14 \times \frac{\pi}{2}} + \frac{2 \times 267.7 + 16 \times 35\frac{3}{4}}{6\frac{1}{2} + 23\frac{1}{2}} + \frac{122.6}{24}$$

$$+ \frac{16 \times 8\frac{3}{4}}{6\frac{1}{2} + 8\frac{3}{4}} = 14.3 + 33.1 + 5.1 + 9.2 = 61.7.$$

$$\lambda = \frac{192 + 61.7}{192} = \frac{253.7}{192} = \mathbf{1.32.}$$

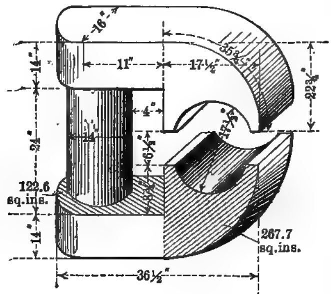


Fig. 180.—Single Magnet Type.

5. *Vertical Double Magnet Type, Fig. 181.*

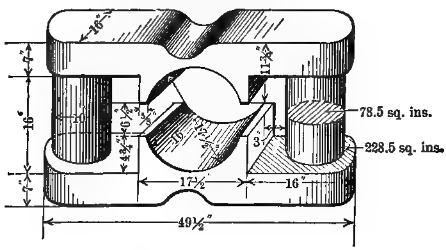


Fig. 181.—Vertical Double Magnet Type.

$\mathfrak{P}_1 = 192.$

By (194):

$$\mathfrak{P}_3 = 2 \times \left\{ \frac{(49\frac{1}{2} + 16) \times 7 + (228.5 - 78.5)}{16} + \frac{16 \times 4\frac{3}{4}}{6\frac{1}{2} + 4\frac{3}{4}} \right\}$$

$$= 2\,(38.2 + 6.8) = 90.$$

$$\lambda = \frac{192 + 90}{192} = \frac{282}{192} = \mathbf{1.47.}$$

6. *Vertical Double Horseshoe Type, Fig. 182.*

$\mathfrak{P}_1 = 192.$

By (177):

$$\mathfrak{P}_2 = \frac{14 \times 16}{7\frac{1}{2}} + 2 \times \frac{5\frac{3}{4} \times 16}{7\frac{1}{2} + 5\frac{3}{4} \times \frac{\pi}{2}} = 29.9 + 11.1 = 41.$$

By (195):

$$\mathfrak{P}_3 = 2 \times \left\{ \frac{4\frac{1}{2} \times 17\frac{1}{2}}{6\frac{1}{2} + 6\frac{3}{4} \times \frac{\pi}{2}} + \frac{\frac{5}{8} \times 16}{6\frac{1}{2}} + 6\frac{1}{2} + \frac{1}{2} \times \frac{\pi}{2} \right\}$$

$$= 2 \times (4.6 + 1.6 + 1.1) = 14.6.$$

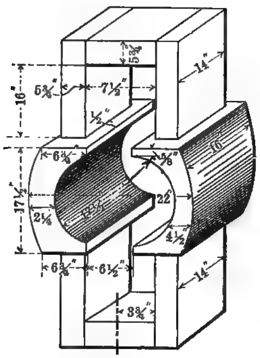


Fig. 182.—Vertical Double Horseshoe Type.

By (201):

$$\mathfrak{P}_4 = \frac{(16 \times 6\frac{3}{4} - 14 \times 5\frac{3}{4}) + 14 \times 3\frac{3}{4}}{16} + \frac{16 \times 17\frac{1}{2}}{16 + \left\{ \frac{17\frac{1}{2}}{2} + 5\frac{3}{4} \right\} \frac{\pi}{4}}$$

$$= 5 + 10.2 = 15.2.$$

$$\lambda = \frac{192 + 41 + 14.6 + 15.2}{192} = \frac{262.8}{192} = \mathbf{1.37}.$$

7. *Horizontal Double Horseshoe Type, Fig. 183.*

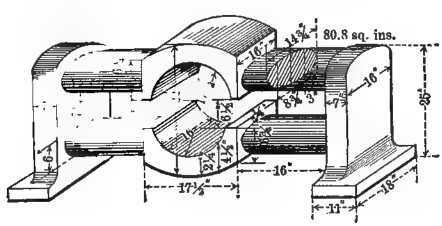


Fig. 183.—Horizontal Double Horseshoe Type.

$\mathfrak{P}_1 = 192.$

By (179):

$$\mathfrak{P}_2 = \frac{8\frac{3}{4} \times 16}{7\frac{1}{4}} + \frac{6 \times \pi \times 16}{7\frac{1}{4} + \frac{3}{4} \times 6} = 19.3 + 25.7 = 45.$$

By (195):

$$\mathfrak{P}_3 = 2 \times \left\{ \frac{4\tfrac{1}{2} \times 17\tfrac{1}{2}}{6\tfrac{1}{2} + 6\tfrac{3}{4} \times \frac{\pi}{2}} + \frac{\tfrac{5}{8} \times 16}{6\tfrac{1}{2}} + \frac{\tfrac{1}{4} \times 16}{6\tfrac{1}{2} + \tfrac{1}{4} \times \frac{\pi}{4}} \right\}$$

$$= 2 \times (4.6 + 1.6 + .6) = 13.6.$$

By (201):

$$\mathfrak{P}_4 = \frac{(16 \times 6\tfrac{3}{4} - 80.8) + 25 \times 16}{16} + \frac{\left\{ \frac{17\tfrac{1}{2}}{2} + 7 \right\} \times \frac{16}{2}}{16 + \left\{ \frac{17\tfrac{1}{2}}{2} + 7 \right\} \frac{\pi}{4}}$$

$$+ \frac{\tfrac{1}{2} \left\{ \frac{17\tfrac{1}{2}}{2} \times 16 + 11 \times 18 \right\}}{16 + \left\{ \frac{17\tfrac{1}{2}}{2} + 11 \right\} \frac{\pi}{4}} = 14.2 + 4.45 + 6.15 = 24.8.$$

$$\lambda = \frac{192 + 45 + 13.6 + 24.8}{192} = \frac{275.4}{192} = \mathbf{1.43.}$$

8. *Horizontal Double Magnet Type, Fig. 184.*

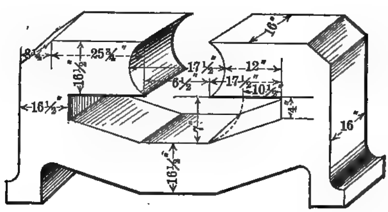


Fig 184.—Horizontal Double Magnet Type.

$\mathfrak{P}_1 = 192.$

By (187):

$$\mathfrak{P}_2 = \frac{16\tfrac{1}{2} \times 17\tfrac{1}{2}}{6\tfrac{1}{2} + 17\tfrac{1}{2} \times \frac{\pi}{2}} + \tfrac{1}{2} \times \frac{16 \times 25\tfrac{3}{4}}{6\tfrac{1}{2} + 25\tfrac{3}{4} \times \frac{\pi}{2}}$$

$$+ \tfrac{1}{2} \times \left\{ 1 + \frac{10\tfrac{1}{2}}{17\tfrac{1}{2}} \right\} \times \frac{16 \times 7}{6\tfrac{1}{2} + 7 \times \frac{\pi}{2}} + \tfrac{1}{2} \times \frac{10\tfrac{1}{2}}{17\tfrac{1}{2}} \times \frac{16 \times 14}{5\tfrac{1}{2}}$$

$$= 8.5 + 5.1 + 5.1 + 12.3 = 31.$$

$$\lambda = \frac{192 + 31}{192} = \frac{223}{192} = \mathbf{1.16.}$$

9. *Bipolar Iron-clad Type, Fig. 185.*

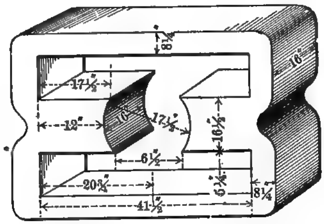


Fig. 185.—Bipolar Iron-clad Type.

$\mathfrak{P}_1 = 192.$

By (184):

$$\mathfrak{P}_2 = \frac{16\tfrac{1}{2} \times 17\tfrac{1}{2}}{6\tfrac{1}{2} + 17\tfrac{1}{2} \times \frac{\pi}{2}} + \left\{1 + \frac{6\tfrac{3}{4}}{17\tfrac{1}{2}}\right\} \times \frac{16 \times 5\tfrac{1}{4}}{6\tfrac{1}{2} + 5\tfrac{1}{4} \times \frac{\pi}{2}} +$$

$$\left\{\frac{6\tfrac{3}{4}}{17\tfrac{1}{2}}\right\} \times \frac{16 \times \left\{17\tfrac{1}{2} - \frac{5\tfrac{1}{4}}{2}\right\}}{1.285 \times 5\tfrac{1}{4}} = 8.5 + 7.9 + 13.6 = 30.$$

$$\lambda = \frac{192 + 30}{192} = \frac{222}{192} = \mathbf{1.15.}$$

10. *Fourpolar Iron-clad Type, Fig. 186.*

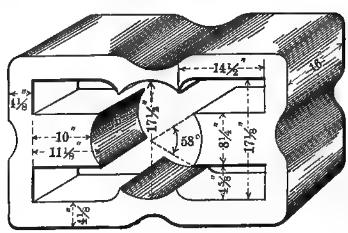


Fig. 186.—Fourpolar Iron-clad Type.

By (167):

$$\mathfrak{P}_1 = \frac{\frac{1}{4}\left\{16\,\pi + 17\tfrac{1}{2}\,\pi \times \frac{58^\circ}{90^\circ}\right\} \times 16}{1.95} = \frac{339}{1.95} = 174.$$

By (185):

$$\mathfrak{P}_2 = \frac{16 \cdot \times (11\tfrac{1}{8} + 14\tfrac{1}{2})}{4\tfrac{5}{8}} + \frac{8\tfrac{1}{4} \times (11\tfrac{1}{8} + 14\tfrac{1}{2})}{4\tfrac{5}{8} + 8\tfrac{1}{4} \times \frac{\pi}{4}}$$

$$= 88.8 + 19 = 107.8.$$

$$\lambda = \frac{174 + 107.8}{174} = \frac{281.8}{174} = \mathbf{1.62}.$$

Taking now the leakage proper, that is, leakage factor minus 1, of the bipolar iron-clad type, which is the smallest found, as unity, we can express the amounts of the stray fields of the remaining types as multiples of this unity, thus obtaining the following comparative leakages of the types considered:

Upright horseshoe type............	0.32	÷ 0.15 =	2.14
Inverted horseshoe type...........	0.255	÷ 0.15 =	1.70
Horizontal horseshoe type.........	0.55	÷ 0.15 =	3.67
Single magnet type................	0.32	÷ 0.15 =	2.13
Vertical double magnet type	0.47	÷ 0.15 =	3.13
Horizontal double horseshoe type..	0.37	÷ 0.15 =	2.46
Vertical double horseshoe type.....	0.43	÷ 0.15 =	2.87
Horizontal double magnet type....	0.16	÷ 0.15 =	1.07
Bipolar iron-clad type	0.15	÷ 0.15 =	1
Fourpolar iron-clad type...........	0.62	÷ 0.15 =	4.14

If, in the latter machine, the stray field of which is somewhat excessive, an armature of larger diameter and smaller axial length would be chosen and the dimensions of the frame altered accordingly, the leakage would be found within the usual limits of the fourpolar iron-clad type.

CHAPTER XIII.

CALCULATION OF LEAKAGE FROM MACHINE TEST.

69. Calculation of Total Flux.

The machine having been built, its actual leakage can be determined from the ordinary machine test. It is only necessary, for this purpose, to run the machine at its normal speed, and to regulate the field current—by changing the series-regulating resistance in a shunt dynamo, or by altering the number of turns in a series machine, or by regulating both in a compound-wound dynamo—until the required output is obtained. Noting then the exciting ampere-turns, we can calculate the total magnetic flux, Φ', through the magnet frame, by a comparatively simple method which is given below; and Φ' divided by the useful flux, Φ, gives the factor λ of the actual leakage.

The observed magnetizing force of AT ampere-turns per magnetic circuit—made up of T_{sh} shunt turns, through which a current of

$$I_{sh} = \frac{E}{r_m} \text{ amperes}$$

(E = potential at terminals, r_m = total resistance of shunt circuit) is flowing, in a shunt machine; or of T_{se} series turns traversed by a current of $I_{se} = I$ amperes (I = current output of dynamo), in a series machine; or partly of the one and partly of the other, in a compound dynamo— is supplying the requisite magnetizing forces used in the different portions of that circuit, viz., the ampere turns needed to overcome the magnetic resistance of the air gaps, of the armature core, and of the field frame, and the magnetizing force required to compensate the reaction of the armature winding upon the magnetic field; hence we have:

$$AT = at_g + at_a + at_m + at_r, \quad \ldots\ldots (206)$$

where AT = total magnetomotive force required per magnetic circuit for normal output, in ampere-turns, observed;

at_g = magnetomotive force used per circuit to overcome the magnetic resistance of the air gaps in ampere-turns, see § 90;

at_a = magnetomotive force used per circuit to overcome magnetic resistance of armature core in ampere-turns, see § 91;

at_m = magnetomotive force used per circuit to overcome magnetic resistance of magnet frame, in ampere-turns, see § 92;

at_r = magnetomotive force required per circuit for compensating armature reactions, in ampere-turns, see § 93.

Since the magnet frame alone carries the total flux generated in the machine, while the air gaps and the armature core are traversed by the useful lines, only the ampere-turns used in overcoming the resistance of the magnet frame depend upon the total magnetic flux, and all others of these partial magnetomotive forces can be determined from the useful flux. The latter, however, is known from the armature data of the machine by virtue of equations (137) and (138), respectively; consequently, from (206) we can determine at_m, and this, in turn, will furnish the value of the total flux, Φ'.

Transposing (206), we obtain:

$$at_m = AT - (at_g + ta_a + at_r), \quad \ldots\ldots(207)$$

in which AT is known from the machine test, at_g and at_a can be calculated from the useful flux, and at_r is given by the data of the armature.

The numerical value of at_m having been found, we can then calculate the total magnetic flux through the machine. In the following, the two cases occurring in practice are considered separately, viz.: (1) but one material, and (2) two different materials being used in building the magnet frame of the machine.

a. Calculation of Total Flux when Magnet Frame Consists of but One Material.

If but one single material—either cast iron, wrought iron, mitis metal, or steel—is used in the magnet frame, the calculation of the total magnetic flux is a very simple operation.

For, if l''_m denotes the length of the magnetic circuit in the magnet frame, from air gap to air gap, and m''_m is the corresponding mean specific magnetizing force, then, according to formula (226), § 88, we have:

$$at_m = l''_m \times m''_m, \quad \ldots\ldots\ldots\ldots(208)$$

from which follows, by substituting the value of at_m from (207):

$$m''_m = \frac{at_m}{l''_m} = \frac{AT - (at_g + at_a + at_r)}{l''_m}. \quad \ldots(209)$$

Dividing the numerical value of at_m, as found by formula (207), by the length, l''_m, of the circuit, we therefore obtain the numerical value of the specific magnetizing force per inch length for the respective material. By means of Table LXXXVIII., p. 336, or Fig. 256, p. 338, then, the density $\mathcal{B}''_m$, corresponding to this particular value of m''_m for the material employed, can be found; and since the density in the magnet frame, $\mathcal{B}''_m$, is the quotient of the total flux per magnetic circuit, Φ'', divided by the mean sectional area, S_m, of one magnetic circuit in the field frame, or

$$\mathcal{B}''_m = \frac{\Phi''}{S_m},$$

we obtain the total magnetic flux per magnetic circuit of the machine from the simple formula

$$\Phi'' = S_m \times \mathcal{B}''_m, \quad \ldots\ldots\ldots\ldots(210)$$

where S_m = mean sectional area of magnet frame, in square inches;

$\mathcal{B}''_m$ = density of lines of force in magnet frame, corresponding to the value of m''_m in Table LXXXVIII., or in Fig. 256, § 88.

b. Calculation of Total Flux when Magnet Frame Consists of Two Different Materials.

In magnet frames made up of two different materials—either of wrought iron cores and cast iron yokes and pole-pieces; or of wrought iron cores and yokes, and cast iron pole-pieces; or of any other combination of two of the various kinds of iron in use for this purpose—the calculation of the total magnetic flux is performed by an indirect method.

Let us assume that the two materials used are wrought and cast iron, and consequently denote

by $l''_{w.i.}$ the length of one circuit in the wrought iron portion of the frame, in inches;

" $l''_{c.i.}$ the length of one circuit in the cast iron portion of the frame, in inches;

" $S_{w.i.}$ the mean area of one circuit in the wrought iron portion, in square inches;

" $S_{c.i.}$ the mean area of one circuit in the cast iron portion, in square inches;

" $\mathfrak{B}''_{w.i.}$ the average magnetic density in the wrought iron, in lines of force per square inch; and

" $\mathfrak{B}''_{c.i.}$ the average magnetic density in the cast iron, in lines of force per square inch;

" $m''_{c.i.}$ the specific magnetizing force per inch of cast iron portion of circuit;

" $m''_{w.i.}$ the specific magnetizing force per inch of wrought iron portion of circuit;

then we have the equation:

$$at_m = l''_{w.i.} \times m''_{w.i.} + l''_{c.i.} \times m''_{c.i.}, \quad \ldots.(211)$$

or, by comparison with formula (207):

$$l''_{w.i} \times m''_{w.i.} + l''_{c.i.} \times m''_{c.i.} = AT - (at_g + at_a + at_r). \quad \ldots\ldots(212)$$

This equation contains two unknown quantities, viz.:

$$m''_{w.i.} \text{ and } m''_{c.i.},$$

and can, consequently, not be solved directly. Table LXXXVIII., p. 336, however, affords the means of calculating the total flux, Φ'', in an indirect manner, as follows:

The useful flux, Φ, being known by virtue of formula (137) or (138), respectively, an assumption can be made of the total flux per circuit, Φ'', by adding to the useful flux per circuit,

$$\frac{\Phi}{n_z},$$

(n_z being the number of the magnetic circuits in the machine), from 10 to 100 per cent., according to the size and the type of the dynamo (see Table LXVIII., p. 263, and Table LXVIII*a*, p. 265). In dividing this approximate value of Φ'' by the areas $S_{\text{w.i.}}$ and $S_{\text{c.i.}}$, respectively, the densities $\mathfrak{B}''_{\text{w.i.}}$ and $\mathfrak{B}''_{\text{c.i.}}$ are obtained, and by means of Table LXXXVIII. (Fig. 256) the corresponding value of $m''_{\text{w.i.}}$ and $m''_{\text{c.i.}}$, respectively. Introducing these values in the equation

$$l''_{\text{w.i.}} \times m''_{\text{w.i.}} + l''_{\text{c.i.}} \times m''_{\text{c.i.}} = Z, \quad \dots . (213)$$

a value Z is produced which, in general, will differ from the value at_{m} obtained by formula (207).

If Z is found smaller than the actual value of at_{m}, then the value of Φ'' was assumed too small; if larger, then Φ'' was taken too large. A second assumption of Φ'' is now made so that the corresponding value of Z obtained in a similar manner from Table LXXXVIII. and formula (213) will be on the other side of at_{m}, *i. e.*, larger than at_{m} in the former, and smaller in the latter case.

By properly interpolating between the first and second assumption, a third assumption of Φ'' is now made which will produce a value of Z very near the actual value of at_{m}. A fourth, or eventually a fifth assumption, will then make the value of Z practically equal to at_{m} from formula (207), and that final value of Φ'', which satisfies the equation (212), is the required total flux per magnetic circuit of the dynamo.

70. Actual Leakage Factor of Machine.

Being thus able to calculate the total flux of magnetic lines through any dynamo from the ordinary machine test, that is,

from its ordinary running conditions, the actual factor of magnetic leakage can be found from

$$\lambda = \frac{\Phi'}{\Phi} = \frac{n_z \times \Phi''}{\Phi}, \quad \ldots\ldots\ldots\ldots\ldots (214)$$

where Φ' = total flux through magnet frame, in lines of force;
Φ'' = total flux per magnetic circuit, calculated from formula (210), or (212), respectively;
Φ = useful flux cutting armature conductors, from (137) or (138), respectively;
n_z = total number of magnetic circuits in machine.

The author, by employing his method of calculating the leakage from the ordinary machine test, § 69, has figured the leakage factors for a great number of practical dynamos[1] of which the test data were at his command, and by combining his results with the researches of Hopkinson,[2] Lahmeyer,[3] Corsepius,[4] Esson,[5] Wedding,[6] Ives,[7] Edser,[8] and Puffer,[9] has averaged the following Table LXVIII. of leakage factors for dynamos of various types and sizes, which is intended as a guide in making the first assumption of the total flux, for solving equation (212), as well as for dimensioning the field magnet frame (see § 60), but which may also be made use of in obtaining an approximate value of the leakage coefficient for rough calculations.

From this table the general fact will be noted that the leakage is the greater the smaller the dynamo, which is due to the difficulty, or rather impossibility, of advantageously dimensioning the magnetic circuit in small machines. In these the length of the air gaps is comparatively much larger, and the relative

[1] For list of machines considered see Preface.

[2] J. and E. Hopkinson, *Phil. Trans.*, 1886, part i.

[3] Lahmeyer, *Elektrotechn. Zeitschr.*, vol. ix. pp. 89 and 283 (1888).

[4] Corsepius, *Elektrotechn. Zeitschr.*, vol. ix. p. 235 (1888).

[5] W. B. Esson, *The Electrician* (London), vol. xxiv. p. 424 (1890); *Journal Inst. El. Eng.*, vol. xix. p. 122 (1890).

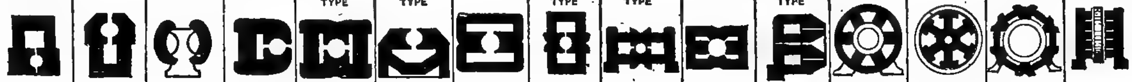
[6] W. Wedding, *Elektrotechn. Zeitschr.*, vol. xiii. p. 67 (1892).

[7] Arthur Stanley Ives, *Electrical World*, vol. xix. p. 11 (January 2, 1892).

[8] Edwin Edser and Herbert Stansfield, *Electrical World*, vol. xx. p. 180 (September 17, 1892).

[9] Puffer, *Electrical Review* (London), vol. xxx. p. 487 (1892).

ND SIZES OF DYNAMOS.*



CAPACITY IN KILO WATTS	FACTOR OF MAGNETIC LEAKAGE, λ															CAPACITY IN KILO WATTS
	1 UPRIGHT HORSESHOE TYPE	2 INVERTED HORSESHOE TYPE	3 HORIZONTAL HORSESHOE TYPE	4 SINGLE MAGNET TYPE	5 VERTICAL DOUBLE MAGNET TYPE	6 HORIZONTAL DOUBLE MAGNET TYPE	7 BIPOLAR IRON CLAD TYPE	8 VERTICAL DOUBLE HORSESHOE TYPE	9 HORIZONTAL DOUBLE HORSESHOE TYPE	10 FOURPOLAR IRON CLAD TYPE	11 SINGLE MAGNET MULTIPOLAR TYPE	12 RADIAL MULTIPOLAR TYPE	13 INNER-POLE TYPE	14 TANGENTIAL MULTIPOLAR TYPE	15 AXIAL MULTIPOLAR TYPE	
1	2·00	1·75	......	1·90	......	1·50	1·50	......	......	......	......	......	......	......	......	·1
·25	1·80	1·60	......	1·75	2·00	1·40	1·40	......	......	......	......	......	......	......	......	·25
·5	1·70	1·50	2·00	1·65	1·90	1·35	1·35	1·80	1·90	......	......	......	......	......	......	·5
1	1·65	1·45	1·90	1·60	1·80	1·30	1·30	1·70	1·75	......	......	......	......	......	......	1
2·5	1·60	1·40	1·80	1·55	1·70	1·28	1·28	1·60	1·65	1·75	1·60	1·50	1·40	1·90	2·00	2.5
5	1·55	1·35	1·75	1·50	1·65	1·25	1·25	1·55	1·60	1·65	1·50	1·40	1·35	1·80	1·90	5
7·5	1·50	1·30	1·70	1·45	1·60	1·22	1·22	1·50	1·55	1·60	1·45	1·35	1·32	1·70	1·80	7.5
10	1·45	1·28	1·65	1·40	1·55	1·20	1·20	1·45	1·50	1·55	1·40	1·32	1·30	1·65	1·70	10
25	1·40	1·25	1·60	1·35	1·50	1·18	1·18	1·40	1·45	1·50	1·35	1·30	1·28	1·60	1·65	25
50	1·35	1·22	1·55	1·32	1·45	1·15	1·15	1·35	1·40	1·45	1·30	1·28	1·25	1·55	1·60	50
100	1·30	1·20	1·50	1·30	1·40	1·12	1·12	1·30	1·35	1·40	1·25	1·25	1·22	1·50	1·55	100
200	1·25	......	......	......	......	......	......	......	......	1·34	1·22	1·22	1·20	1·45	1·50	200
300	1·20	......	......	......	......	......	......	......	......	1·35	1·20	1·20	1·18	1·40	1·45	300
500	......	......	......	......	......	......	......	......	......	......	......	1·18	1·15	1·35	1·40	500
1000	......	......	......	......	......	......	......	......	......	......	......	1·16	1·12	1·30	1·35	1000
2000	......	......	......	......	......	......	......	......	......	......	......	1·15	1·10	1·25	1·30	2000

*The values of λ in this table, for the purpose of being on the safe side in case of rough calculations, are taken with regard to the most unfavorable conditions, namely, for a *smooth-core armature*, for a massive magnet frame with *cast-iron polepieces*, and for *high speed;* and in the bipolar types, columns 1 to 9, have reference to *drum* armatures. In order not to obtain too great a total flux when dimensioning the field frame, therefore, the leakage factor is to be taken correspondingly less than given above for each and every improvement in the leakage conditions, as follows: For dynamos with *toothed* or *perforated armatures* decrease the leakage proper, that is, the leakage factor minus 1, by from *20* to *50* per cent., according to size, the smaller percentage referring to large outputs; if the type is such that the *employment of wrought iron or cast steel* for the field frame has the effect of materially reducing the areas of surfaces of opposite magnetic polarity, without much affecting their distance apart, a further allowance of from *10* to *20* per cent. may be made in estimating the leakage; in case the machine is designed for *low speed*, its magnet frame will be proportionately large, its leakage, therefore, is reduced to that of a correspondingly larger size, and $(\lambda - 1)$ may be taken from *5* to *15* per cent. smaller than given above; and for bipolar dynamos with *smooth-ring* armatures, finally, *20* to *30* per cent. less can be taken than for similar machines of same output having drum armatures.

distances of the leakage surfaces much smaller than in large dynamos; the permeance of the air gaps, therefore, is relatively much smaller, while the permeances of the leakage paths are considerably larger, comparatively, than in large machines, and formula (157), in consequence, will produce a high value of the leakage coefficient for a small dynamo.

It further follows from Table LXVIII. that the leakage factor for various types and sizes of dynamos varies within the wide range of from 1.10 to 2.00, which result agrees with observations of Mavor,[1] who, however, seems not to have considered capacities over 100 KW.

By comparing the values of λ for any one capacity, the relative merits of the various types considered may be deduced.

Thus it is learned that, as far as magnetic leakage is concerned, the Horizontal Double Magnet Type (column 6) and the Bipolar Iron-clad Type (column 7) are superior to any of the other types, which undoubtedly is due to the common feature of these types of having the cores of opposite magnetic potential in line with each other on opposite sides of the armature, thus reducing the magnetic leakage between them to a minimum.

Next in line, considering bipolar dynamos, are the Inverted Horseshoe Type (column 2), the Single Magnet Type (column 4), the Upright Horseshoe Type (column 1), and the Vertical Double Horseshoe Type (column 8).

Of multipolar machines the two best forms, magnetically, are, respectively, the Innerpole Type (column 13), and the Radial Multipolar Type (column 12). In the first named of these types the magnet cores form a star, having a common yoke in the centre and the polepieces at the periphery; thus the distances of the leakage paths increase the direct proportion to the difference of magnetic potential, a feature which is most desirable, and which accounts for the low values of λ for the type in question.

The most leaky of all types seem to be the Horizontal Single Horseshoe Type (column 3), and the Axial Multipolar Type (column 15).

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[1] Mavor, *Electrical Engineer* (London), April 13, 1894; *Electrical World*, vol. xxiii. p. 615, May 5, 1894.

In the former type the excessive leakage is due to the magnetic circuit being suspended over an iron surface extending over its entire length, while in the latter type it is due to the comparatively close relative proximity of a great number of magnet cores (two for each pole) parallel to each other.

When making the allowances for improvements referred to in the note to Table LXVIII., the following Table LXVIII*a* is obtained, which gives the usual limits of the leakage factor for various sizes of the most common types of continuous current dynamos:

TABLE LXVIII*a*.—USUAL LIMITS OF LEAKAGE FACTOR FOR MOST COMMON TYPES OF DYNAMOS.

Capacity of Dynamo in Kilowatts.	Ordinary Horseshoe Type. 	Inverted Horseshoe Type. 	Double Magnet Type. 	Bipolar Iron Clad Type. 	Fourpolar Iron Clad Type. 	Multipolar Ring Type. 
.1	1.50 to 2.00	1.40 to 1.75	....	1.25 to 1.50	....	....
.25	1.45 " 1.80	1.35 " 1.60	1.45 to 2.00	1.22 " 1.40	....	....
.5	1.40 " 1.70	1.30 " 1.50	1.40 " 1.90	1.20 " 1.35	....	....
1	1.35 " 1.65	1.25 " 1.45	1.35 " 1.80	1.18 " 1.30	....	....
2.5	1.30 " 1.60	1.20 " 1.40	1.30 " 1.70	1.16 " 1.28	1.35 to 1.75	1.20 to 1.50
5	1.25 " 1.55	1.18 " 1.35	1.25 " 1.60	1.14 " 1.25	1.32 " 1.65	1.18 " 1.40
10	1.22 " 1.50	1.16 " 1.30	1.20 " 1.55	1.12 " 1.22	1.30 " 1.60	1.16 " 1.35
25	1.20 " 1.45	1.14 " 1.25	1.18 " 1.50	1.10 " 1.20	1.28 " 1.55	1.15 " 1.32
50	1.18 " 1.40	1.12 " 1.22	1.16 " 1.45	1.09 " 1.18	1.26 " 1.50	1.14 " 1.30
100	1.16 " 1.35	1.10 " 1.20	1.15 " 1.40	1.08 " 1.15	1.24 " 1.45	1.13 " 1.28
200	1.14 " 1.30	....	....	....	1.22 " 1.40	1.12 " 1.25
300	1.12 " 1.25	....	....	....	1.20 " 1.35	1.11 " 1.22
500	....	....	....	....	....	1.10 " 1.20
1,000	....	....	....	....	....	1.09 " 1.18
2,000	....	....	....	....	....	1.08 " 1.15

PART IV.

DIMENSIONING OF FIELD MAGNET FRAME.

CHAPTER XIV.

FORMS OF FIELD MAGNETS.

71. Classification of Field Magnet Frames.

With reference to the type of the field magnet frame modern dynamos may be classified as follows:

I.—Bipolar Machines.

1. Single Horseshoe Type.
   - *a.* Upright single horseshoe type (Fig. 187).
   - *b.* Inverted single horseshoe type (Fig. 188).
   - *c.* Horizontal single horseshoe type (Fig. 189).
   - *d.* Vertical single horseshoe type (Fig. 190).
2. Single Magnet Type.
   - *a.* Horizontal single magnet type (Figs. 191 and 192).
   - *b.* Vertical single magnet type (Fig. 193).
   - *c.* Single magnet ring type (Fig. 194).
3. Double Magnet Type.
   - *a.* Horizontal double magnet type (Figs. 195 and 197).
   - *b.* Vertical double magnet type (Figs. 196 and 199).
   - *c.* Inclined double magnet type (Fig. 198).
   - *d.* Double magnet ring type (Fig. 200).
4. Double Horseshoe Type.
   - *a.* Horizontal double horseshoe type (Fig. 201).
   - *b.* Vertical double horseshoe type (Fig. 202).
5. Iron-clad Type.
   - *a.* Horizontal iron-clad type (Figs. 203 and 204).
   - *b.* Vertical iron-clad type.
     - *α.* Single magnet vertical iron-clad type (Figs. 205 and 206).
     - *β.* Double magnet vertical iron-clad type (Fig. 207).

II.—Multipolar Machines.

1. Radial Multipolar Type.
   - *a.* Radial outerpole type (Fig. 208).
   - *b.* Radial innerpole type (Fig. 209).

2. Tangential Multipolar Type.
   a. Tangential outerpole type (Fig. 210).
   b. Tangential innerpole type (Fig. 211).
3. Axial Multipolar Type (Fig. 212).
4. Radi-tangent Multipolar Type (Fig. 213).
5. Single Magnet Multipolar Type.
   a. Axial pole single magnet multipolar type (Fig. 214).
   b. Outer-innerpole single magnet multipolar type (Fig. 215).
6. Double Magnet Multipolar Type (Fig. 216).
7. Multipolar Iron-clad Type (Fig. 217).
   Horizontal fourpolar iron-clad type (Figs. 218 and 220).
   Vertical fourpolar iron-clad type (Fig. 219).
8. Multiple Horseshoe Type (Figs. 221 and 222).
9. Fourpolar Double Magnet Type (Fig. 223).
10. Quadruple Magnet Type (Fig. 224).

72. Bipolar Types.

The simplest form of field magnet frame is that resembling the shape of a horseshoe. Such a horseshoe-shaped frame may be composed of two magnet cores joined by a yoke, or may be formed of but one electromagnet provided with suitably shaped polepieces. The former is called the *single horseshoe type,* the latter the *single magnet type.*

A *single horseshoe frame* may be placed in four different positions with reference to the armature, the two cores either being *above* or *below* the armature, or situated *symmetrically.* one on each side, in a *horizontal* or in a *vertical* position.

The *upright single horseshoe type,* Fig. 187, is the realization of the first named arrangement, having the armature below the cores, and is therefore often called the "*undertype.*" This form is now used in the Edison dynamo,[1] built by the General Electric Co., Schenectady, N. Y., in the motors of the "C & C" (Curtis & Crocker) Electric Co.,[2] New York, and is further employed by the Adams Electric Co., Worcester, Mass.;

[1] *Electrical Engineer,* vol. xiii. p. 391 (1891); *Electrical World,* vol. xix. p. 220 (1892).

[2] Martin and Wetzler, " The Electric Motor," third edition, p 230.

by the E. G. Bernard Company, Troy, N. Y.; by the Detroit Electrical Works,[1] Detroit, Mich. ("King" dynamo); the Com-

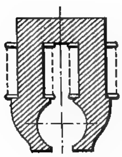
FIG. 187

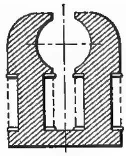
FIG. 188

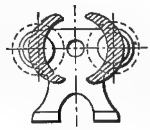
FIG. 189

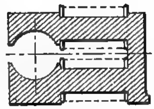
FIG. 190

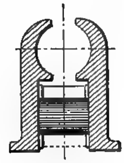
FIG. 191

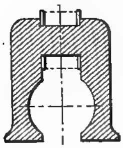
FIG. 192

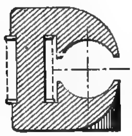
FIG. 193

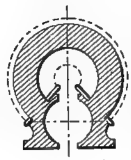
FIG. 194

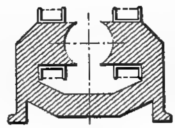
FIG. 195

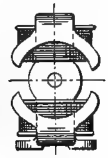
FIG. 196

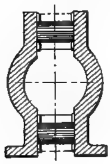
FIG. 197

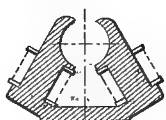
FIG. 198

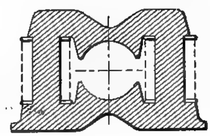
FIG. 199

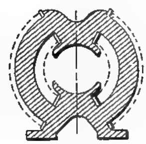
FIG. 200

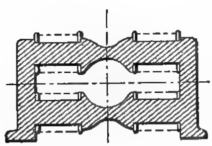
FIG. 201

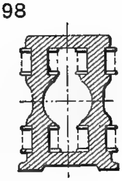
FIG. 202

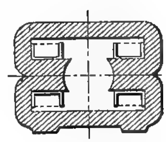
FIG. 203

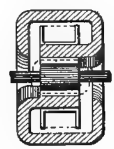
FIG. 204

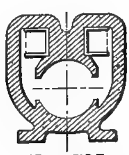
FIG. 205

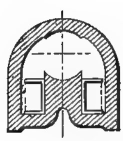
FIG. 206

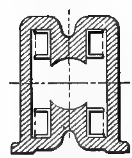
FIG. 207

Figs. 187 to 207.—Types of Bipolar Fields.

mercial Electric Co.[2] (A. D. Adams), Indianapolis, Ind.; the Novelty Electric Co.,[3] Philadelphia, Pa.; the Elektron Manu-

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[1] *Electrical World*, vol. xxi. p. 165 (1893).

[2] *Electrical World*, vol. xx. p. 430 (1892).

[3] *Electrical World*, vol. xvi. p. 404 (1890).

facturing Co.[1] (Perret), Springfield, Mass.; by Siemens Bros.,[2] London, Eng.; Mather & Platt[3] (Hopkinson), Manchester, Eng.; the India-rubber, Guttapercha and Telegraph Works Co.,[4] Silvertown, Eng., and by Clarke, Muirhead & Co., London.

The *inverted horseshoe type*, Fig. 188, having the armature above the cores, is also called the "*overtype.*" Of this form are the General Electric Co.'s "Thomson-Houston Motors," the standard motors of the Crocker-Wheeler Electric Co.,[5] Ampère, N. J.; further, machines of the Keystone Electric Co.,[6] Erie, Pa.; the Belknap Motor Co.,[7] Portland, Me.; the Holtzer-Cabot Electric Co.,[8] Boston, Mass.; the Card Electric Motor and Dynamo Co.,[9] Cincinnati, O.; the La Roche Electrical Works,[10] Philadelphia, Pa.; the Excelsior Electric Co.,[11] New York; the Zucker & Levett Chemical Co.,[12] New York (American "Giant" dynamo); the Knapp Electric and Novelty Co.,[13] New York; the Aurora Electric Co.,[14] Philadelphia, Pa.; the Detroit Motor Co.,[15] Detroit, Mich.; the National Electric Manufacturing Co.,[16] Eau Claire, Wis.; Patterson &

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[1] *Electrical Engineer*, vol. xiii. p. 8 (1892).

[2] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 509.

[3] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, pp. 519 and 522.

[4] *Electrical World*, vol. xiii. p. 84 (1889).

[5] *Electrical World*, vol. xvii. p. 130 (1891); *Electrical Engineer*, vol. xiv. p. 199 (1892).

[6] *Electrical World*, vol. xix. p. 220 (1892).

[7] *Electrical World*, vol. xxi. p. 470 (1893); *Electrical Engineer*, vol. xiv. p. 210 (1892).

[8] *Electrical Engineer*, vol. xvii. p. 291 (1894).

[9] *Electrical World*, vol. xxiii. p. 499 (1894); *Electrical Engineer*, vol. xi. p. 13 (1891). (This company is now the Bullock Electric Manufacturing Company.)

[10] *Electrical World*, vol. xvii. p. 17 (1893); *Electrical Engineer*, vol. xiv. p. 559 (1892); vol. xv. p. 491 (1893).

[11] *Electrical Engineer*, vol. xiv. p. 240 (1892).

[12] *Electrical Engineer*, vol. xiv. p. 187 (1892); *Electrical World*, vol. xxii. p. 210 (1893). (Now the Zucker, Levett & Loeb Company.)

[13] *Electrical World*, vol. xxi. pp. 286, 306, 471 (1893).

[14] *Electrical World*, vol. xv. p. 11 (1890).

[15] *Electrical World*, vol. xvi. p. 437 (1890); *Electrical Engineer*, vol. x. p. 695 (1890).

[16] *Electrical World*, vol. xvi. pp. 121, 419 (1890); vol. xxiv. p. 220 (1894); *Electrical Engineer*, vol. xviii. p. 178 (1894).

Cooper[1] (Esson), London; Johnson & Phillips[2] (Kapp), London; Siemens & Halske,[3] Berlin, Germany; Ganz & Co.,[4] Budapest, Austria; Allgemeine Elektricitäts Gesellschaft,[5] Berlin; Berliner Maschinenbau Actien-gesellschaft, vorm. L. Schwartzkopff,[6] Berlin; and Zuricher Telephon Gesellschaft,[7] Zurich, Switzerland.

Machines of the *horizontal single horseshoe type*, Fig. 189, in which the centre lines of the two magnet cores and the axis of the armature lie in the same horizontal plane, are built by the Jenney Electric Co.,[8] New Bedford, Mass. ("Star" dynamo), by the Great Western Manufacturing Co.[9] (Bain), Chicago, Ill., and by O. L. Kummer & Co.,[10] Dresden, Germany.

The *vertical single horseshoe type*, Fig. 190, finally, having the axes of magnet cores and armature in one vertical plane, is employed by the Excelsior Electric Co.[11] (Hochhausen), New York, and by the Donaldson-Macrae Electric Co.,[12] Baltimore, Md.

Single core horseshoe frames may be designed by placing the magnet either in a *horizontal* or in a *vertical* position, or by joining two polepieces of suitable shape by a magnet of circular form. The types thus obtained are the *horizontal single magnet type*, the *vertical single magnet type*, and the *single magnet ring type*.

In the *horizontal single magnet type*, Figs. 191 and 192 respectively, the armature may either be situated *above* or *below* the core. Machines of the former type (Fig. 191) are built by the

[1] S. P. Thompson, "Dynamo-Electric Machinery," plate v.

[2] S. P. Thompson, "Dynamo-Electric Machinery," plates i and ii.

[3] *Elektrotechn. Zeitschr.*, vol. vii. p. 13 (1886); Kittler, "Handbuch," vol. i. p. 851.

[4] *Zeitschr. f. Electrotechn.*, vol. vii. p. 78 (1889); Kittler, "Handbuch," vol. i. p. 930.

[5] Grawinkel and Strecker, "Hilfsbuch," fourth edition (1895), p. 287.

[6] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 288.

[7] Grawinkel and Stricker, "Hilfsbuch," fourth edition, p. 328.

[8] *Electrical World*, vol. xix. p. 172 (1892); *Electrical Engineer*, vol. xiii. p. 182 (1892).

[9] *Electrical Engineer*, vol. xvii. p. 421 (1894). (Now the Western Electric Co.)

[10] Kittler, "Handbuch," vol. i. p. 949.

[11] *Electrical Engineer*, vol. xvii. p. 465 (1894).

[12] *Electrical Engineer*, vol. xiii. p. 397 (1892).

Jenney Electric Motor Co.,[1] Indianapolis, Ind.; the Porter Standard Motor Co., New York; the Fort Wayne Electric Corp.,[2] Fort Wayne, Ind.; the United States Electric Co., New York; the Holtzer-Cabot Electric Co.,[3] Boston; the Card Electric Motor and Dynamo Co.,[4] Cincinnati, O.; the Simpson Electrical Manufacturing Co.,[5] Chicago; the Chicago Electric Motor Co.,[6] Chicago; the Bernstein Electric Co.,[7] Boston; and by the Premier Electric Co.,[8] Brooklyn. The latter type, Fig. 192, is employed by the Elektron Manufacturing Co.,[9] Springfield, Mass.; by the Riker Electric Motor Co.,[10] Brooklyn; and by the Actiengesellschaft Elektricitätwerke, vorm. O. L. Kummer & Co.,[11] Dresden.

The vertical single magnet type, Fig. 193, is used by the "D. & D." Electric Manufacturing Company,[12] Minneapolis, Minn.; the Packard Electric Company,[13] Warren, O.; the Boston Motor Company,[14] Boston; the Elbridge Electric Manufacturing Company, Elbridge, N. Y.; the Woodside Electric Works[15] (Rankin Kennedy), Glasgow, Scotland; by Greenwood & Batley,[16] Leeds, England; by Goolden & Trotter[17] (Atkinson), England; and by Naglo Bros.,[18] Berlin.

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[1] *Electrical Engineer*, vol. xiii. p. 182 (1892.)

[2] *Electrical Engineer*, vol. xiii. p. 408 (1892); *Electrical World*, vol. xxviii. p. 394 (1896).

[3] *Electrical World*, vol. xix. p. 107 (1892).

[4] *Electrical World*, vol. xxiii. p. 499 (1894).

[5] *Electrical World*, vol. xxii. p. 30 (1893).

[6] *Electrical World*, vol. xxii. p. 31 (1893).

[7] *Electrical World*, vol. xix. p. 283 (1892).

[8] *Electrical World*, vol. xix. p. 186 (1892).

[9] *Electrical Engineer*, vol. xv. p. 540 (1893).

[10] *Electrical Engineer*, vol. xvi. p. 436 (1893).

[11] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 277.

[12] *Electrical World*, vol. xx. p. 183 (1892); *Electrical Engineer*, vol. xiv. p. 272 (1892).

[13] *Electrical World*, vol. xx. p. 265 (1892), *Electrical Engineer*, vol. xiv. p. 414 (1892).

[14] *Electrical World*, vol. xxi. p. 471 (1893).

[15] *The Electrician* (London), March 1, 1889; *Electrical World*, vol. xiii., April, 1889.

[16] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 531.

[17] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 615.

[18] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 314.

Fig. 194 shows the *single magnet ring type*, which is employed by the Mather Electric Company,[1] Manchester, Conn.

Two magnets, instead of forming the limbs of a horseshoe, can also be set in line with each other, one on each side of the armature, or may be arranged so as to be symmetrical to the armature, but with like poles pointing to the same direction, instead of forming a single magnetic circuit with *salient* poles; the frame will then constitute a double circuit with *consequent* poles in the yokes joining the respective ends of the magnet cores. In both of these cases the cores may be put in a horizontal or vertical position, and in consequence we obtain two *horizontal double magnet types*, Figs. 195 and 197, and two *vertical double magnet types*, Figs. 196 and 199.

The *salient pole horizontal double magnet type*, Fig. 195, is employed by Naglo Bros.,[2] Berlin, and by Fein & Company, Stuttgart, Germany; and the *salient pole vertical double magnet type*, Fig. 196, by the Edison Manufacturing Company,[3] New York; and by Siemens & Halske,[4] Berlin.

The *consequent pole horizontal double magnet type*, Fig. 197, is used in the Feldkamp motor, built by the Electrical Piano Company,[5] Newark, N. J.; and in the fan motor of the De Mott Motor and Battery Company;[6] and the *consequent pole vertical double magnet type*, Fig. 199, by the Columbia Electric Company,[7] Worcester, Mass.; the Keystone Electric Company, Erie, Pa.; the Akron Electrical Manufacturing Company,[8] Akron, O.; the Mather Electric Company,[9] Manchester, Conn.; the Duplex Electric Company,[10] Corry, Pa.; the Gen-

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[1] *Electrical Engineer*, vol. xvii. p. 181 (1894).

[2] Kittler, "Handbuch," vol. i. p. 908; Jos. Krämer, "Berechnung der Dynamo Gleichstrom Maschinen."

[3] "Composite" Fan Motor, *Electrical Engineer*, vol. xiv. p. 140 (1893); *Electrical World*, vol. xxviii. p. 375 (1896); *Electrical Age*, vol. xix. p. 269 (1897).

[4] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 326.

[5] *Electrical World*, vol. xxi. p. 240 (1893).

[6] *Electrical World*, vol. xxi. p. 395 (1893).

[7] *Electrical World*, vol. xxiii. p. 849 (1894).

[8] *Electrical World*, vol. xx. p. 264 (1892).

[9] *Electrical World*, vol. xxiv. p. 112 (1894); *Electrical Engineer*, vol. xviii. p. 99 (1894).

[10] *Electrical World*, vol. xix. pp. 107, 171 (1892); *Electrical Engineer*, vol. xiii. p. 198 (1892).

eral Electric Traction Company (Snell), England; Mather & Platt (Hopkinson),[1] Manchester, England; Immish & Company,[2] England; Oerlikon Works (Brown),[3] Zurich, Switzerland; Helios Company,[4] Cologne; and by Naglo Bros.,[5] Berlin.

If in the latter form the magnets are made of circular shape, the *double magnet ring type*, Fig. 200, is obtained, which is built by the "C & C" Electric Company,[6] New York, and which has been used in the Griscom motor[7] of the Electro-dynamic Company, Philadelphia.

The *inclined double magnet type*, illustrated in Fig. 198, forms the connecting link between the double magnet and the single horseshoe types; it is employed by the Baxter Electrical Manufacturing Company,[8] Baltimore, Md.; by Fein & Company,[9] Stuttgart; and by Schorch[10] in Darmstadt.

The combination of two horseshoes with common polepieces furnishes two further forms of field magnet frames. Fig. 201 shows the *horizontal double horseshoe type*, and Fig. 202 the *vertical double horseshoe type*.

Machines of the former type (Fig. 201) are built by the United States Electric Company[11] (Weston), New York; the Brush Electric Company,[12] Cleveland, O.; the Ford-Washburn Storelectric Company, Cleveland, O.; the Western Electric Company,[13] Chicago, Ill.; the Fontaine Crossing and Electric Company (Fuller), Detroit, Mich.; by Crompton & Company,[14] London, England; by Lawrence, Paris & Scott, England, and by Schuckert & Company, Nuremberg, Germany.

The latter form (Fig. 202) is employed in dynamos of Fort

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[1] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 496.
[2] Gisbert Kapp, "Transmission of Energy," p. 272.
[3] Kittler, "Handbuch," vol. i. p. 921.
[4] Kittler, "Handbuch," vol. i. p. 904.
[5] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 312.
[6] *Electrical World*, vol. xxii. p. 247 (1892).
[7] Martin and Wetzler, "The Electric Motor," third edition, p. 126.
[8] Martin and Wetzler, "The Electric Motor," third edition, p. 228.
[9] Kittler, "Handbuch," vol. i. p. 944.
[10] Jos. Krämer, "Berechnung der Gleichstrom Dynamo Maschinen."
[11] Kittler, "Handbuch," vol. i. p. 879.
[12] *Electrical Engineer*, vol. xiv. p. 50 (1892).
[13] *Electrical Engineer*, vol. xvi. p. 323 (1893).
[14] Kapp, "Transmission of Energy," p. 292.

Wayne Electric Corporation[1] (Wood), Fort Wayne, Ind.; La Roche Electric Works,[2] Philadelphia; Granite State Electric Company,[3] Concord, N. H.; Onondaga Dynamo Company, Syracuse, N. Y.; Electric Construction Corporation[4] (Elwell-Parker); and Crompton Company,[5] London, England.

If one or both the polepieces of a consequent pole double magnet type are prolonged in the axial direction, that is, toward the armature, and the winding is transferred from the cores to these elongated polepieces, then a type is obtained in which the magnet frame forms a closed iron wrappage with inwardly protruding poles. Forms of this feature are known as *iron-clad types*, and, according to the number of magnets and to their position, are single magnet and double magnet, horizontal and vertical iron-clad types.

Fig. 203 shows the *horizontal iron-clad type*, having two horizontal magnets. It is used by the General Electric Company,[6] Schenectady, N.Y. (Thomson-Houston Arc Light type), Detroit Electric Works,[7] Detroit, Mich.; Eickemeyer Company,[8] Yonkers, N. Y.; Fein & Company,[9] Stuttgart; and Aachen Electrical Works[10] (Lahmeyer), Aachen, Germany.

A modification of this type consists in letting the poles project parallel to the shaft, one above and one below, or one on each side of the armature; the only magnetizing coil required in this case will completely surround the armature. This special horizontal iron-clad form, which is illustrated in Fig. 204, is realized in the Lundell machine,[11] built by the Interior Conduit and Insulation Company, New York.

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[1] *Electrical World*, vol. xxiii. p. 845 (1894); vol. xxviii. p. 390 (1896); *Electrical Engineer*, vol. xvii. p. 598 (1894).

[2] *Electrical Engineer*, vol. xiii. p. 439 (1892).

[3] *Electrical Engineer*, vol. xvi. p. 45 (1893).

[4] *Electrical Engineer*, vol. xv. p. 166 (1893).

[5] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 486.

[6] Silv. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 465.

[7] *Electrical World*, vol. xx. p. 46 (1892); *Electrical Engineer*, vol. xiv. p. 27 (1892).

[8] Kittler, "Handbuch," vol. i. p. 941.

[9] Kittler, "Handbuch," vol. i. p. 944.

[10] Kittler, "Handbuch," vol. i. p. 917.

[11] *Electrical World*, vol. xx. pp. 13, 381 (1892); vol. xxiii. p. 32 (1894); *Electrical Engineer*, vol. xiii. p. 643 (1892); vol. xiv. p. 544 (1892); vol. xvii. p. 17 (1894.)

In Figs. 205 and 206 the two possible cases of the *vertical single magnet iron-clad type* are depicted, the magnet being placed *above* the armature in the former and *below* the armature in the latter case. The *single magnet iron-clad overtype*, Fig. 205, is adopted in the street-car motors of the General Electric Company, Schenectady, N. Y.; in the machines of the Muncie Electrical Works,[1] Muncie, Ind.; of the Lafayette Engineering and Electric Works,[2] Lafayette, Ind., and in the battery fan motor of the Edison Manufacturing Company,[3] New York. Machines of the *single magnet iron-clad undertype*, Fig. 206, are built by the Brush Electrical Engineering Company[4] (Mordey), London, and by Stafford and Eaves,[5] England.

The *vertical double magnet iron-clad type*, Fig. 207, having two vertically projecting magnets, one above and one below the armature, is employed in the machines of the Wenstrom Electric Company,[6] Baltimore; the Triumph Electric Company,[7] Cincinnati, O.; the Shawhan-Thresher Electric Company,[8] Dayton, O.; the Card Motor Company,[9] Cincinnati, O.; the Johnson Electric Service Company,[10] Milwaukee, Wis.; the Erie Machinery Supply Company,[11] Erie, Pa.; O. L. Kummer & Company,[12] Dresden; Deutsche Elektrizitäts-Werke[13] (Garbe, Lahmeyer & Co.), Aachen; Schuckert & Company,[14] Nuremburg, Germany; Oerlikon Works,[15] Zurich; and the Zurich Telephone Company,[16] Zurich, Switzerland.

There are various other bipolar types, which, however,

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[1] *Electrical Engineer*, vol. xv. p. 606 (1893).
[2] *Western Electrician*, vol. xviii. p. 273 (1896).
[3] *Electrical World*, vol. xxi. p. 347 (1893).
[4] *Elektrotechn. Zeitschr.*, vol. xi. p. 135 (1890).
[5] S. P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 202.
[6] *Elektrotechn. Zeitschr.*, vol. xi. p. 122 (1890).
[7] *Electrical Engineer*, vol. xvii. p. 314 (1894).
[8] *Electrical World*, vol. xxiii. p. 191 (1894).
[9] *Electrical World*, vol. xxii. p. 15 (1893).
[10] *Electrical Engineer*, vol. xvii. p. 290 (1894).
[11] *Electrical World*, vol. xix. p. 283 (1892).
[12] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 278.
[13] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 293.
[14] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 299.
[15] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 320.
[16] *Elektrotechn. Zeitschr.*, vol. ix. pp. 181, 347, 410 and 485 (1888).

mostly are out of date, and, therefore, of very little practical importance. These can easily be regarded as special cases of the types enumerated above.

73. Multipolar Types.

Multipolar field magnet frames can have one or two magnets for every pole, or each magnet can independently supply

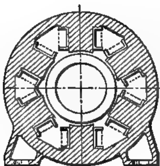
FIG. 208

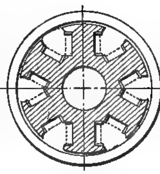
FIG. 209

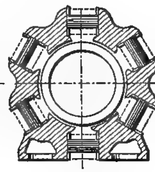
FIG. 210

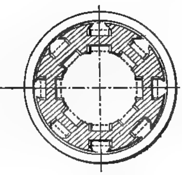
FIG. 211

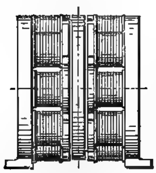
FIG. 212

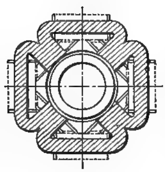
FIG. 213

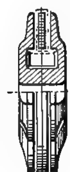
FIG. 214

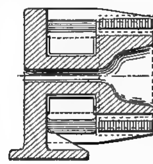
FIG. 215

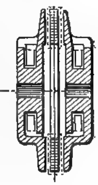
FIG. 216

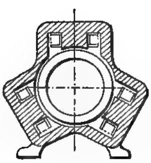
FIG. 217

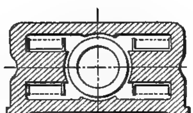
FIG. 218

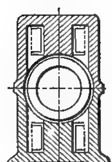
FIG. 219

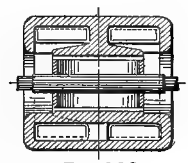
FIG. 220

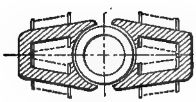
FIG. 221

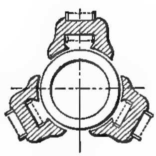
FIG. 222

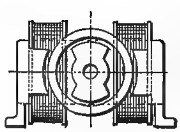
FIG. 223

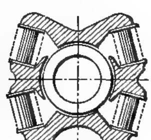
FIG. 224

Figs. 208 to 224.—Types of Multipolar Fields.

two poles, or one single magnet, or two magnets, may be provided with polepieces of such shape as to form the desired number of poles of opposite polarity.

If the number of magnets is identical with the number of poles, the magnets may either be placed in a *radial*, a *tangetial*, or an *axial* position with reference to the armature, and in the two first-named cases they may be put either *outside* or *inside* of the armature.

The *Radial Outerpole Type* is shown in Fig. 208; this form has been adopted as the standard type for large dynamos of the General Electric Company,[1] Schenectady, N. Y.; of the Westinghouse Electric and Manufacturing Company,[2] Pittsburg, Pa.; the Crocker-Wheeler Electric Company,[3] Ampère, N. J.; the Riker Electric Motor Company,[4] Brooklyn; the Stanley Electric Manufacturing Company,[5] Pittsfield, Mass.; the Fort Wayne Electric Company,[6] Fort Wayne, Ind.; the Eddy Electric Manufacturing Company,[7] Windsor, Conn.; the Belknap Motor Company,[8] Portland, Me.; the Shawhan-Thresher Electric Company,[9] Dayton, O.; the Great Western Electric Company[10] (Bain), Chicago; the Walker Manufacturing Company,[11] Cleveland, O.; the Mather Electric Company,[12] Manchester, Conn.; the Claus Electric Company,[13] New York; the Commercial Electric Company,[14] Indianapolis;

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[1] *Electrical World*, vol. xxi. p. 335 (1893); vol. xxiv. pp. 557 and 652 (1894); *Electrical Engineer*, vol. xiii. p. 165 (1892); vol. xiv. p. 562 (1892); vol. xviii. pp. 426, 507 (1894).

[2] *Electrical World*, vol. xxi. p. 91 (1893); vol. xxiv. p. 421 (1894); *Electrical Engineer*, vol. xviii. p. 330 (1894).

[3] *Electrical World*, vol. xxiii. p. 307 (1894); *Electrical Engineer*, vol. xvii. p. 193 (1894).

[4] *Electrical World*, vol. xxiii. p. 687 (1894); *Electrical Engineer*, vol. xvii. p. 442 (1894).

[5] *Electrical World*, vol. xxiii. p. 815 (1894); *Electrical Engineer*, vol. xvii. p. 507 (1894).

[6] *Electrical World*, vol. xxiii. p. 878 (1894); vol. xxviii. p. 395 (1896).

[7] *Electrical World*, vol. xxv. p. 34 (1895).

[8] *Electrical Engineer*, vol. xvii. p. 502 (1894).

[9] *Electrical Engineer*, vol. xvii. p. 463 (1894).

[10] *Electrical World*, vol. xxiii. p. 161 (1894).

[11] *Electrical World*, vol. xxiii. pp. 475 and 785 (1894); vol. xxviii. p. 423 (1896); *Electrical Age*, vol. xviii. p. 605 (1896).

[12] *Electrical Engineer*, vol. xiv. p. 364 (1892).

[13] *Electrical Engineer*, vol. xvi. p. 3 (1893).

[14] *Electrical World*, vol. xxiv. p. 627 (1894); vol. xxviii. p. 437 (1896); *Electrical Engineer*, vol. xviii. p. 506 (1894).

the Zucker, Levitt & Loeb Company,[1] New York; the Allgemeine Electric Company[2] (Dobrowolsky), Berlin, Germany; O. L. Kummer & Company,[3] Dresden; Garbe, Lahmeyer & Company,[4] Aachen; Elektricitäts Actien-Gesellschaft, vormals W. Lahmeyer & Company,[5] Frankfurt a. M.; Schuckert & Company,[6] Nuremburg; C. & E. Fein,[7] Stuttgart; Naglo Bros.,[8] Berlin; the Zurich Telephone Company,[9] Zurich; the Oerlikon Machine Works,[10] Zurich, Switzerland; R. Alioth & Company,[11] Basel, Switzerland; the Berlin Electric Construction Company (Schwartzkopff),[12] Berlin, Germany; and numerous others.

In Fig. 209 is represented the *Radial Innerpole Type*, which is used by the Siemens & Halske Electric Company,[13] Chicago, Ill., and Berlin, Germany; by the Alsacian Electric Construction Company,[14] Belfort, Alsace; by Naglo Bros.,[15] Berlin, Germany; by Fein & Co.,[16] Stuttgart, Germany; and by Ganz & Co.,[17] Budapest, Austria.

The *Tangential Outerpole Type*, Fig. 210, is employed by the Riker Electric Motor Company, Brooklyn; by the Baxter Motor Company,[18] Baltimore, Md.; the Mather Electric Company,[19] Manchester, Conn.; the Dahl Electric Motor Com-

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1 "Improved American Giant Dynamo," *Electrical Age*, vol. xviii. p. 600 (Oct. 17, 1896).

2 *Electrical Engineer*, vol xii. p. 596 (1891); vol. xvi. p. 103 (1893)

3 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 278.

4 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 291.

5 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 294.

6 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 299.

7 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 304.

8 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 311.

9 Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 327.

10 *Electrical Engineer*, vol. xii. p. 597 (1891).

11 Kittler, "Handbuch," vol. i. p. 934.

12 Kittler, "Handbuch," vol. i. p. 939.

13 *Electrical World*, vol. xxii., p. 61 (1893); *Electrical Engineer*, vol. xii. p. 572 (1891); vol. xiv. p. 313 (1892).

14 *L'Electricien*, vol. i. p. 33 (1891).

15 Kittler, "Handbuch," vol. i. p. 916.

16 *Zeitschr. f. Elektrotechn.*, vol. v. p. 545 (1887).

17 *Electrotechn. Zeitschr.*, vol. viii. p. 233 (1887).

18 Hering, "Electric Railways," p. 294.

19 *Electrical World*, vol. xxiv. p. 134 (1894); *Electrical Engineer*, vol. xviii. p. 177 (1894).

pany,[1] New York; the Electrochemical and Specialty Company,[2] New York ("Atlantic Fan Motor"), and by Cuénod, Sauter & Co.[3] (Thury), Geneva, Switzerland; generators of this type are further used in the power station of the General Electric Company,[4] Schenectady, N. Y., and in the Herstal,[5] Belgium, Arsenal.

Machines of the *Tagential Innerpole Type*, Fig. 211, are built by the Helios Electric Company,[6] Cologne, Germany.

In the *Axial Multipolar Type*, Fig. 212, there are usually two magnets for each pole, one on each side of the armature, in order to produce a symmetrical magnetic field. This form is used by the Short Electric Railway Company,[7] Cleveland, O.; Schuckert & Co.,[8] Nuremberg, Germany; Fritsche & Pischon,[9] Berlin, Germany; Brush Electric Engineering Company,[10] London, England ("Victoria" Dynamo); by M. E. Desroziers,[11] Paris, and by Fabius Henrion,[12] Nancy, France. The type recently brought out by the C. & C. Electric Company,[13] New York, has but one magnet per pole, and the polepieces are arranged opposite the external circumference of the armature.

Fig. 213 shows the *Raditangent Multipolar Type*, which is a combination of the Radial and Tangential Outerpole Types, Figs. 208 and 210 respectively, and which is employed by the Standard Electric Company,[14] Chicago, Ill.

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[1] *Electrical World*, vol. xxi. p. 213 (1893).

[2] *Electrical World*, vol. xxi. p. 394 (1893).

[3] Kittler, "Handbuch," vol. i. p. 936.

[4] Thompson, "Dynamo-Electric Machinery," fourth edition, p. 517.

[5] 500 HP. Generator, *Electrical World*, vol. xx. p. 224 (1892).

[6] Kittler, "Handbuch," vol. i. p. 905.

[7] *Electrical World*, vol. xviii. p. 165 (1891).

[8] *Elektrotechn. Zeitschr.*, vol. xiv. p. 513 (1893); *Electrical Engineer*, vol. xii. p. 595 (1891).

[9] *Electrical World*, vol. xx. p. 308 (1892); *Electrical Engineer*, vol. xii. p. 572 (1891).

[10] Thompson, "Dynamo Electric Machinery," fourth edition, p. 498.

[11] *Electrical Engineer*, vol. xiv. p. 259 (1892); vol. xv. p. 340 (1893).

[12] Grawinkel and Strecker, "Hilfsbuch," fourth edition, p. 317.

[13] *Electrical World*, vol. xxviii. p. 372 (1896).

[14] *Electrical World*, vol. xxiii. pp. 342, 549 (1894); *Electrical Engineer*, vol. xvii. pp. 189, 379 (1894).

If only one magnet is used in multipolar fields, the pole-pieces may be so shaped as to face the armature in an *axial* or in a *radial* direction. In the former case the *Axial-Pole Single Magnet Multipolar Type*, Fig. 214, is obtained, which is used by the Brush Electrical Engineering Company[1] (Mordey), London, England, and by the Fort Wayne Electric Company[2] (Wood), Fort Wayne, Ind.

In the latter case the *Outer-Inner Pole Single Magnet Type*, Fig. 215, results, in which the polepieces may either *all* be opposite the *outer* or the *inner* armature surface, or *alternately outside* and *inside* of the armature; the latter arrangement, which is the most usual, is illustrated in Fig. 215, and is employed by the Waddell-Entz Company,[3] Bridgeport, Conn., and by the Esslinger Works,[4] Wurtemberg, Germany; the all outerpole arrangement is employed in the direct connected multipolar type of the C & C Electric Company,[5] New York.

If two magnets furnish the magnetic flux, they are placed concentric to the armature, and the two sets of polepieces so arranged that adjacent poles on either side of the armature are of unlike polarity, but that poles facing each other on opposite sides of the armature have the same polarity. Such a *Double Magnet Multipolar Type* is shown in Fig. 216; it is that designed by Lundell,[6] and built by the Interior Conduit and Insulation Company, New York.

In giving the yoke of the Radial Multipolar Type (Fig. 208) such a shape as to form a polepiece between each two consecutive magnets, an iron-clad form is obtained having alternate salient and consequent poles, and requiring but one-half the number of magnets as a radial multipolar machine of same number of poles.

Fig. 217 shows a field frame of the *Multipolar Iron-clad Type*, having six poles, which is the form employed in the gearless street car motor of the Short Electric Railway Com-

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[1] Thompson, "Dynamo Electric Machinery," fourth edition, p. 678.

[2] *Electrical Engineer* vol. xv. p. 46 (1893).

[3] *Electrical World*, vol. xix. p. 13 (1892); vol. xxii. p. 120 (1893).

[4] Kittler, "Handbuch," vol. i. p. 945.

[5] *Electrical World*, vol. xxv. p. 33 (1895).

[6] *Electrical World*, vol. xx. p. 85 (1892).

pany,[1] Cleveland, O. In Figs. 218 and 219, two special cases of this type are depicted, both representing *Fourpolar Iron-clad Types*, and differing only in the position of the magnets. The *Horizontal Fourpolar Iron-clad Type*, Fig. 218, is used in the Edison Iron-clad Motor[2] (General Electric Company), and in the dynamos of the Wenstrom Electric Company,[3] Baltimore, Md. The *Vertical Fourpolar Iron-clad Type*, Fig. 219, is employed by the Elliott-Lincoln Electric Company,[4] Cleveland, O.

Fig. 220 shows a special case of the Horizontal Fourpolar Iron-clad Type, obtained by symmetrically doubling the frame illustrated in Fig. 204, and providing four poles instead of two. The cores are so wound that the centre of the cylindrical iron wrappage has one polarity and the ends the opposite polarity. Two oppositely situated polepieces are joined to the middle, and the two sets of intermediate ones to the ends of the magnet frame; the lower half of Fig. 220, consequently, is a section taken at right angles to the upper half, the diametrically opposite section being identical. This type has been developed by the Storey Motor and Tool Company,[5] New York.

Multipolar fields may also be formed by a number of independent horseshoes arranged symmetrically around the outer armature periphery. Figs. 221 and 222 show two such *Multiple Horseshoe Types*, double magnet horseshoes being employed in the former, and single magnet horseshoes in the latter type. Multiple horseshoe machines of the double magnet form (Fig. 221) have been designed by Elphinstone & Vincent, and by Elwell-Parker Electric Construction Corporation,[6] England; while the single-magnet form (Fig. 222) is employed by the Electron Manufacturing Company[7] (Perret), Springfield, Mass.

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[1] *Electrical World*, vol. xx. p. 241 (1892); *Electrical Engineer*, vol. xiv. p. 395 (1895).

[2] *Electrical Engineer*, vol. xii. p. 598 (1891).

[3] *Electrical World*, vol. xxiv. p. 183 (1894).

[4] *Electrical World*, vol. xxi. p. 193 (1893); vol. xxii. p. 484 (1893).

[5] *Electrical World*, vol. xxi. p. 214 (1893); *Electrical Engineer*, vol. xv. p. 263 (1893).

[6] *The Electrician* (London), vol. xxi. p. 183 (1888).

[7] *Electrical Engineer*, vol. x. p. 592 (1890); vol. xiii. p. 2 (1892).

Further forms of multipolar fields can be derived from the bipolar horizontal and vertical double magnet types respectively. If, in the Vertical Double Magnet Type, Fig. 196, an additional polepiece is provided at the centre of the frame so as to face the internal surface of the armature at right angles to the outer polepieces, the *Fourpolar Vertical Double Magnet Type* is created, which, when laid on its side, will constitute the *Fourpolar Horizontal Double Magnet Type*, Fig. 223. If, in the Vertical Double Magnet Type, Fig. 199, the two cores are cut in halves and additional polepieces inserted at right angles to the existing ones, the *Vertical Quadruple Magnet Type*, Fig. 224, is obtained; the same operation performed with the Horizontal Double Magnet Type, Fig. 197, will give the *Horizontal Quadruple Magnet Type*.

Fourpolar Horizontal Double Magnet Dynamos, Fig. 223, are built by the Zurich Telephone Company,[1] Zurich, Switzerland; and Vertical Quadruple Magnet Machine, Fig. 224, by the Duplex Electric Company,[2] Corry, Pa.

Numerous other multipolar types have been invented and patented, but either are of historical value only, or have not yet come into practical use.

74. Selection of Type.

If the type is not specified, the field magnet frame for a large output machine should be chosen of one of the *multipolar types*, as in these the advantage of a better proportioning and a higher efficiency of the armature winding, and the possibility of a symmetrical arrangement of the magnetic frame, results in a saving of copper as well as of iron; while for smaller machines—below 10 KW capacity—the *bipolar forms* are preferable on account of the great complication caused by the increased number of armature sections, commutator-divisions, field coils, etc., necessary in multipolar machines, and on account of the narrowness of the neutral or non-sparking space on a multipolar commutator.

The field, moreover, should have as few separate magnetic

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[1] Kittler, "Handbuch," vol. i. p. 947.

[2] *Electrical World*, vol. xx. p. 14 (1892); *Electrical Engineer*, vol. xiv. p. 1 (1892).

circuits as possible; thus, in the case of a bipolar type, it should be a single magnetic circuit rather than the consequent pole type which is formed by two or more magnetic circuits, of one or two magnets each, in parallel, because the former is more economical in wire and in current required for excitation. In two-circuit consequent pole machines, for instance, such as the double magnet types, Figs. 197, 199, and 200, and the double horseshoe types, Figs. 201 and 202, according to Table LXIX., § 75, there is 1.41 times the length of wire, and consequently also 1.41 times the energy of magnetization required than in a single circuit, round cores being used in both cases, and the single circuit having exactly twice the area of each of the two parallel circuits in the consequent pole machines. Triple and quadruple magnetic circuits, *i. e.*, 3 or 4 cores, or sets of cores, magnetically in parallel, are still more objectionable, requiring, when the cores are of circular cross-section, 1.73 and 2.00 times as much wire, respectively, as a single magnetic circuit having a round core of equal total sectional area.

If a machine has several magnetic circuits, each of which, however, passes through all the magnets in series, then the frame is to be considered as consisting of but one single circuit, for the subdivision only takes place in the yokes, and it is immaterial as to the length of exciting wire whether the return path of a single circuit is formed by one yoke, or by a number of yokes magnetically in parallel. The above-named objection to divided circuit types, consequently, does not apply in the case of the iron-clad forms, Figs. 203 to 207.

According to Table LXVIII., § 70, the horizontal double magnet type, Fig. 195, and the horizontal iron-clad type, Fig. 203, are the best bipolar forms, magnetically. The iron-clad types, furthermore, possess the mechanical advantage of having the field windings and the armature protected from external injuries by the frame of the machines, which makes them eminently adaptable to motors for railway, mining, and similar work.

The inverted horseshoe type, Fig. 188, which ranks very highly, as far as its magnetic qualities are concerned, has the centre of its armature at a comparatively very great distance from the base, requiring very high pillow-blocks, which have

to carry the weight as well as the downward thrust of the armature inherent to the inverted forms having the field windings below the centre of revolution; see § 42. The side pull of the belt with a high centre line of shaft tends to tip the machine, and the changes in the pull due even to the undulations of the belt will cause a tremor in the frame which jars the brushes, and, eventually, loosens their holders, and which has a disastrous influence upon the wearing of the commutator. On this account the inverted forms, or "under-types," can only be used for small and medium-sized machines, in which the height of the pillow-blocks remains within practical limits.

In selecting a *multipolar* type, Table LXVIII. shows that the radial innerpole type, Fig. 209, offers the best advantage with regard to the magnetical disposition; with this type, however, are connected some mechanical difficulties, due to the necessity of supporting the frame from one of its ends, laterally, and the armature from the other.

In the outerpole types the armature core can be supported centrally from the inner circumference, and the frame suitably provided with external lugs or flanges resting upon the foundation, a most desirable arrangement for mechanical strength and convenience. The most favorite of the outerpole forms is the radial outerpole type, Fig. 208, on account of its superiority, magnetically, over the tangential and axial multipolar types.

In all dynamo designs the consideration is especially to be borne in mind that the whole machine as well as its various parts should be easily accessible for inspection, and so arranged that they can conveniently be removed for repair or exchange. A large number of machines owe their popularity chiefly to their good disposition in this respect.

The shape of the frame in all cases is preferably to be so chosen that the length of the magnetic circuit in the same is as short as possible.

Advantages and Disadvantages of Multipolar Machines.

The *advantages* of multipolar over bipolar dynamos can be summarized as follows:

(1) By the multipolar construction a saving in weight of

material is effected both in the armature and in the field magnet, due to the subdivision of the magnetic circuit.

(2) Multipolar machines have a more compact and symmetrical form, because the component parts of the magnet frame are much smaller than a corresponding bipolar magnet, and are evenly spaced around the armature.

(3) Since there are as many openings around the armature as there are poles, the ventilation of the armature is much better; and since a number of small cores have a greater surface than one large core of equal cross-section and length, the dissipation of heat is facilitated, so that under the same conditions multipolar machines run cooler than bipolar ones.

(4) For armatures of the same diameter, the individual parts of the field frame are much smaller and more easily handled in the multipolar than in the bipolar type.

The *disadvantages* of multipolar machines are:

(1) Greater complication in constructing, fitting, winding, and connecting, owing to the increased number of parts and consequent larger number of magnetic and electrical circuits.

(2) Strong magnetic side pull on the armature in case of eccentricity of field; much greater than in bipolar machines.

(3) Greater difficulty in balancing field; with multiple-circuit armature winding, the flux must be exactly the same for each pole, otherwise the E. M. F. of the circuits will be unequal, producing wasteful currents in them which, in turn, cause excessive sparking and heating. This difficulty, however, is not present in two-circuit windings.

Comparison of Bipolar and Multipolar Types.

The saving in material effected by the multipolar construction is seen by comparing the three designs shown in Fig. 224*a*, representing armatures of equal size in bipolar, four-pole, and eight-pole fields. Assuming that in all three cases the poles cover a like portion of the armature periphery, and that the gap induction is the same, then evidently the total armature flux is the same in the three designs. In practice, the gap density is usually less in bipolar than in multipolar machines, but this difference is not a necessity, and need not be considered in this comparison.

In the bipolar type, it will be seen that the armature must

carry $\frac{1}{2}$ the total flux between its shaft and periphery, while in the four-pole machine the armature cross-section is only required to carry $\frac{1}{4}$ the total flux, hence the radial depth B of the armature core in the four-pole machine need be only one-half as great as A in the bipolar type, thus reducing its weight. The flux passing through each core of the four-pole machine is $\frac{1}{2}$, and that passing through its yoke ring is $\frac{1}{4}$ of the corresponding bipolar flux, as is indicated by the dotted lines, each of which represents $\frac{1}{8}$ of the total flux. Owing to this subdivision of the flux, the weight of the multipolar magnet frame is less than that of the bipolar.

The difference is still more marked when we compare the eight-pole machine shown in Fig. 224a with the bipolar and

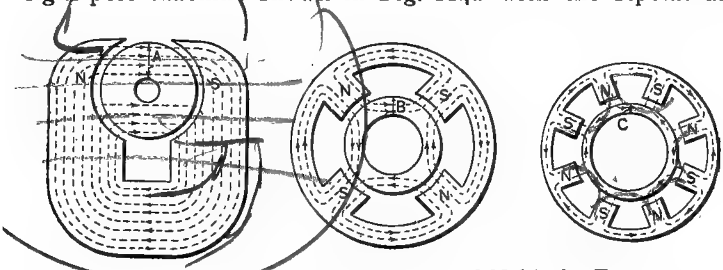

Fig. 224a.—Comparison of Bipolar and Multipolar Types.

four-pole types. The radial thickness of the field ring in the eight-pole machine is only $\frac{1}{2}$ as much as in the four-pole machine, and $\frac{1}{8}$ as great as the thickness of the yoke in the bipolar machine, and the armature may be hollowed out to a radial thickness C of only $\frac{1}{4}$ that of the bipolar armature, A, as shown, the total flux being the same for all three designs.

Proper Number of Poles for Multipolar Field Magnets.

If a multipolar field has been adopted, the best number of poles must next be decided upon. This is a question, partly, of selecting a size and number of field cores, coils, etc., convenient for making and handling, but it is chiefly a matter of the number of magnetic cycles occurring in the armature core. The number of cycles per second, as stated on p. 111, is

$$N_1 = \frac{N}{60} \times n_p,$$

if N is the number of revolutions per minute, and n_p the number of pairs of magnet poles.

Direct-current machinery is designed, generally, so that N is between 10 and 35 cycles per second. This limits the number of poles, the object being to reduce the core losses due to *hysteresis* and *eddy currents*, the former, as we have seen in § 32, being proportional to N_1, whereas the latter increase with the square of N_1, see § 33. The lower frequency of about 10 or 15 cycles applies to *low-speed* machines for direct connection to engines, and the higher frequency of 30 or 35 is adopted in *high-speed* belt-connected generators and motors.

By transformation of the above formula for N_1, it therefore follows that, for instance, the maximum speed of a four-pole machine, having 2 pairs of poles, should be

$$N = \frac{60 \times N_1}{n_p} = \frac{60 \times 35}{2} = 1050 \text{ revs. p. min.}$$

In some instances, four-pole machines are run at higher speeds than this, for example 1200 revs. per min., which gives 40 cycles. But this is rather too high a frequency, and should not be adopted except for special reasons.

If the values $n_p = 1, 2, 3, 4$, etc., and $N_1 = 10, 15, 25$, and 35, respectively, be inserted into the above formula for N, the following table of dynamo speeds for various numbers of poles is obtained, from which the proper number of poles for any particular speed can be taken:

TABLE LXVIII*b*.—Number of Magnet Poles for Various Speeds.

Number of Poles.	Limits of Speed.		
	Low-Speed Machines. $N_1 = 10$ to 15 cycles	Medium-Speed Machines. $N_1 = 15$ to 25 cycles.	High-Speed Machines. $N_1 = 25$ to 35 cycles.
2	600 to 900	900 to 1500	1500 to 2100
4	300 " 450	450 " 750	750 " 1050
6	200 " 300	300 " 500	500 " 700
8	150 " 225	225 " 375	375 " 525
10	120 " 180	180 " 300	300 " 420
12	100 " 150	150 " 250	250 " 350
14	85 " 130	130 " 215	215 " 300
16	75 " 115	115 " 190	190 " 260
18	70 " 100	100 " 170	170 " 230
20	60 " 90	90 " 150	150 " 210

Generally speaking, the *bipolar type* is used for high speeds, such as 1500 revs. per min. or more. A further reason for selecting the bipolar type for very high speeds is the fact that machines running at 1500 revolutions, or more, are usually quite small. For speeds between 400 and 900 revs. per min., the *four-pole* construction is especially suitable, giving frequencies from $13\frac{1}{3}$ to 30 cycles. This includes nearly all belted dynamos and motors, from the large sizes of 200 KW, or more, down to about 10 KW, below which the *bipolar* form is generally used for the reasons given. Between 200 and 400 revs. per min., *six poles* are commonly adopted, the corresponding frequencies being from 10 to 20 cycles. This range of speed comprises practically all generators directly connected to high-speed engines, from the largest to the smallest, excepting combinations with very high speed engines of 600 revs. per min., or more, for which *four poles* would be preferable. When a dynamo is directly driven by a steam turbine, at an extremely high speed without reducing gear, the field should be of the *bipolar* type. For speeds below 200 revs. per min. the number of poles is generally increased to *eight, or more.* This applies to most generators directly connected to low-speed engines.

In some cases motors of various sizes, even down to 1 or 2 HP, are required to run at low speeds in order to be connected directly to the machines which they drive. For this purpose *six poles* are suitable for speeds from 200 to 400 revs. per min., and *eight, or more, poles,* if the speed is below 200 revs. per min. Railway motors are nearly always constructed with *four poles,* the speed being very variable, but having a maximum value in most cases of about 800 revs. per min. This gives a rather high frequency of $26\frac{2}{3}$ cycles, but as the maximum speed is rarely maintained for more than a few minutes at a time, the heating due to hysteresis and eddy currents in the armature core does not rise above the limit allowed. The average speed corresponds to a moderate frequency of 15 to 20 cycles.

CHAPTER XV.

GENERAL CONSTRUCTION RULES.

75. Magnet Cores.

a. Material.

The field cores should preferably be of *wrought iron*, or of *cast steel*, in order to economize in magnet wire, for the use of *cast iron*, on account of its low permeability, would require cores of at least $1\frac{2}{3}$, *i. e.*, almost twice the cross-section, and therefore a much greater length of wire, to obtain the necessary magnetizing force. With the smaller wrought-iron cores the leakage would also be less.

In spite of the decided advantage of wrought-iron cores, *cast-iron* field magnets are very common, since the temptation to use castings instead of forgings is very great. Where weight and bulk are of no consequence, a cast-iron field magnet may prove nearly as economical as one of wrought iron costing considerably more, but the former requires from $\frac{1}{3}$ to $\frac{1}{2}$ times more wire to encircle it than a wrought-iron one of similar magnetic density, in case of circular cross-section, and it is evident that this, by introducing additional electrical resistance, will prove a constant source of unnecessary running expense.

As to the use of *steel* in dynamos, H. F. Parshall, in a paper delivered before the Franklin Institute,[1] states that magnet frames made of cast steel are 25 per cent. cheaper than those of cast iron, but possess the disadvantage of being not as uniform in magnetic qualities as cast iron. He further asserts that good cast steel should not have greater percentages of impurities than .25 per cent. of carbon, .6 per cent. of manganese, .2 per cent. of silicon, .08 per cent. of phosphorus, and .05 per cent. of sulphur. The effect of carbon is to lessen the magnetic continuity and to greatly reduce the permeability;

[1] *Electrical World*, vol. xxiii. p. 214, February 17, 1894.

carbon, therefore, is the most objectionable impurity, and, if possible, should be restricted to smaller amounts than the maximum above quoted. Manganese, in quantities larger than stated, seriously reduces the magnetic susceptibility of the steel, a 12 per cent. mixture having scarcely greater susceptibility than air. Silicon is objectionable through facilitating the formation of blowholes, and from its hardening effect.

E. Schulz,[1] in comparing two dynamos differing only in the material of the field frame and in the magnet winding, finds that the weight of a cast-steel magnet frame is about one-half of that of cast iron, and that the weight of the copper for the magnets, on account of the smaller cross-section and the greater permeability of the cast steel, is reduced to somewhat less than one-half. The price of the frame will accordingly be about $1\frac{1}{4}$ times that of the cast-iron one, but, on account of the reduction of the copper weight, the cost of the whole machine will be less for a cast-steel than for a cast-iron frame, the total weight being less than one-half in the former case.

According to Professor Ewing[2] the permeability of good cast steel at low magnetic forces is less than that of wrought iron, but the reverse is the case with high forces. In a specially good sample tested by G. Kapp and Professor Ewing, a magnetic density of 18,000 lines per square centimetre (= 116,000 lines per square inch) was reached, with but little more than one-half the magnetizing force as is necessary for the same induction in ordinary wrought iron.

b. Form of Cross-Section.

The best form of cross-section for a magnet-core is undoubtedly that which possesses the smallest circumference for a given area, and this most economical section is the *circle.* It is, however, often preferable on account of reducing the dimension of the machine perpendicular to the armature shaft, to use cores of other than circular section; in this case either rectangular, elliptical, or oval cores are employed, or several

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[1] *Elektrisches Echo,* August 11, 1894; *Electrical World,* vol. xxiv., p. 238 (September 8, 1894).

[2] *Electrical Engineer,* London, October 5, 1894; *Electrical World,* vol. xxiv. p. 446 (October 27, 1894).

round cores are placed side by side and connected in parallel to each other, magnetically. The latter method, however, is not recommendable for the reason that the magnetizing effects of the neighboring coils partly neutralize each other, because of the currents of equal polarity flowing in opposite lateral directions in the parts of the coils facing each other, as indicated by arrows in Fig. 225. There is, consequently, a double

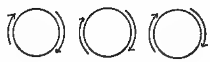


Fig. 225.—Direction of Current in Parallel Magnet Cores of same Polarity.

loss connected with this arrangement, a larger expenditure of copper, connected with higher magnet resistance, and decrease of the magnetizing effects by mutual influence of the coils.

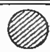
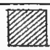
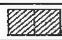
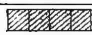
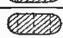
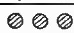
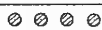
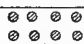
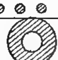
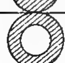
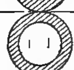
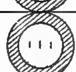
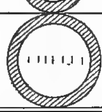
Besides the forms mentioned, also square cores and hollow magnets of ring-section are frequently used.

An idea of the economy of the form of cross-section to be chosen can be formed by means of the following Table LXIX., which gives the circumferences for unit area of the various forms of cross-sections employed in modern machines, and compares the same with the circumference of the most economical form, the circle. In the case of *rectangular* and *elliptical* cores, four forms each are considered, the lengths being, respectively, 2, 3, 4, and 8 times the width of the sections. For *oval* cores three sections are examined, the semicircular end portions being attached to a centre portion formed of 1, 2, and 4 adjacent squares, respectively. Next come four sections consisting of *several round cores in parallel*, namely, 2, 3, 4, and 8 *separate circles*. Of *hollow* cores, finally, five cases are considered, the internal diameter being, respectively, 1, 2, 3, 4, and 8 times the radial thickness of the cross-section.

Hollow Magnets are used in some special types, such as shown in Figs. 84, 94, 95, 96, and 100, where large circumferences of the cores are required but not the total area inclosed by these circumferences, and where the armature or its shaft has to pass through the centre of the magnet.

As to the use of *hollow* magnets in place of *solid* ones, Profes-

TABLE LXIX.—CIRCUMFERENCE OF VARIOUS FORMS OF CROSS-SECTIONS OF EQUAL AREA.

Form of Cross-Section	Description	Circumference for Unit Area	Relative Circumference (Circle=1)
	Circle	3.545	1
	Square	4.000	1.13
	Rectangle, 1:2	4.243	1.20
	" 1:3	4.62	1.305
	" 1:4	5.00	1.41
	" 1:8	6.364	1.80
	Ellipse, 1:2	3.87	1.09
	" 1:3	4.35	1.23
	" 1:4	4.84	1.37
	" 1:8	6.53	1.84
	Oval, 1 squ., 2 Semi-circles	3.85	1.085
	" 2 " 2 "	4.28	1.21
	" 4 " 2 "	5.09	1.44
	2 Circles	5.01	1.41
	3 "	6.14	1.73
	4 "	7.09	2.00
	8 "	10.03	2.83
	Ring, 1:1	3.85	1.085
	" 1:2	4.09	1.155
	" 1:3	4.43	1.25
	" 1:4	4.76	1.34
	" 1:8	5.91	1.67

sor Grotrian[1] states that with weak magnetizing forces only the outer layers of the iron, next to the winding, are magnetized.

[1] *Elektrotechn. Zeitschr.*, vol. xv. p. 36 (January 18, 1894); *Electrical World*, vol. xxiii. p. 216 (February 17, 1894).

E. Schulz,[1] however, showed by practical experiments that the magnetization is exactly proportional to the area of the core-section, even at the low induction due to the remanent magnetism; from this can be concluded that Professor Grotrian's results do not apply to the case of dynamo magnets under practical conditions. A. Föppl[2] claims that the theory of Professor Grotrian is correct, *i. e.*, that the flux gradually penetrates the magnet from its circumference, and that under certain circumstances it may not reach the centre of the core, but he admits that this theory has no practical bearing upon such magnets as are now used in practical dynamo design.

c. Ratio of Core-area to Cross-section of Armature.

The relation between the cross-section of iron in the magnet cores to that of the armature core is a very important one, as on its proper adjustment depends the attainment of maximum output per pound of wire with minimum weight of iron.

According to tests made at the Cornell University under the direction of Professor Dugald C. Jackson,[3] the best area of cross-section of the magnet cores for drum machines is $1\frac{1}{3}$ times that of least cross-section of armature, if the cores are of good *wrought iron*, or about $2\frac{1}{2}$ times the minimum armature section if *cast iron* cores are used.

According to Table XXII., § 26, the maximum core density in ring armatures is from $1\frac{1}{3}$ to $1\frac{2}{3}$ times that of drum armatures; for equal amounts of active wire, therefore, the former require $1\frac{1}{3}$ to $1\frac{2}{3}$ times as great a magnetic flux as the latter, and the cross-sections of the magnet cross, consequently, have to be taken correspondingly greater in case of ring machines, namely, $1\frac{3}{4}$ to $2\frac{1}{4}$ times the minimum armature section in case of *wrought iron* cores, and 3 to 4 times the armature section for *cast iron* field magnets.

Professor S. P. Thompson, in his "Manual on Dynamo-

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[1] *Elektrotechn. Zeitschr.*, vol. xv. p. 50 (February 8, 1894); *Electrical World*, vol. xxiii. p. 337 (March 10, 1894).

[2] *Elektrotechn. Zeitschr.*, vol. xv. p. 206 (April 12, 1894); *Electrical World*, vol. xxiii. p. 680 (May 19, 1894).

[3] *Transactions Am. Inst of El. Eng.*, vol. iv. (May 18, 1887); *Electrical Engineer*, vol. iii. p. 221 (June, 1887).

Electric Machinery,"[1] gives 1.25 for wrought iron and 2.3 for cast iron as the usual ratio in drum machines, and 1.66 and 3 respectively, in ring-armature dynamos.

In the experiments conducted by Professor Jackson, ten different armatures, all of same length and same external diameter, but of different bores, were used in the same field, thus including a range of from .5 to 1.4 for the ratio of least armature section to core area. The curves obtained show that the total induction through the armature increased quite rapidly when the armature was increased in area from .5 of that of the magnets to about .75 of the core area. From .75 to .9 there is still an increase of induction with increase of armature section, though comparatively small, and beyond .9 the increase is of no practical importance.

76. Polepieces.

a. Material.

The polepieces, if the shape and the construction of the magnet frame permits, should be of *wrought iron* or *cast steel*, in order to reduce their size, and therefore their magnetic leakage, they being of the highest magnetic potential of any part of the magnetic circuit. In forging, care should be taken that the "grain" or texture of the iron runs in the direction of the lines of force. The polepieces, however, usually have to embrace from .7 to .8 of the armature surface (compare §15), and are, therefore, particularly in the case of bipolar machines, often comparatively large. If in such a case their cross-section, in order to give sufficient mechanical strength, is to be far in excess of the area needed for the magnetic flux, there is no gain in using wrought iron or cast steel, and the polepieces should be made of cast iron. The *cast iron* used should be as soft and free from impurities as possible. It is preferable, whenever practicable, to have it annealed, and, if not too large in bulk, to have it converted into malleable iron; this is especially to be recommended for small machines.

An admixture of *aluminum* has been found to increase the permeability of the cast iron; by adding $1\frac{1}{2}$ per cent., by weight, of aluminum, the maximum carrying capacity of the

[1] S. P. Thompson, "Dynamo-Electric Machinery,' fifth edition, p. 378.

cast iron is increased about 5 per cent.; by 3 per cent. admixture it is increased 7 per cent.; and by adding 6 per cent. of aluminum, the induction increases about 9 per cent.; above 7 per cent. of admixture the permeability decreases, and at 12 per cent. addition of aluminum the gain in magnetic conductivity falls down to 7 per cent. From this it follows that an addition of from 6 to 7 per cent., by weight, of aluminum is the proper admixture for the purpose of improving the magnetic qualities of cast iron, which is explained by the fact that the latter percentage is the limit from which up the hardening influence of the aluminum upon the cast iron becomes appreciable.

In large multipolar machines combination frames consisting of wrought-iron magnet cores, cast-iron yokes, and cast-steel polepieces give excellent results, having the advantages of the high permeability and uniformity in the magnetic qualities of the wrought iron, of cheapness of the cast iron, and of reduction in size of the cast-steel polepieces, and being easier to machine, requiring less chipping, and being more easily finished than a magnet frame made entirely of cast steel.

A material which a few years ago was quite a favorite with dynamo builders, but which since has to a great extent been displaced by the cheaper cast steel, is the so-called "*Mitis metal*," or *cast wrought iron*, obtained by melting down scrap wrought iron in crucibles, and by rendering it fluid by the addition of a small quantity of aluminum. The trouble with this material was that a great many extra precautions had to be taken to procure sound castings, and that as a rule the castings were rough and difficult to work on account of their toughness. The magnetic value of Mitis iron differs very little from that of cast steel, its permeability at the inductions used in practice being but a trifle lower than that of the latter.

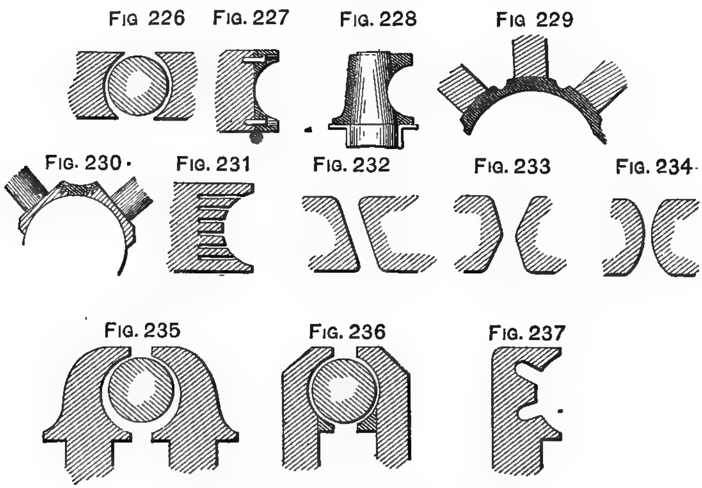
Edges and sharp corners are to be avoided as much as possible, for if they protrude sufficiently they will act to a certain extent as poles, and give cause to a source of loss. In castings thin projections are apt to chill while being cast, thus making them quite hard and destroying their magnetic qualities; when necessary for mechanical reasons, they should, therefore, be cast quite thick and massive, and may afterward be planed or turned down to the required size.

b. Shape.

The polepieces have for their object the transmission to the armature of the magnetic flux set up by the field magnet, and the establishment of a magnetic field space around the armature. The shape to be given to them must, therefore, effect the concentration of the lines of force upon the armature, and not their diffusion through the air. This, in general, is achieved by making the polar surfaces as large as possible, and bringing them as near to the armature as mechanical considerations permit, and by reducing the leakage areas of the free pole surfaces as much as possible. For practical rules of fixing the distance between the pole corners and the clearance between armature surface and polepieces for various kinds and sizes of armatures, see Tables LX. and LXI., § 58, respectively.

Since eddy currents are produced in all metallic masses, either by their motion through magnetic fields or by variations in the strength of electric currents flowing near them, the polepieces of a dynamo-electric machine are seats of such currents, which form closed circuits of comparatively low resistance, and thereby cause undue heating. These currents are strongest where the changes in the intensity of the magnetic field or of the electric current are the greatest and the most sudden; this is the case, and consequently the eddy currents are strongest at those corners of the polepieces from which the armature is moved in its rotation, for, owing to the distortion of the magnetic field by the revolving armature, a density greater than the average is created at the corners where the armature leaves the polepieces, and a density smaller than the average at the corners where it enters. In order to reduce and eventually to avoid the generation of these eddy currents in the polepieces, as well as in the armature conductors, it is therefore necessary to prevent the crowding of the magnetic lines toward the tips of the polepieces, and to so arrange the poles that the magnetic field does not suddenly fall off at the pole corners, but gradually decreases in strength toward the neutral zone. This object in a smooth armature machine can be attained (1) by gradually increasing the air gap from the centres of the poles toward the

neutral spaces in boring the polar faces to a diameter larger than their least diametrical distance apart, thus giving an *elliptical* shape to the field space, as illustrated in Fig. 226; (2) by providing wrought iron polepieces with *cast iron tips* forming the pole corners and terminating the arcs embraced by the pole faces (see Figs. 227 and 228); or (3) by establishing a magnetic shunt between two neighboring poles in connecting the polepieces, either by a cast-iron ring of small sectional



Figs. 226 to 237.—Types of Polepieces.

area (Dobrowolsky's *pole-bushing*) or by placing thin *bridges* across the neighboring pole corners, as shown in Figs. 229 and 230, respectively.

The *ellipsity* of the field space has the advantage that it confines the lines of force within the sphere of the pole faces by proportionately increasing the reluctance toward the pole corners, thus preventing an increase of the magnetic density at any particular portion of the polepiece. The application of *cast-iron pole tips* with wrought iron (or cast-steel) polepieces does not prevent the crowding of the lines at the pole corners, but, by reason of the low permeability of the cast iron, reduces their density to a figure below that in the wrought iron, and consequently effects a graduation of the field strength near the neutral space, the maximum density being in the

wrought iron at the point where the cast-iron tips are joined. In the *pole bushing* or its equivalent, the *pole bridges*, the reach of the magnetic field is greatly increased, the percentage of the polar arc being practically = 100, and also a more or less gradual decrease of the field strength at the neutral point is obtained, but the length of the non-sparking space is greatly reduced and thereby its uncertainty increased, thus making the proper setting of the brushes a very difficult operation.

It has also been recommended to *laminate* both the polepieces and the magnet cores in the direction parallel to the armature shaft, in order to prevent the production of eddy currents, but this can only be applied to small dynamos, as the additional cost connected with such a lamination in large machines would be in no proportion to the small gain obtained. Besides, there is another reason against lamination: a laminated magnet frame is very sensitive to the fluctuations in the load of the machine, which naturally react upon the magnetic field, and in following these fluctuations an unsteady magnetization is produced, which, in turn, again tends to increase the fluctuations causing its variability; while in a solid magnet frame the eddy currents induced by the changes of magnetization caused by the fluctuations of the load tend to counteract the very changes producing them, and therefore exercise a steadying influence upon the field, thus reducing the fluctuations in the external circuit of the machine.

An expedient sometimes used instead of laminating the polepieces is to cut narrow longitudinal slots in the polepieces, Fig. 231, thus laminating a portion of the polepieces only. These slots at the same time serve to increase the length of the path traversed by the lines of force set up by the action of the armature current, and to thus reduce the armature reaction upon the magnetic field, checking the sparking connected therewith.

When the commutator brushes, after having short-circuited an armature coil, break this short circuit, the sudden reversal of the current in the same, produced in passing the neutral line of the field, together with the self-induction set up by the extra current on breaking, causes a spark to appear at the brushes, which may be considerable, since in the comparatively low resistance of the short-circuited coil a small electromotive

force is sufficient to produce a heavy current. If a dynamo, therefore, is otherwise well designed, that is, if the armature is subdivided into a sufficient number of sections, if the field is strong enough so as not to be overpowered by the armature, and if the thickness of the brushes is so chosen as to not short-circuit more than one or two armature sections each simultaneously, and as not to leave one commutator-bar before making connection with the next strip, then the sparking at the commutator can be reduced to a practically unappreciable degree by so shaping the pole surfaces as to give a suitable fringe of magnetic field of graduated intensity, thus not only causing the current in the short-circuited coils to die out by degrees, but also compelling the coils to enter the field of opposite polarity gradually. This is achieved by giving the pole corners an *oblique*, or a *double conical*, or a *hyperbolical* form, as illustrated by top views in Figs. 232, 233, and 234, respectively.

For the purpose of counteracting the magnetic pull due to the armature thrust in bipolar machines, see § 42, the pole-pieces are often mounted *eccentrically*, leaving a smaller gap-space at the side averted from the field coils than at the side toward the same, Fig. 235, or in case of wrought-iron or steel polepieces, *cast-iron pole tips* are used at the side toward the exciting coils, and wrought-iron or steel tips at the other, Fig. 236. Both the *eccentricity of the pole faces* and the *cast-iron pole tips*, if suitably dimensioned, have the effect of increasing the reluctance of the stronger side of the field in the same proportion as the density rises on account of the dissymmetry of the field, thus making the product of density and permeance the same in both halves.

In a very instructive paper, entitled "On the Relation of the Air Gap and the Shape of the Poles to the Performance of Dynamo-electric Machinery," Professor Harris J. Ryan[1] has demonstrated the importance of making the polepieces of such shape that saturation at the pole corners cannot occur even at full load; for, the armature ampere turns cannot change the total magnetization established by the field when the pole corners are unsaturated. He further proved by experiment that for a sparkless operation at all loads of a constant current

[1] *Transactions A. I. E. E.*, vol. viii. p. 451 (September 22, 1891); *Electrical World*, vol. xviii. p. 252 (October 3, 1891).

generator, it is necessary that the air gap be made of such a depth that the ampere turns required to set up the magnetization through the armature without current, and for the production of the maximum E. M. F. of the machine, shall be a little more than the ampere turns of the armature when it furnishes its normal current. As long as the brushes were kept under the pole faces the non-sparking point was wherever the brushes were placed, no matter whether the armature core was saturated or not.

In order to enable currents to be taken from a machine at various voltages, Rankine Kennedy[1] has proposed to subdivide the pole faces by deep, wide slots parallel to the armature shaft, Fig. 237, thus providing a number of neutral points on the commutator, at which brushes may be placed without sparking. If, for instance, there are two such grooves in each polepiece, the total voltage of the machine is divided into three equal parts, and by employing an intermediate brush at one of the additional neutral spaces, two circuits can be supplied by the machine, one each between the intermediate brush and one of the main brushes, one having two-thirds and the other one-third of the total voltage furnished by the dynamo.

77. Base.

The *base* is the only part of the machine where weight is not only not objectionable but very beneficial, and it should therefore be a heavy iron casting, especially as the extra cost of plain cast iron is insignificant as compared with the entire cost of the machine. A heavy base brings the centre of gravity low, and consequently gives great stability and strength to the whole machine.

Besides this mechanical argument in favor of a massive casting, there is a magnetical reason which applies to all types in which the base constitutes a part of the magnetic circuit, as is the case in the inverted horseshoe type, Fig. 188, in the vertical single-magnet type, Fig. 193, in the inclined and vertical double-magnet types, Figs. 198 and 199, respectively, in the iron-clad types, Figs. 203, 205, 206, 207, 218, and 219, respectively, and in the vertical quadruple magnet machine, Fig. 224.

[1] English Patent No. 1640, issued April 4, 1892.

In these and similar types a heavy base of consequent high permeance reduces the reluctance of the entire magnetic circuit, and effects a saving in exciting power which usually is sufficient to repay the extra expense involved, and often even reduces the total cost of the machine.

If the base forms a part of the magnetic circuit of the machine, constituting either the yoke or one of the polepieces, its least cross-section perpendicular to the flow of the magnetic lines should be dimensioned by the rules given for cast-iron magnets—that is, it should be at least $1\frac{2}{3}$ to 2 times the area of the magnet cores, if the latter are of wrought iron or cast steel, and at least of equal area if they are of the same material as the base, *i. e.*, of cast iron.

78. Zinc Blocks.

In some forms of machines, such as the upright horseshoe type, Fig. 187, the horizontal single-magnet types, Figs. 191 and 192, the consequent pole, horizontal double magnet type, Fig. 197, the tangential multipolar type, Fig. 210, etc., the magnet frame rests upon two polepieces of opposite polarity, and if these were joined by the iron base, the latter would constitute a stray path of very much lower reluctance than the useful path through air gaps and armature, and the lines of force emanating from these two polepieces would thus be shunted away from the armature, instead of forming a magnetic field for the conductors. In order to prevent such a short-circuiting of the magnetic lines it is necessary either to use material different from iron for the base, or to interpose blocks of a non-magnetic substance between the polepieces and the bed-plate. The former method can be applied to small machines only, and in this case the magnet frame is mounted upon a base of either *wood* or *brass*. For large machines a wooden base would be too weak and too light, and a brass one too expensive, and resort has to be taken to the second method of interposing a non-magnetic block, *zinc* being most usually employed. These zinc blocks must be of the necessary strength, not only to carry the weight of the frame, but also to withstand the tremor of the machine, and must be made high enough to introduce a sufficient amount of reluctance into the path of leakage through the base. The reluctance

required in that path must be at least four times, and preferably should be up to ten or twelve times that of the air gaps; that is, its relative permeance calculated from formula (161), § 62, according to the size of the machine, should range between $\frac{1}{4}$ and $\frac{1}{12}$ of the relative permeance of the air gaps, as found from formula (167) or (168), § 64, the amount of leakage through the iron base being thereby limited to 25 per cent. of the useful flux in small dynamos, and to 8 per cent. in the largest machines.

This condition is fulfilled if the height of the zinc blocks, according to the kind and the size of the machine, is from three to fifteen times greater than the radial length of the gap-space. The following Tables, LXX., LXXI., and LXXII., give the value of this ratio, the consequent height of the zinc blocks, and the corresponding approximate leakage through the base for high-speed dynamos with smooth-core drum armatures, for high-speed dynamos with smooth-core ring armatures, and for low-speed machines with toothed and perforated armatures, respectively:

TABLE LXX.—HEIGHT OF ZINC BLOCKS FOR HIGH-SPEED DYNAMOS WITH SMOOTH-CORE DRUM ARMATURES.

CAPACITY IN KILOWATTS.	Diameter of Armature Core (from Table X.)	Height of Winding Space (from Table XVIII.)	Height of Binding.	Radial Clearance (from Table LXI.)	Radial Length of Gap-Space. Inch.	Ratio of Height of Zinc Block to Length of Gap-Space.	Height of Zinc Blocks. Inches.	Approximate Leakage through Base in p. c. of Useful Flux.
1	$3\frac{1}{4}$"	.3"	.03"	.045	.375"	5	$1\frac{7}{8}$"	25%
2	$3\frac{3}{4}$	.325	.03	.045	.4	5	2	25
3	$4\frac{1}{2}$	.35	.03	.045	.425	$5\frac{1}{4}$	$2\frac{1}{4}$	25
5	$5\frac{1}{4}$	.375	.03	.045	.45	$5\frac{1}{2}$	$2\frac{1}{2}$	20
10	6	.4	.04	.06	.5	$5\frac{1}{2}$	$2\frac{3}{4}$	20
15	$6\frac{3}{4}$	.425	.04	.06	.525	6	$3\frac{1}{8}$	18
20	$7\frac{1}{2}$	.45	.04	.06	.55	$6\frac{1}{4}$	$3\frac{1}{2}$	18
25	$8\frac{1}{4}$	.475	.04	.06	.575	7	4	16
30	9	.5	.05	.075	.625	$7\frac{1}{4}$	$4\frac{1}{2}$	16
50	$10\frac{1}{2}$	.525	.05	.075	.65	$7\frac{3}{4}$	5	15
75	$12\frac{1}{2}$	.55	.06	.09	.7	$8\frac{1}{2}$	6	14
100	15	.6	.06	.09	.75	$8\frac{5}{8}$	$6\frac{1}{2}$	14
150	$18\frac{1}{2}$	.65	.065	.125	.84	9	$7\frac{1}{2}$	12
200	$22\frac{1}{2}$	.7	.07	.16	.93	$9\frac{3}{4}$	9	10
300	28	.8	.07	.19	1.06	$10\frac{1}{2}$	11	10

TABLE LXXI.—HEIGHT OF ZINC BLOCKS FOR HIGH-SPEED DYNAMOS WITH SMOOTH-CORE RING ARMATURES.

CAPACITY IN KILOWATTS.	Diameter of Armature Core (from Table XI.)	Height of Winding Space (from Table XVIII.)	Height of Binding.	Radial Clearance (from Table LXII.)	Radial Length of Gap-Space. Inch.	Ratio of Height of Zinc Block to Length of Gap-Space.	Height of Zinc Blocks. Inches.	Approximate Leakage through Base in p. c. of Useful Flux.
1	7"	.25"	.03"	.045"	.325"	8	$2\frac{1}{2}$	15%
2	8	.25	.03	.045	.325	9	3	14
3	$9\frac{1}{2}$	.275	.04	.06	.375	$9\frac{1}{2}$	$3\frac{1}{2}$	14
5	11	.3	.04	.06	.4	10	4	12
10	14	.325	.05	.075	.45	11	5	12
15	15	.325	.05	.075	.45	11	5	12
20	16	.35	.06	.09	.5	12	6	10
25	18	.35	.06	.09	.5	12	6	10
30	20	.375	.07	.13	.575	12	7	10
50	24	.4	.07	.13	.6	$13\frac{1}{2}$	8	9
75	28	.425	.07	.155	.65	$14\frac{1}{2}$	$9\frac{1}{2}$	9
100	32	.45	.07	.155	.675	$15\frac{1}{2}$	$10\frac{1}{2}$	8
150	36	.475	.07	.18	.725	16	$11\frac{1}{2}$	8

TABLE LXXII.—HEIGHT OF ZINC BLOCKS FOR LOW-SPEED DYNAMOS WITH TOOTHED AND PERFORATED ARMATURE.

CAPACITY IN KILOWATTS.	Diameter of Armature Core (from Table XII.)	Height of Winding Space (from Table XVIII.)	Radial Clearance (from Table LXI.)	Maximum Radial Length of Gap-Space. Inch.	Ratio of Height of Zinc Block to Maximum Length of Gap-Space	Height of Zinc Blocks. Inches.	Approximate Leakage through Base in p. c. of Useful Flux.
2	12"	$1\frac{1}{8}$"	$\frac{3}{64}$"	$1\frac{11}{64}$"	3	$3\frac{1}{2}$"	15%
3	15	$1\frac{1}{4}$	$\frac{1}{16}$	$1\frac{5}{16}$	3	4	15
5	17	$1\frac{3}{8}$	$\frac{1}{16}$	$1\frac{7}{16}$	$3\frac{1}{2}$	5	12
10	21	$1\frac{1}{2}$	$\frac{5}{64}$	$1\frac{37}{64}$	$3\frac{3}{4}$	6	12
15	23	$1\frac{5}{8}$	$\frac{5}{64}$	$1\frac{45}{64}$	4	$6\frac{3}{4}$	10
20	25	$1\frac{11}{16}$	$\frac{3}{32}$	$1\frac{23}{32}$	$4\frac{1}{4}$	$7\frac{1}{2}$	10
25	27	$1\frac{3}{4}$	$\frac{3}{32}$	$1\frac{27}{32}$	$4\frac{3}{8}$	8	10
30	30	$1\frac{13}{16}$	$\frac{1}{8}$	$1\frac{15}{16}$	$4\frac{1}{2}$	$8\frac{3}{4}$	8
50	36	$1\frac{7}{8}$	$\frac{1}{8}$	2	$4\frac{3}{4}$	$9\frac{1}{2}$	8

From the comparison of the above Tables LXX., LXXI. and LXXII., it follows that the height of the zinc blocks increases in a nearly direct proportion with the diameter of the armature

core, and that, for the same armature diameter, a smooth-drum machine requires a higher, and a toothed or perforated armature machine a lower zinc than a smooth-ring dynamo. By compiling the results of Tables LXX., LXXI., and LXXII., the following Table, LXXIII., is obtained, from which it can be seen that the heights of zinc blocks for smooth-ring machines are from 18 to 30 per cent. less than for smooth-drum dynamos, and those for machines with toothed and perforated armatures are from 11 to 20 per cent. less than for smooth-ring armature dynamos:

TABLE LXXIII.—COMPARISON OF ZINC BLOCKS FOR DYNAMOS WITH VARIOUS KINDS OF ARMATURE.

DIAMETER OF ARMATURE CORE.	HEIGHT OF ZINC BLOCKS.		
	Smooth Armature.		Toothed or Perforated Armature.
	Drum.	Ring.	
Inches.	Inches.	Inches.	Inches.
3	$1\frac{3}{4}$	..	..
4	2	..	..
6	$2\frac{3}{4}$	2	$1\frac{3}{4}$
8	4	3	$2\frac{1}{2}$
10	5	$3\frac{1}{2}$	3
12	$5\frac{3}{4}$	$4\frac{1}{2}$	$3\frac{1}{2}$
15	$6\frac{1}{2}$	5	4
18	$7\frac{1}{2}$	6	5
21	$8\frac{3}{4}$	7	6
24	$9\frac{3}{4}$	8	7
27	11	9	8
30	..	10	$8\frac{3}{4}$
36	..	$11\frac{1}{2}$	$9\frac{1}{2}$

79. Pedestals and Bearings.

In the design of the base, especially when the portion of the field frame above the armature centre cannot be lifted off, care should be taken that the armature can easily be withdrawn longitudinally by removing one of the bearing pedestals, which, therefore, should be a separate casting. In machines where the lowest point of the armature periphery is at a considerable height above the base, as for instance in dynamos of

the overtypes, Figs. 188, 191, 198, and 206, respectively, further of the vertical double types, Figs. 197, 202, 207, 219, and 224, respectively, and of the radial and tangential outerpole types, Figs. 208 and 210, respectively, it is preferable that the pedestals should be made of two parts, the upper part, which should have a depth from the shaft centre a little in excess of the radius of the finished armature, being removable, while the lower portion, which may be cast in one with the base, will form a convenient resting place for the armature in removal.

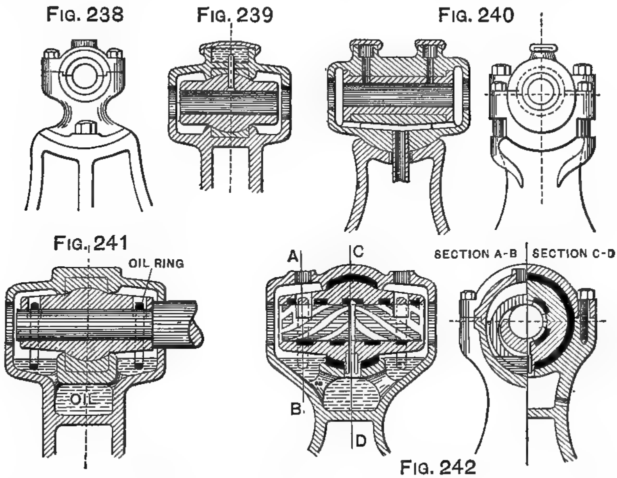
In most cases this problem of making high pedestals of two parts can practically be solved by boring out the pedestal seats together with the polepieces, thus providing a cylindrical seat for the pillow blocks, as shown in Fig. 238. This design is particularly advantageous also for machines in which the base forms one of the polepieces, as for example, the forms shown in Figs. 193, 199 and 219, as in this case, outside of the finishing of the core seats, this boring to a uniform radius is the only tooling necessary for the base.

If the field frame is symmetrical with reference to the horizontal plane through the armature centre, the frame of the machine is usually made in halves, and the armature, in case of repair, can be removed by lifting it from its bed without disturbing the bearing pedestals. The bearing boxes must for this purpose be made divided so that all parts of the machine above the shaft centre are removable. This design affords the further advantage that the bearing caps can be taken off at any time and the bearings inspected, and it has for this reason become a general practice in dynamo design to employ split bearings, even for types in which the armature cannot be lifted.

It is, further, of great importance that the bearing should not only be exactly concentric, but that they also should be accurately in line with each other; for large machines it is therefore advisable to effect automatic alignment by providing the bearings with spherical seats. This can be attained either by giving the enlarged central portion of the shell a spherical shape, Fig. 239, or in providing the bottom part of the box with a spherical extension fitting into a spherical recess in the pedestal, Fig. 240.

In order to prevent heating of the bearings, the shells in modern dynamos are usually furnished with some automatic

oiling device, the most common form of which, shown in Fig. 241, consists of a brass ring or chain dipping into the oil chamber of the box and resting upon and turning with the shaft, thereby causing a continuous supply of oil at the top of the shaft. A further improvement of this self-oiling arrangement, patented in 1888 by the Edison General Electric Company, is illustrated in Fig. 242. In this the interior of the



Figs. 238 to 242.—Pedestals and Bearings.

shell is provided with spiral grooves filled with soft metal and forming channels for conveying oil from each end of the bearing to a circumferential groove which surrounds the shaft at the centre of the shell, and which communicates with the oil chamber beneath the bearing. These grooves not only effect a steady supply, but a continuous *circulation* of oil, the latter being lifted from the reservoir into the shell by the oiling rings, thence forced by the spiral channels into the central groove, from where it flows back into the oil chamber.

80. Joints in Field Magnet Frame.

a. Joints in Frames of One Material.

Magnet frames consisting of but one material may either be formed of one single piece or may be composed of several parts. If the frame is of cast iron or cast steel, in small

dynamos usually the former is the case, *i. e.*, the whole frame is cast in one, while in large machines it generally consists of two castings; if, however, wrought iron is used, it is, as a rule, much more convenient to forge each part separately and to build up the frame by butt-jointing the parts. In so jointing a magnet frame, it is of the utmost importance to accurately adjust and finish the surfaces to be united, so as to make the joint as *perfect* as possible, for every poorly fitted joint, by reduction of the sectional area at that point, introduces a considerable reluctance in the magnetic circuit. If, however, the contact between the two surfaces is as good as planing and scraping can make it, a practically perfect joint is obtained, and the additional reluctance, which then only depends upon the degree of magnetization, is entirely inappreciable for such high magnetic densities as are employed in modern dynamos. Experiments have shown that at low densities the additional magnetomotive force required to overcome the reluctance of a joint is very much greater, comparatively, than at high inductions, which is undoubtedly due to the pressure created by the magnetic attraction of the two surfaces across the joint, this pressure being proportional to the square of the density. The following Table LXXIV. shows the influence of the density of magnetization upon the effect of a well-fitted joint in a wrought iron magnet frame, the induction in the iron ranging from 10,000 to 120,000 lines per square inch, and indicates that the reluctance of the joint becomes the less significant the nearer saturation of the iron is approached.

At a magnetic density of $\mathfrak{B}''_m = 10{,}000$ lines of force per square inch, each joint in the circuit is equivalent to an air space of .0016 inch, or has a reluctance equal to that of an additional length of 3 inches of wrought iron; at $\mathfrak{B}''_m = 100{,}000$ lines per square inch, the thickness of an equivalent air space is only .00065 inch, which corresponds to the reluctance of .22 inch of wrought iron at that density; and at or above $\mathfrak{B}''_m = 120{,}000$, finally, a good joint is found to have no effect whatever upon the reluctance of the circuit.

b. Joints in Combination Frames.

For magnet frames consisting of two or three different materials the same rule as for frames of one material holds good as

to the *nature* of the joint, but since the ordinary butt-jointing would limit the capacity of the joint to that of the inferior magnetic material, it is essential in the case of combination frames to increase the *area of contact* in the proportion of the relative permeabilities of the two materials joined. Thus, if wrought and cast iron are butt-jointed, the capacity of the joint is reduced to that of the cast iron, whereby the advantage of the high permeability of the wrought iron is destroyed and the permeance of the circuit is considerably increased; and in order to have the full benefit of the wrought iron, the contact area of the joint must be increased proportionally to the ratio of the permeability of the wrought iron to that of the cast iron at the particular density employed.

TABLE LXXIV.—INFLUENCE OF MAGNETIC DENSITY UPON THE EFFECT OF JOINTS IN WROUGHT IRON.

DENSITY OF MAGNETIZATION. $\mathcal{B}''_m$ Lines per sq. in.	PRESSURE ON JOINT DUE TO MAGNETIC ATTRACTION. $\frac{\mathcal{B}''^2_m}{72{,}134{,}000}$ lbs. per. sq. in.	MAGNETIZING FORCE REQUIRED FOR 1 INCH.		DIFFERENCE DUE TO JOINT, $\mathcal{H} = \mathcal{H}_2 - \mathcal{H}_1$ Amp. turns.	EQUIVALENT OF JOINT	
		Solid. $\mathcal{H}_1$ Amp. turns.	Jointed. $\mathcal{H}_2$ Amp. turns.		Air Space, $\frac{\mathcal{H}}{.3133 \times \mathcal{B}''_m}$ Inch.	Length of Iron, $\frac{\mathcal{H}}{\mathcal{H}_1}$ Inch.
10,000	1.4	1.7	6.7	5	.0016	3.0
20,000	5.5	3.2	12.6	9.4	.00155	2.9
30,000	12 5	5	19.1	14.1	.0015	2.8
40,000	22	7	25.2	18.2	.00145	2.6
50,000	35	9.5	31.4	21.9	.0014	2.3
60,000	50	12.7	38.1	25.4	.00135	2.0
70,000	68	18.3	45.7	27.4	.00125	1.5
80,000	89	27.6	55.2	27.6	.0011	1.0
90,000	112	50 8	76.2	25.4	.0009	0.5
95,000	125	68	91.8	23.8	.0008	.35
100,000	139	90	110	20	.00065	.22
105,000	153	134	150	16	.0005	.12
110,000	168	288	300	12	.00035	.04
112,500	176	391	400	9	.00025	.023
115,000	183	500	506	6	.00016	.012
117,500	192	600	603	3	.00008	.005
120,000	200	700	700	0	.00000	.000

For a density in wrought iron of 100,000 lines of force per square inch, for example, a magnetomotive force of 90 ampere-turns is required per inch length of the circuit, and the same

specific magnetomotive force is capable of setting up about 40,000 lines per square inch in cast iron; the contact area of a joint between wrought iron and cast iron in this case must therefore be increased in the ratio of 100,000 : 40,000, or must be made $2\frac{1}{2}$ times the cross-section of the wrought iron in order to reduce the permeability of the joint to that of the wrought iron.

In practice this problem of providing a sufficiently large contact area between a wrought and a cast iron part of the mag-

FIG. 243

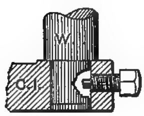


FIG. 244

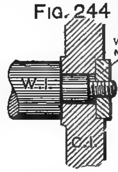


FIG. 245

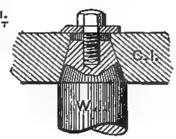


FIG. 246

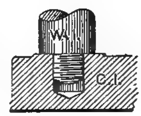


FIG. 247

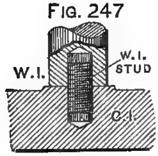


FIG 248

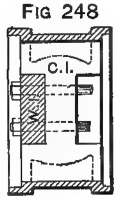


FIG. 249

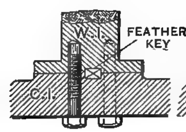
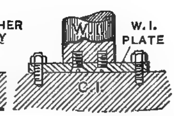


FIG 250



Figs. 243 to 250.—Joints in Magnetic Circuits.

netic circuit may be solved either by setting the wrought iron into the cast iron, or by extending the surface of the wrought iron part near the joint by means of flanges; or, finally, by inserting an intermediate wrought-iron plate into the joint. In Figs. 243, 244, 245 and 246 are shown four methods of increasing the area of the joint by means of projecting the wrought-iron core into the cast-iron yoke or polepiece, differing only in the manner of securing a good contact between the parts, the first one employing a set-screw, the second one a wrought-iron nut, and the third one using a conical fit with draw-screw for this purpose, while in the fourth one the threaded projection of the core itself forms the tightening screw. Fig. 247 illustrates a modification of the method shown in Fig. 246, a separate screw-stud being used instead of the threaded extension of the wrought-iron core. In case of rectangular magnet cores the arrangement shown by plan in Fig. 248 effects an excellent

joint; in this the cores are inserted into the base from the sides, thus offering three surfaces to form the contact area. The manner of supplying the necessary joint surface by flanged extensions of the wrought-iron core is illustrated in Fig. 249, which shows the method of fastening employed in large multipolar machines, feather-keys being used to secure exact relative position of the cores. In Fig. 250, finally, a joint is shown in which a wrought-iron contact plate is inserted between the wrought-iron core and the cast-iron yoke or polepiece with the object of increasing the area of the joint and of spreading the lines of force gradually from the smaller area of the wrought iron to the larger of the cast iron.

CHAPTER XVI.

CALCULATION OF FIELD MAGNET FRAME.

81. Permeability of the Various Kinds of Iron.—Absolute and Practical Limits of Magnetization.

The field magnet of a dynamo has the function of supplying to the interpolar space in which the armature conductors revolve magnetic lines of force in a number sufficient either to cause the generation of the required electromotive force, in case of a generator, or to produce a motion of the desired power, in case of a motor. The cross-sections of the various parts of the field magnet frame, that is, of the iron structure constituting the path or paths, for the flow of these magnetic lines, consequently, must be dimensioned with reference to the number of lines of force to be carried, and to the magnetic conductivity of the material used.

The number of lines which by a certain exciting power or magnetomotive force can be passed through a portion of a magnetic circuit depends upon the area of the cross-section and on the magnetic conductivity of the material of that part of the circuit. The various magnetic materials, according to their hardness, have a different capability of conducting magnetic lines, the softest material being the best magnetic conductor. The specific magnetic conductance of air being taken as unity, the relative magnetic conductance, or the relative *permeance*, of the various magnetic materials is indicated by the ratio of the number of lines of force produced in unit cross-section of these materials to the number of lines set up by the same magnetizing force in unit cross-sections of air. This ratio, or coefficient of magnetic induction, is called the *magnetic conductivity*, or the *permeability* of the material.

The number of lines per square centimetre of sectional area set up by a certain magnetizing force in *air* is conventionally designated by $\mathcal{H}$, that in *iron* by $\mathcal{B}$, and the permeability by

the symbol μ; between these three quantities, therefore, exists the relation

$$\mu = \frac{\mathfrak{B}}{\mathfrak{H}}, \text{ or } \mathfrak{B} = \mu \times \mathfrak{H} \qquad (215)$$

Since for air the permeability $\mu = 1$, the number of lines of force per square centimetre of air is numerically equal to the magnetizing force in magnetic measure, *i. e.*, in current-turns. Permeability is therefore often also defined as the ratio of the magnetization produced to the magnetizing force producing it.

TABLE LXXV.—PERMEABILITY OF DIFFERENT KINDS OF IRON AT VARIOUS MAGNETIZATIONS.

Density of Magnetization.		Permeability, μ.			
Lines per sq. inch $\mathfrak{B}''$	Lines per cm². $\mathfrak{B}$	Annealed Wrought Iron.	Commercial Wrought Iron.	Gray Cast Iron.	Ordinary Cast Iron.
20,000	3,100	2,600	1,800	850	650
25,000	3,875	2,900	2,000	800	700
30,000	4,650	3,000	2,100	600	770
35,000	5,425	2,950	2,150	400	800
40,000	6,200	2,900	2,130	250	770
45,000	6,975	2,800	2,100	140	730
50,000	7,750	2,650	2,050	110	700
55,000	8,525	2,500	1,980	90	600
60,000	9,300	2,300	1,850	70	500
65,000	10,100	2,100	1,700	50	450
70,000	10,850	1,800	1,550	35	350
75,000	11,650	1,500	1,400	25	250
80,000	12,400	1,200	1,250	20	200
85,000	13,200	1,000	1,100	15	150
90,000	14,000	800	900	12	100
95,000	14,750	530	680	10	70
100,000	15,500	360	500	9	50
105,000	16,300	260	360	....	....
110,000	17,400	180	260	....	....
115,000	17,800	120	190	....	....
120,000	18,600	80	150	....	....
125,000	19,400	50	120	....	...
130,000	20,150	30	100	....	....
135,000	20,900	20	85	....	....
140,000	21,700	15	75	....	....

While the permeability of air and of all non-magnetic substances is a constant (for air it is 1, and for diamagnetic materials slightly less than 1) for all stages of magnetization, that of magnetic materials varies with the degree of saturation.

The more lines a certain cross-section of iron carries, the less permeable is it for additional lines, as is evident from the preceding Table LXXV. containing the average permeabilities of different kinds of iron at various degrees of magnetization.

At a certain limit, for every kind of iron, a very material increase in the magnetizing forces does not appreciably increase the magnetization induced, and the iron is then *saturated* with lines of force. This limit of magnetization in *annealed wrought iron* is reached at a density of about $\mathcal{B}''=130,000$ lines per square inch, or $\mathcal{B}=20,200$ lines per square centimetre; in *soft steel* at $\mathcal{B}''=127,500$ lines per square inch, or $\mathcal{B} \doteq 19,800$ lines per square centimetre; in *mitis iron* at $\mathcal{B}''=122,500$ lines per square inch, or $\mathcal{B}=19,000$ lines per square centimetre; in *cast iron* with a 6.5 per cent. admixture of *aluminum* at $\mathcal{B}''=87,500$ lines per square inch, or $\mathcal{B}=13,500$ lines per square centimetre, and in *ordinary cast iron* at $\mathcal{B}''=77,500$ lines per square inch, or $\mathcal{B}=12,000$ lines per square centimetre of cross-section. The magnetizations, however, to which these materials are subjected in practical electromagnets are taken far below the actual limits of absolute saturation, since "saturation curves" indicating the variation of the induction $\mathcal{B}$, with increasing magnetizing force, $\mathcal{H}$, show that from a certain point, the "knee" of the curve, the magnetization increases much slower than the magnetizing force which causes it. In wrought iron, for instance, an induction of $\mathcal{B}=13,580$ requires a magnetizing force of $\mathcal{H}=25$, an induction of $\mathcal{B}=16,000$ a magnetomotive force of $\mathcal{H}=50$, and density of $\mathcal{B}=16,500$ necessitates an exciting power of $\mathcal{H}=100$; and an increase of 100 per cent. in the magnetizing force, consequently causes a rise in density of 18½ per cent. at the lower magnetization, while again doubling the magnetomotive force at the higher induction only causes an increase in magnetic density of about 3 per cent. In practice, therefore, the limits of the magnetic densities of the different materials are to be fixed with regard to the relative economy of iron and copper. Taking the practical limit of saturation too low means a small saving of copper at a large expense of iron, while too high a density effects a comparatively small saving of iron at a large expense of copper. Since copper costs many times more than iron, the densities should be limited rather low, the tendency toward the former extreme

being preferable to that toward the latter. With this point in view, the "*Practical Working Densities*" given in the following Table LXXVI. are recommended for use in dynamo designing, while under the heading of "*Practical Limits of Magnetization*" the highest densities are tabulated that the author would advise to allow in magnet frames of dynamo-electric machines. For sake of completeness the "*Absolute Saturation*" of the various materials, as given above, are added in Table LXXVI.:

TABLE LXXVI.—PRACTICAL WORKING DENSITIES AND LIMITS OF MAGNETIZATION FOR VARIOUS MATERIALS.

MATERIAL.	PRACTICAL WORKING DENSITY.		PRACTICAL LIMIT OF MAGNETIZATION.		ABSOLUTE SATURATION.	
	Lines p. sq. inch. $\mathfrak{B}''_m$	Lines p. sq. cm. $\mathfrak{B}_m$	Lines p. sq. inch.	Lines p. sq. cm.	Lines p. sq. inch.	Lines p. sq. cm.
Wrought Iron	90,000	14,000	105,000	16,300	130,000	20,200
Cast Steel............	85,000	13,200	100,000	15,500	127,500	19,800
Mitis Iron	80,000	12,400	95,000	14,750	122,500	19,000
Cast Iron, containing 6.5% Aluminum......	45,000	7,000	55,000	8,500	87,500	13,500
Cast Iron, ordinary ...	40,000	6,200	50,000	7,750	77,500	12 000

82. Sectional Area of Magnet Frame.

The magnet frame carries the total flux generated in the machine; according to § 81, consequently, the cross section of any portion of it must be

$$S''_m = \frac{\Phi'}{\mathfrak{B}''_m} = \frac{\lambda \times \Phi}{\mathfrak{B}''_m}; \quad \dots\dots\dots\dots (216)$$

S''_m = Area of magnet frame, in square inches;
Φ' = Total flux generated in machine, from formula (156), § 60;
Φ = Useful flux necessary to produce the required E. M. F., from formula (137), § 56;
λ = Factor of magnetic leakage, preliminary value from Table LXVIII., § 70, final value from formula (157), § 61;
$\mathfrak{B}''_m$ = Magnetic density in magnet frame, from Table LXXVI., § 81.

If only one material is used the value found from formula (216) is the uniform cross-section of the whole frame, *i. e.*, of the cores, the yoke, and the polepieces; in case of combination frames, however, the area for each material must be calculated separately:

For Wrought iron, $$S''_m = \frac{\lambda \times \Phi}{90{,}000}; \qquad (217)$$

" Cast steel, $$S''_m = \frac{\lambda \times \Phi}{85{,}000}; \qquad (218)$$

" Mitis iron, $$S''_m = \frac{\lambda \times \Phi}{80{,}000}; \qquad (219)$$

" Cast iron, containing 6.5% of aluminum, $$S''_m = \frac{\lambda \times \Phi}{45{,}000}; \qquad (220)$$

" Ordinary cast iron, $$S''_m = \frac{\lambda \times \Phi}{40{,}000}; \qquad (221)$$

In combining the averages for the useful flux, taken from Table LXIV., § 59, for the practical conductor velocities given in Tables X., XI. and XII., respectively, with the leakage coefficients compiled in Table LXVIII., § 70, the average total flux, Φ, for dynamos of various kinds and sizes is obtained, and, then by applying formulæ (217) to (221), the sectional areas of the field frame for various kinds and sizes of machines can be found. In this manner the following Tables LXXVII., LXXVIII., and LXXIX., have been prepared, which give the cross-sections of field magnet frames of different materials for high-speed drum machines, high-speed ring dynamos, and low speed ring machines, respectively.

The figures given for the areas directly apply to single circuit bipolar dynamos only; for double circuit bipolar, and for multipolar machines they represent the total cross-section of all the magnetic circuits in parallel, or for frames of only one material, the total area of all the cores of same free polarity, the cross-sections of the various portions of the field magnet frame are therefore obtained in dividing these figures by the number of magnetic circuits, *i. e.*, by the number of pairs of magnet poles:

TABLE LXXVII.—SECTIONAL AREA OF FIELD MAGNET FRAME FOR HIGH-SPEED DRUM DYNAMOS.

Capacity in Kilowatts.	Conductor Velocity (Table X). Ft. per second.	Average Useful Flux. Table LXIV. Lines of force.	Av'age Leak'ge Coeffi-cient. Table LXVIII	Average total flux, Φ'. Lines of force.	Area of Field Magnet Frame. Wr'ght Iron, $S_m = \frac{\Phi'}{90,000}$ sq. in.	Cast Steel, $S_m = \frac{\Phi'}{85,000}$ sq. in.	Mitis Iron, $S_m = \frac{\Phi'}{80,000}$ q. in.	Cast Iron, 6.5% Al. $S_m = \frac{\Phi'}{45,000}$ sq. in.	Cast Iron, ordin'y $S_m = \frac{\Phi'}{40,000}$ sq. in.
.1	25	200,000	2.00	400,000	4.5	4.7	5	9	10
.25	30	333,000	1.90	630,000	7	7.4	7.9	14	15.8
.5	32	550,000	1.80	990,000	11	11.7	12.4	22	24.8
1	34	880,000	1.75	1,640,000	17.1	18.1	19.3	34.2	38.6
2	36	1,530,000	1.70	2,600,000	28 9	30.6	32.5	57.8	65
3	40	1,875,000	1.65	3,100,000	34.5	36.5	38.8	69	77.6
5	45	2,550,000	1.60	4,080,000	45.5	48	51	91	102
10	50	4,000,000	1.55	6,200,000	69	73	77.5	138	155
15	50	5,700,000	1.50	8,550,000	95	101	107	190	214
20	50	7,200,000	1.45	10,400 000	115.5	122	130	231	260
25	50	8,500,000	1.40	11,900,000	132	140	149	264	298
30	50	9,900,000	1.40	13,850,000	154	163	173	308	346
50	50	15,500,000	1.35	20,900,000	232	246	261	464	522
75	50	22,000,000	1.35	29,700,000	330	350	371	660	742
100	50	28,000,000	1.30	36,400,000	405	430	455	810	910
150	50	39,500,000	1.30	51,400,000	572	605	643	1,144	1,286
200	50	50,000,000	1.25	62,500,000	695	735	782	1,390	1,564
300	50	70 000,000	1.20	84,000,000	933	990	1,050	1,866	2,100

TABLE LXXVIII.—SECTIONAL AREA OF FIELD MAGNET FRAME FOR HIGH-SPEED RING DYNAMOS.

Capacity in Kilowatts.	Conductor Velocity (Table XI). Ft. per second.	Average Useful Flux. (Table LXIV.) Lines of force.	Av'age Leak'ge Coeffi-cient. Table LXVIII	Average Total Flux, Φ'. Lines of force.	Area of Field Magnet Frame. Wr'ght Iron, $S_m = \frac{\Phi'}{90,000}$ sq. in.	Cast Steel, $S_m = \frac{\Phi'}{85,000}$ sq. in.	Mitis Iron, $S_m = \frac{\Phi'}{80,000}$ sq. in.	Cast Iron, 6.5% Al. $S_m = \frac{\Phi'}{45,000}$ sq. in.	Cast Iron, ordin'y $S_m = \frac{\Phi'}{40,000}$ sq. in.
.1	50	100,000	1.80	180,000	2	2.1	2.2	4	4 5
.25	55	182,000	1.70	310.000	3.5	3.7	3.9	7	7.8
.5	60	292,000	1.60	467,000	5.2	5.5	5.8	10.4	11.6
1.	65	462,000	1.55	715,000	8	8.4	8.9	16	17.8
2.5	70	930,000	1.50	1,400,000	15.5	16.5	17 5	31	35
5	75	1,500,000	1.45	2,180,000	24.2	25.6	27.3	48.4	54.5
10	80	2,500,000	1.40	3,500,000	39	41.2	43.8	78	87.5
25	80	5,320,000	1.35	7,200,000	80	85	90	160	180
50	85	9,120,000	1.30	11,900,000	132	140	149	264	298
75	85	13,000,000	1.25	16,250,000	180	191	203	360	406
100	85	16,500,000	1.22	20,100,000	224	236	251	448	502
200	88	28,400,000	1.20	34,000,000	378	400	425	756	850
300	90	39,000,000	1.18	46,000,000	512	542	575	1,024	1,150
400	92	47,800,000	1.18	56,500,000	628	665	707	1,256	1,415
600	95	62,000,000	1.17	72,500,000	806	855	905	1,612	1,810
800	95	74,200,000	1.17	87,000,000	967	1,025	1,085	1,935	2,170
1,000	95	84,200,000	1.16	97,700,000	1,085	1,150	1,240	2,170	2,480
1,500	100	97,500,000	1.16	113,000,000	1,255	1,330	1,410	2,510	2,820
2,000	100	110,000,000	1.15	126,500,000	1,400	1,490	1,580	2,800	3,160

TABLE LXXIX.—SECTIONAL AREA OF FIELD MAGNET FRAME FOR LOW-SPEED RING DYNAMOS.

Capacity in Kilowatts.	Conductor Velocity (Table XII.) Ft. per second.	Average Useful Flux. (Table LXIV.) Lines of force.	Av'age Leak'ge Coeffi-cient. Table LXVIII	Average Total Flux. Φ'. Lines of force.	AREA OF FIELD MAGNET FRAME.				
					Wr'ght Iron, $S_m = \frac{\Phi'}{90,000}$ sq. in.	Cast Steel, $S_m = \frac{\Phi'}{85,000}$ sq. in.	Mitis Iron, $S_m = \frac{\Phi'}{80,000}$ sq. in.	Cast Iron, 6.5% Al. $S_m = \frac{\Phi'}{45,000}$ sq. in.	Cast Iron, ordin'y $S_m = \frac{\Phi'}{40,000}$ sq. in.
2.5	25	2,600,000	1.50	3,900,000	43.3	46	48.7	86.6	97.5
5	26	4,420,000	1.45	6,400,000	71.2	75.3	80	142.4	160
10	28	7,150,000	1.40	10,000,000	111	117.5	125	222	250
25	30	14,200,000	1.35	19,200,000	213.5	226	240	417	480
50	32	24,200,000	1.30	31,500,000	350	360	394	700	788
75	33	33,500,000	1.25	42,000,000	467	495	525	934	1,050
100	35	40,000,000	1.22	48,800,000	543	575	610	1,086	1.220
200	40	62,500,000	1.20	75,000,000	833	883	938	1,666	1,875
300	42	83,300,000	1.18	98,500,000	1,095	1,160	1,230	2,190	2,460
400	44	100,000,000	1.18	118,000,000	1,310	1,390	1,475	2,620	2,950
600	45	131,000,000	1.17	153,500,000	1,725	1,810	1,940	3,450	3,880
800	45	157,000,000	1.17	184,000,000	2,050	2,165	2,300	4,100	4,600
1,000	45	178,000.000	1.16	206,500,000	2,300	2,430	2,580	4,600	5,160
1,500	45	217,000,000	1.16	252,000,000	2,800	2,970	3,150	5,600	6.300
2,000	45	245,000,000	1.15	282,000,000	3,140	3,320	3,525	6,280	7,050

For cases of practical design, in which the fundamental conditions materially differ from those forming the base for the above tables, the areas obtained by formula (216) may also widely vary from the figures given, but, by proper consideration, these tables will answer even for such a case, and will be found useful for comparing the results of calculations.

83. Dimensioning of Magnet Cores.

The sectional area of the magnet cores being found by means of the formulæ and tables given in § 82, their *length* and their *relative position* must be determined.

a. Length of Magnet Cores.

In the majority of types the length of the magnet cores has a more or less fixed relation to the dimensions of the armature, and definite rules can only be laid down for such cases where the length of the magnets is not already limited by the selection of the type.

Two points have to be considered in dimensioning the length of the magnets. The longer the cores are made, the less height will be taken up by the magnet winding; the mean length of a convolution of the magnet wire, and, consequently, the total length of wire required for a certain magnetomotive force will, therefore, be smaller the greater the length of the

core. On the other hand, the shorter the cores are chosen the shorter will be the magnetic circuit of the machine, and, in consequence, the less magnetomotive force will be required to set up the necessary magnetic flux.

Of these two considerations—economy of copper at the expense of additional iron on the one hand, and saving in magnetomotive force and in weight of iron on the other—the latter predominates over the former, from which fact follows the general rule to make the cores as short as is possible without increasing the height of the winding space to an undue amount.

In order to enable the proper carrying out of this rule, the author has compiled the following Table LXXX., which gives practical values of the height of the winding space for magnets of various types, shapes and sizes:

TABLE LXXX.—HEIGHT OF WINDING SPACE FOR DYNAMO MAGNETS.

Size of Core.				Bipolar Types.				Multipolar Types.			
				Cylindrical Cores.		Rectangular or Oval Cores.		Cylindrical Cores.		Rectangular or Oval Cores.	
Diameter of Circular Cross-Section.		Area of Rectangular or Oval Section.		Height of Winding Space.	Ratio of Winding Height to Diameter of Core.	Height of Winding Space.	Ratio of Winding Height to Diam. of Equal Circular Section.	Height of Winding Space.	Ratio of Winding Height to Diameter of Core.	Height of Winding Space.	Ratio of Winding Height to Diam. of Equal Circular Section.
Ins.	cm.	Sq. ins.	Sq. cm.	h_m Inch.		h_m Inch.		h_m Inch.		h_m Inch.	
1	2.5	.8	4.9	½	.50	....	....	¾	.75	....	....
2	5.1	3.1	20.4	¾	.375	1	.50	1¼	.625	1½	.75
3	7.6	7.1	45.4	1	.33	1¼	.42	1¾	.58	2	.67
4	10.2	12.6	81.7	1¼	.31	1½	.38	2	.50	2½	.625
6	15.3	28.3	184	1½	.25	2	.33	2¼	.375	2¾	.46
8	20.3	50.3	324	1¾	.22	2½	.31	2½	.31	3	.375
10	25.5	78.5	511	1⅞	.19	2¾	.275	2¾	.28	3¼	.33
12	30.5	113.1	731	2	.17	3	.25	3	.25	3½	.29
15	38.1	176.7	1140	2⅛	.14	3¼	.22	3¼	.22	3¾	.25
18	45.7	254.5	1640	2¼	.125	3½	.20	3½	.20	4	.22
21	53.3	346	2231	2⅜	.113	3¾	.18	3¾	.18	4½	.215
24	61.	452	2922	2½	.104	4	.17	4	.17	5	.21
27	68.6	573	3606	2⅝	.097	4¼	.16	4¼	.16	5½	.205
30	76 2	707	4560	2¾	.092	4½	.15	4½	.15	6	.20
33	83.8	855	5515	2⅞	.087	5	.15	4¾	.145	6½	.197
36	91.5	1018	6576	3	.083	5½	.15	5	.14	7	.195

In *bipolar* machines, such as the various horseshoe types, in which the length of the magnet cores is not limited by the form of the field magnet frame, the radial height of the magnet winding in case of *cylindrical magnets* varies from one-half to one-twelfth the core diameter, according to the size of the magnets, and in case of *rectangular* or *oval* magnets, is made from .5 to .15 of the diameter of the equivalent circular cross-section. For *multipolar* types, in which the length of the magnets is of a comparatively much greater influence upon size and weight of the machine, it is customary to set the limit of the winding height considerably higher, in order to reduce the length necessary for the magnet winding. For *cylindrical* magnets to be used in multipolar machines, therefore, the practical limit of winding height ranges from .75 to .14 of the core diameter, and for *rectangular* or *oval* magnets, from .75 to .195 of the diameter of the equivalent circular area, according to the size.

In case of emergency the figures given for *rectangular* cores may be used in calculating *circular* magnets, or those given for *multipolar* types may be employed for *bipolar* machines.

In order to keep the winding heights within the limits given in Table LXXX. the lengths of *cylindrical* magnets have to be made from 3 to 1 times the core diameter for *bipolar* types, and from 1 to $\frac{1}{2}$ the core diameter for *multipolar* types; those of *rectangular* magnets from $1\frac{2}{3}$ to $\frac{3}{4}$ the equivalent diameter for *bipolar* types, and from $1\frac{1}{4}$ to $\frac{2}{3}$ the equivalent diameter for *multipolar* types; and the lengths of *oval* magnets, finally, from $1\frac{1}{2}$ to $\frac{3}{4}$ the diameter of the equivalent circular area for *bipolar* types, and from $1\frac{1}{4}$ to $\frac{5}{8}$ the equivalent diameter for *multipolar* types.

In the following Tables LXXXI., LXXXII., LXXXIII., and LXXXIV., the dimensions of *cylindrical* magnet cores for *bipolar* types, of *cylindrical* magnet cores for *multipolar* types, of *rectangular* magnet cores, and of *oval* magnet cores, respectively, have been calculated. In the former two of these tables the lengths and corresponding ratios are given for *cast-iron* as well as for *wrought-iron* and *cast-steel* cores; in the latter two for *wrought iron* and *cast steel* only. From Tables LXXXI. and LXXXII. it follows that *cast-iron* cores are made from 20 to 10 per cent. longer, according to the size, than *wrought-iron*

or *cast-steel* ones of the same diameter, the lengths of cast-iron cores of rectangular or oval cross-section can therefore be easily deduced from the figures given in Tables LXXXIII. and LXXXIV.

TABLE LXXXI.—DIMENSIONS OF CYLINDRICAL MAGNET CORES FOR BIPOLAR TYPES.

Total Flux, in Webers.	Dimensions of Magnet Cores, in Inches.					
	Wrought Iron and Cast Steel.			Cast Iron.		
	Diam. d_m	Length. l_m	Ratio $l_m : d_m$	Diam. d_m	Length. l_m	Ratio $l_m : d_m$
70,000	1	3	3.0	1½	4½	3.0
150,000	1½	3¾	2.5	2¼	5¾	2.56
275,000	2	4½	2.25	3	7	2.33
425,000	2½	5	2.0	3¾	8¼	2.20
600,000	3	5¾	1.92	4½	9½	2.11
850,000	3½	6½	1.86	5¼	10¾	2.05
1,100,000	4	7½	1.87	6	12	2.0
1,700,000	5	9	1.80	7½	14¼	1.9
2,500,000	6	10½	1.75	9	16½	1.83
3,300,000	7	12	1.72	10½	18	1.71
4,500,000	8	13½	1.70	12	20	1.67
5,500,000	9	15	1.67	13½	22	1.63
7,000,000	10	16	1.60	15	24	1.60
8,500,000	11	17	1.55	16½	25½	1.55
10,000,000	12	18	1.50	18	27	1.50
15,000,000	15	22	1.46	22½	32	1.42
22,500,000	18	25	1.39	27	37	1.37
30,000,000	21	28	1.33	31½	41	1.30
40,000,000	24	31	1.29	36	45	1.25
50,000,000	27	34	1.26	....	....	....
60,000,000	30	36	1.20	....	....	....
75,000,000	33	38	1.15	....	....	....
90,000,000	36	40	1.11	....	....	....

b. Relative Position of Magnet Cores.

The majority of types having two or more magnets, the relative position of the magnet cores is next to be considered. In a great number of forms, having the magnets arranged symmetrically with reference to the armature circumference, the exact relative position of the magnet cores is given by the shape of the field magnet frame; in other types, however, having parallel

magnets on the same side of the armature, diametrically or axially, the shape of the frame does not fix their relative position, and the *distance* between them is to be properly determined.

This is done by limiting the magnetic leakage across the cores to a certain amount, according to the size of the machine, namely, from about 33 per cent. of the useful flux in small machines, to 8 per cent. in large dynamos.

The relative amount of the leakage across the magnet cores is determined by the ratio of the permeance between the cores to the permeance of the useful path, and the percentage of the core leakage is kept within the limits given above, if the average permeance of the space between the magnet cores does not exceed one-third of the permeance of the gap-space in small machines, and one-twelfth of the gap permeance in large dynamos, or if the reluctance across the core is at least three to twelve times, respectively, that of the gaps.

TABLE LXXXII.—DIMENSIONS OF CYLINDRICAL MAGNET CORES FOR MULTIPOLAR TYPES.

Total Flux per Magnetic Circuit, in Maxwells.	Dimensions of Magnet Cores, in Inches.					
	Wrought Iron and Cast Steel.			Cast Iron.		
	Diam. d_m	Length. l_m	Ratio. $l_m : d_m$	Diam. d_m	Length. l_m	Ratio. $l_m : d_m$
275,000	2	2	1.00	3	3½	1.17
600,000	3	2¾	.92	4½	4½	1.00
1,100,000	4	3½	.875	6	5½	.92
1,700,000	5	4	.80	7½	6¾	.90
2,500,000	6	4½	.75	9	8	.89
4,500,000	8	6	.75	12	10½	.875
7,000,000	10	7½	.75	15	13	.87
10,000,000	12	9	.75	18	15	.83
15,000,000	15	11	.73	22½	18	.80
22,500,000	18	13	.72	27	20	.74
30,000,000	21	14½	.69	31½	22	.70
40,000,000	24	16	.67	36	24	.67
50,000,000	27	17	.63	....	....	....
60,000,000	30	18	.60	....	....	....
75,000,000	33	19	.58	....	....	....
90,000,000	36	20	.56	....	....	....

TABLE LXXXIII.—DIMENSIONS OF RECTANGULAR MAGNET CORES. (WROUGHT IRON AND CAST STEEL.)

Total Flux per Magnetic Circuit, in Maxwells.	Cross-Section.				Length.			
					Bipolar Types.		Multipolar Types.	
	Breadth, Inches.	Width, Inches.	Area, Square Inches.	Diam. of Equiv. Circular Area d_m	Length l_m	Ratio $l_m : d_m$	Length l_m	Ratio $l_m : d_m$
500,000	2	3	6	2¾	4½	1.64	3½	1.27
700,000	2	4	8	$3\frac{3}{16}$	5	1.57	4	1.25
1,000,000	2	6	12	$3\frac{15}{16}$	5½	1.40	4½	1.14
1,400,000	2	8	16	4½	6	1.33	5	1.11
1,200,000	3	4½	13.5	$4\frac{3}{16}$	5½	1.31	4½	1.08
1,600,000	3	6	18	$4\frac{13}{16}$	6¼	1.30	5	1.04
2,400,000	3	9	27	5⅞	7½	1.28	6	1.02
3,200,000	3	12	36	6¾	8½	1.26	6½	.96
2,000,000	4	6	24	$5\frac{9}{16}$	7	1.26	5¾	1.03
2,750,000	4	8	32	6⅜	8	1.26	6¼	.98
4,250,000	4	12	48	7⅞	9¾	1.24	7½	.95
5,500,000	4	16	64	9	10¾	1.20	8½	.95
4,750,000	6	9	54	8⅜	10	1.20	8	.96
6,500,000	6	12	72	9⅝	11½	1.20	9	.94
9,500,000	6	18	108	11¾	13½	1.15	11	.94
12,500,000	6	24	144	13½	15½	1.15	12½	.93
8,500,000	8	12	96	11	13	1.18	10½	.96
11,000,000	8	16	128	12¾	15	1.18	12	.94
17,000,000	8	24	192	15⅝	17½	1.12	14	.90
13,000,000	10	15	150	13⅞	16	1.15	12½	.90
17,500,000	10	20	200	16	18	1.12	14	.875
26,000,000	10	30	300	19½	20	1.03	16	.82
19,000,000	12	18	216	16⅝	18	1.08	15	.90
25,000,000	12	24	288	19⅛	20	1.05	16	.84
38,000,000	12	36	432	23½	22	.94	18	.77
30,000,000	15	22½	337.5	20¾	20	.96	17	.82
40,000,000	15	30	450	24	22	.92	18	.75
50,000,000	15	37½	562.5	26¾	24	.90	19	.71
38,000,000	18	24	432	23½	22	.94	18	.765
47,500,000	18	30	540	26¼	24	.915	19	.74
57,000,000	18	36	648	28¾	25	.87	20	.70
55,000,000	21	30	630	28⅜	25	.85	20	.71
66,000,000	21	36	756	31	26	.83	21	.68
77,000,000	21	42	882	33½	27	.81	22	.66
75,000,000	24	36	864	33⅛	27	.81	22	.66
90,000,000	24	42	1008	35⅞	28	.80	23	.66
100,000,000	24	48	1152	38¼	30	.785	24	.64

The area of the cross section and the length of the cores being given, the reluctance of the space between them depends upon the shape of their cross section and upon the distance between them. In case of *round* cores the shape is given by

TABLE LXXXIV.—DIMENSIONS OF OVAL MAGNET CORES. (WROUGHT IRON AND CAST STEEL.)

Total Flux per Magnetic Circuit, in Maxwells.	Cross-Section.				Length.			
	Breadth, Inches.	Width, Inches.	Area, Square Inches.	Diameter of Equiv. Circular Area d_m	Bipolar Types. Length l_m Inches.	Bipolar Types. Ratio $l_m : d_m$	Multipolar Types. Length l_m Inches.	Multipolar Types. Ratio $l_m : d_m$
600,000	2	4	7.14	3	4½	1.50	3½	1.17
1,000,000	2	6	11.14	3¾	5¼	1.40	4¼	1.13
1,300,000	2	8	15.14	4⅜	6	1.37	5	1.14
1,400,000	3	6	16.06	4⅜	6	1.33	5	1.11
2,200,000	3	9	25.06	5⅝	7½	1.33	6	1.07
3,000,000	3	12	34.06	6⅝	8½	1.28	7	1.05
2,500,000	4	8	28.56	6	7¾	1.29	6	1.00
3,900,000	4	12	44.56	7½	9¾	1.27	7½	1.00
5,250,000	4	16	60.56	8¾	11	1.26	8½	.97
5,500,000	6	12	64.26	9	11	1.22	8½	.945
8,750,000	6	18	100.26	11¼	13	1.16	10½	.93
12,000,000	6	24	136.26	13⅛	15	1.14	12	.915
10,000,000	8	16	114.14	12	14	1.17	11	.92
15,000,000	8	24	178.14	15	16	1.065	13	.87
15,000,000	10	20	178.5	15	16	1.065	13	.87
24,000,000	10	30	278.5	18¾	20	1.065	16	.85
22,500,000	12	24	257	18	18	1.00	15	.835
35,000,000	12	36	401	22½	21	.93	17	.755
35,000,000	15	30	400	22½	21	.93	17	.755
55,000,000	15	45	625	28¼	25	.885	20	.71
50,000,000	18	36	578	27⅛	24	.885	19	.70
60,000,000	18	42	686	29½	25	.85	20	.68
70,000,000	21	42	787	31⅝	26	.82	21	.66
80,000,000	21	48	913	34⅛	27	.79	22	.645
90,000,000	24	48	1028	36⅛	28	.78	22½	.625
100,000,000	24	54	1172	38⅝	30	.78	24	.62

the area of the cross-section, and the reluctance of the path from core to core, in consequence, only depends upon their distance apart, directly increasing with the same. The reluctance of the air gaps is determined by the diameter and length of the armature, by the percentage of polar embrace, and by the radial length of the gap-space, decreasing with the area of the gap and increasing with its length. The cross-section of the cores and the gap area, both depending upon the output of the machine, have a more or less fixed relation to each other—varying with the type, the voltage, the speed, and the

kind of armature—and the relation between the reluctance across the cores to that of the air gaps can approximately be expressed by the ratio of the average distance apart of the cores to the radial length of the gap-space. In dynamos with *smooth-drum* armature this ratio is made from 6 to 16, in *smooth-ring* machines from 8 to 20, and for *toothed* and *perforated* armatures the distance apart of the cores is taken from 3 to 6 times the maximum radial length of the gap-space, *i. e.*, from 3 to 6 times the distance between pole face and bottom of armature slot. The following Table LXXXV. gives the average distance between cylindrical magnet cores for various kinds and sizes of armatures, the ratio of this distance to the radial length of the gap-space, and the corresponding approximate leakage between the magnet cores, expressed in per cent. of the useful flux:

TABLE LXXXV.—DISTANCE BETWEEN CYLINDRICAL MAGNET CORES.

Diameter of Armature, Inches.	Smooth Core Armature.								Toothed or Perforated Armature.			
	Drum.				Ring.							
	Radial Length of Gap Space.	Least Distance between Cores.	Ratio of Distance Apart to Length of Gap.	Approx. Leakage between Cores in p. c. of Useful Flux.	Radial Length of Gap Space.	Least Distance between Cores.	Ratio of Distance Apart to Length of Gap.	Approx. Leakage between Cores in p. c. of Useful Flux.	Max. Radial Length of Gap Space.	Least Distance between Cores.	Ratio of Distance Apart to Max. Length of Gap.	Approx. Leakage between Cores in p. c. of Useful Flux.
2	11/32"	2"	5.8	33%	..	..	..	..	..	..	..	..
3	3/8	2 3/8	6.3	30	..	..	..	..	..	..	..	..
4	7/16	3	6.9	25	5/16"	2 1/2"	8.0	20%	11/16"	2"	2.9	15%
6	1/2	3 3/4	7 5	22	11/32	3	8.7	18	13/16	2 1/2	3 1	14
8	9/16	4 1/2	8.0	20	3/8	3 3/4	10.0	16	15/16	3 1/4	3.5	13
10	5/8	5 1/2	8.8	18	13/32	4 3/4	11.7	14	1 1/16	4	3.8	12
12	11/16	7	10.2	16	7/16	5 3/4	13.1	12	1 3/16	4 3/4	4.0	11
15	3/4	8 1/2	11.3	14	15/32	7	14.9	11	1 5/16	5 1/2	4.2	10
18	13/16	10	12.3	12	1/2	8 1/4	16.5	10	1 1/2	6 1/2	4.3	10
21	7/8	12	13.5	10	9/16	9 1/2	16.9	9 1/2	1 5/8	7 1/2	4.6	9 1/2
25	15/16	14	15	9	5/8	11	17.6	9	1 3/4	8 1/2	4.9	9
30	1	16	16	8	11/16	13	18.9	8 1/2	1 7/8	10	5.3	8 1/2
40	1 1/8	18	16	8	3/4	15	20	8	2	12	6	8

In case of *inclined* cylindrical magnets the figures given in Table LXXXV. for the least distances apart are to be considered as the *mean* least distances, taken across the magnets midway between their ends. (Compare formula 180, § 65.)

In dynamos with *rectangular* and *oval* cores the leakage across, for the same distance apart, is greater than in case of circular cores of equal sectional area, increasing in proportion to the ratio of the width of the cores to their breadth. For *rectangular* and *oval* cores, therefore, the distance apart is to be made greater than for round cores in order to limit the leakage between them to the same amount; and the distance must be the greater the wider the cores are in proportion to their thickness. The following Table LXXXVI. gives the minimum, average and maximum values of the ratio of the distance across rectangular and oval cores of various shapes of cross-sections to the distance which, between round cores of equal sectional area, effects approximately the same leakage, in small, in medium-sized, and in large dynamos, respectively:

TABLE LXXXVI.—DISTANCE BETWEEN RECTANGULAR AND OVAL MAGNET CORES.

Ratio of Thickness to Width of Cores.	Distance between Rectangular and Oval Magnet Cores, as compared with that between Round Cores of Equal Area, causing approximately the same leakage across.		
	Minimum. (Small Machines.)	Average.	Maximum. (Large Machines.)
1 : 1	1.0	1.0	1.0
3 : 4	1.05	1.07	1.1
2 : 3	1.1	1.15	1.2
1 · 2	1.15	1.22	1.3
1 3	1.2	1.3	1.4
1 4	1.25	1.37	1.5
1 5	1.3	1.45	1.6
1 · 6	1.35	1.55	1.75
1 · 7	1.4	1.65	1 9
1 : 8	1.5	1.75	2.05
1 : 9	1.6	1.9	2.25
1 : 10	1.7	2.1	2.5

In order to determine the proper distance apart of rectangular and oval magnet cores, the corresponding distance between round cores of equal cross-section is taken from Table LXXXIII., in multiplying the radial length of the gap-space by the ratio of distance apart to length of gap for the particular size of armature. The distance thus obtained is then multiplied by the respective figure found for the shape in question from Table LXXXVI.

84. Dimensioning of Yokes.

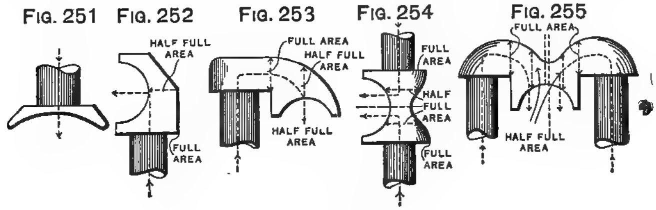
In *bipolar* types—the dimensions of the magnet cores being given by Tables LXXXI., LXXXIII. or LXXXIV., § 83, and their least distance apart by Table LXXX. or LXXXVI., § 83, thus fixing the length of the yoke, and the sectional area of the yokes being found from formula (216), § 82—the dimensioning of the yoke consists in arranging its cross-section with reference to the shape of the section of the cores, and, for the case that its material is different from that of the cores, in providing a sufficient contact area, conforming to the rules given in § 80.

In *multipolar* types the total cross-section found for the frame from formula (216), § 82, is to be divided by the total number of magnetic circuits in the machine and multiplied by the number of circuits passing through any part of the yoke in order to obtain the sectional area required for that part of the yoke; otherwise the above rules also govern the dimensioning of the yokes for multipolar machines.

85. Dimensioning of Polepieces.

In dimensioning the polepieces, three cases have to be considered: (1) the path of the lines of force leaving the polepieces has the same direction as their path through the magnets (Fig. 251); (2) the path of the lines leaving the polepieces makes a right angle to that through the cores (Fig. 252); and (3) the path of the lines leaving the polepieces is parallel but of opposite direction to that through the cores, making two turns at right angles in the polepieces (Fig. 253).

FIG. 251 FIG. 252 FIG. 253 FIG. 254 FIG. 255



Figs. 251 to 255.—Various Kinds of Polepieces.

In the first case, Fig. 251, which occurs in dynamos of the iron-clad, the radial and the axial multipolar types, the shape

of the cross-section is fixed by the form of the magnet core at one end and by the axial length or the radial width of the armature, respectively, and the percentage of polar arc at the other, while the *height*, in the direction of the lines of force, is to be made as small as possible, in order not to increase the total length of the magnetic circuit more than necessary.

TABLE LXXXVII.—DIMENSIONS OF POLEPIECES FOR BIPOLAR HORSESHOE TYPE DYNAMOS.

Capacity, in Kilowatts.	Drum Armature.						Ring Armature.				
	Average Total Flux of Bipolar Horseshoe Drum Type. Maxwells.	Diameter of Armature. Inches.	Dimensions of Polepiece.				Average Total Flux of Bipolar Horseshoe Ring Type. Maxwells.	Diameter of Armature. Inches.	Dimensions of Polepiece.		
			Height. Inches.	Length (= Length of Armature.)	Thickness in Centre. Inches.				Height. Inches.	Area in Centre Square Inches.	
					Wrought Iron.	Cast Iron.				Wrought Iron.	Cast Iron.
.1	350,000	$1\frac{3}{4}$	$2\frac{3}{8}$	$3\frac{1}{2}$	$\frac{1}{2}$	1	175,000	4	$4\frac{3}{4}$	$\frac{7}{8}$	$1\frac{3}{4}$
.25	500,000	$2\frac{1}{4}$	3	$4\frac{1}{2}$	$\frac{9}{16}$	$1\frac{1}{8}$	250,000	5	$5\frac{3}{4}$	$1\frac{1}{4}$	$2\frac{1}{2}$
.5	650,000	$2\frac{3}{4}$	$3\frac{1}{2}$	$5\frac{1}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	325,000	6	$6\frac{3}{4}$	$1\frac{5}{8}$	$3\frac{1}{4}$
1	900,000	$3\frac{1}{4}$	4	6	$\frac{3}{4}$	$1\frac{1}{2}$	450,000	7	$7\frac{3}{4}$	$2\frac{1}{4}$	$4\frac{1}{2}$
2	1,400,000	$3\frac{3}{4}$	$4\frac{1}{2}$	7	1	2	675,000	8	9	$3\frac{3}{8}$	$6\frac{3}{4}$
3	1,800,000	$4\frac{1}{2}$	$5\frac{3}{8}$	$8\frac{1}{4}$	$1\frac{1}{8}$	$2\frac{1}{4}$	900,000	$9\frac{1}{2}$	$10\frac{1}{2}$	$4\frac{1}{2}$	9
5	2,500,000	$5\frac{1}{4}$	$6\frac{1}{8}$	$9\frac{1}{2}$	$1\frac{3}{8}$	$2\frac{5}{8}$	1,300,000	11	12	$6\frac{1}{2}$	13
10	3,500,000	6	7	$10\frac{1}{2}$	$1\frac{3}{4}$	$3\frac{3}{8}$	2,100,000	14	15	$10\frac{1}{2}$	21
15	5,000,000	$6\frac{3}{4}$	$7\frac{3}{4}$	12	$2\frac{1}{8}$	$4\frac{1}{4}$	2,800,000	15	16	14	28
20	6,000,000	$7\frac{1}{2}$	$8\frac{5}{8}$	13	$2\frac{5}{16}$	$4\frac{5}{8}$	3,500,000	16	17	$17\frac{1}{2}$	35
25	7,000,000	$8\frac{1}{4}$	$9\frac{3}{8}$	14	$2\frac{1}{2}$	5	4,200,000	18	19	21	42
30	8,000,000	9	$10\frac{1}{4}$	15	$2\frac{3}{4}$	$5\frac{3}{8}$	4,800,000	20	21	24	48
50	12,500,000	$10\frac{1}{2}$	$11\frac{5}{8}$	18	$3\frac{1}{2}$	7	7,000,000	24	25	35	70
75	16,500,000	$12\frac{1}{2}$	$13\frac{7}{8}$	20	$4\frac{1}{8}$	$8\frac{1}{4}$	9,500,000	28	$29\frac{1}{4}$	$47\frac{1}{2}$	95
100	21,000,000	15	$16\frac{1}{2}$	22	$4\frac{3}{4}$	$9\frac{1}{2}$	12,000,000	32	$33\frac{1}{2}$	60	120
150	30,000,000	$18\frac{1}{2}$	$20\frac{1}{4}$	26	$5\frac{3}{4}$	$11\frac{1}{2}$	17,000,000	36	$37\frac{1}{2}$	85	170
200	38,000,000	$22\frac{1}{2}$	$24\frac{1}{2}$	31	$6\frac{1}{8}$	$12\frac{1}{4}$	21,500,000	40	42	$107\frac{1}{2}$	215
300	57,000,000	28	$30\frac{1}{2}$	38	$7\frac{1}{2}$	15	30,000,000	46	48	150	300

In the second case, Fig. 252, met with in bipolar and multiple horseshoe and in tangential multipolar types, the height of the polepieces is determined by the diameter, and the length of the polepiece by the length of the armature, while the area of the cross-section, perpendicular to the flow of the lines, is to be made of the size obtained by formula (216) at the end next to the magnet core, and to be gradually decreased in amount from that end to the opposite end or to the centre of

the polepiece, respectively, according to whether there is but one magnetic circuit, or whether two circuits are passing through the same polepiece. Since, in bipolar machines, the lines of force are supposed to divide equally between the two halves of the armature, only one-half of the total flux passes the centre of the polepieces, in order to reach the half of the armature opposite the magnets, and the area in the centre of the polepiece consequently needs to be but one-half that at the end next to the core. In case the two circuits passing through each polepiece, Fig. 254, the same applies to the cross-section of the polepiece, at one-quarter the height from either end. For ready use, in the preceding Table LXXXVII., the dimensions of wrought- and cast-iron polepieces for various sizes of bipolar horseshoe type dynamos are calculated for drum and ring armatures, by combining the respective data given in former tables.

In the third case, Fig. 253, finally, which is found in single and double magnet types, the length of the magnetic circuit in the polepiece is determined by the diameter of the armature, by the cross-section of the magnet core, and by the height of their winding space; the width, parallel to the armature shaft of the polepiece near the magnet, is given by the width of the magnet core, and that near the armature by the axial length of the latter. The heights, parallel to the axis of the magnet core, in case of a single circuit, are to be so chosen that all of the cross-sections, up to that in line with the pole corner next to the magnet core, have an area at least equal in amount to that obtained by formula (216), and that the section in line with the armature centre has an area of one-half that amount. In case of two circuits meeting at the polepieces (consequent pole types), Fig. 255, the full area has to be provided from either end of the polepiece to the sections in line with the pole corners, half the full area at quarter distance from each pole corner, that is, midway between each pole corner and the pole centre, and sufficient cross-section for mechanical strength only is needed at the centre of the polepiece.

PART V.

CALCULATION OF MAGNETIZING FORCES.

CHAPTER XVII.

THEORY OF THE MAGNETIC CIRCUIT

86. Law of the Magnetic Circuit.

The magnetic flux through the various parts of the magnetic circuit being known by means of formulæ (137), § 56, and (156), § 60, respectively, and the dimensions of the magnet frame being determined by the rules and formulæ given in Chapters XV. and XVI., the magnetomotive force necessary to drive the required flux through the circuit of given reluctance can now be calculated by virtue of the "*Law of the Magnetic Circuit.*"

For the magnetic circuit a law holds good similar to *Ohm's Law* of the electric circuit; in the electric circuit:

$$\text{Current (or Electric Flux)} = \frac{\text{Electromotive Force}}{\text{Resistance}},$$

and analogously, in the magnetic circuit:

$$\text{Magnetic Flux} = \frac{\text{Magnetomotive Force}}{\text{Reluctance}},$$

from which follows:

$$\text{Magnetomotive Force} = \text{Magnetic Flux} \times \text{Reluctance.} \quad \ldots\ldots (222)$$

The *Reluctance* of a magnetic circuit, similar to the electric case of *resistance*, can be expressed by the specific reluctance, or *reluctivity*, of the material, and the dimensions of the magnetic conductor, thus:

$$\text{Reluctance} = \text{Reluctivity} \times \frac{\text{Length}}{\text{Area}}.$$

But the *reluctivity* of a magnetic material is the reciprocal of its *permeability* (similarly as the *resistivity* of an electric conducting material is the reciprocal of its *conductivity*), and consequently we have:

$$\text{Reluctance} = \frac{\text{Length}}{\text{Permeability} \times \text{Area}} \quad \ldots\ldots (223)$$

Combining (222) and (223), we obtain:

$$\text{Magnetomotive Force} = \frac{\text{Magnetic Flux}}{\text{Area}} \times \frac{\text{Length}}{\text{Permeability}},$$

and since the quotient of magnetic flux by area is the *magnetic density*, we have:

$$\text{Magnetomotive Force} = \frac{\text{Magnetic Density}}{\text{Permeability}} \times \text{Length}.$$

The permeability of magnetic materials depending upon the magnetic density employed in the circuit, see Table LXXV., § 81, the quotient of magnetic density and permeability also depends upon the density, and has a fixed value for every degree of saturation and for each material. But this quotient multiplied by the length of the circuit gives the magnetomotive force required for that circuit, and consequently represents the magnetomotive force per unit of length, or the *specific magnetomotive force* of the circuit. In order to obtain the M.M.F. required for any material, any density and any length, therefore, the specific M. M. F. for the respective material at the density employed is to be multiplied by the length of the circuit:

$$\text{Magnetomotive Force} = \text{Specific M. M. F.} \times \text{Length.} \qquad (224)$$

87. Unit Magnetomotive Force.—Relation Between Magnetomotive Force and Exciting Power.

An infinitely long solenoid of unit cross-sectional area (1 square centimetre), having unit magnetizing force or exciting power (1 current-turn) per unit of length (1 centimetre) possesses poles of unit strength at its ultimate extremities. If the exciting power per centimetre length, therefore, is 1 ampere-turn, *i. e.*, $\frac{1}{10}$ of a current-turn (the ampere being the tenth part of the absolute unit of current-strength), the poles produced at the ends of the solenoid will be of the strength of $\frac{1}{10}$ of a unit pole.

Since a unit pole disperses 4π lines of force, or maxwells, see § 55, the magnetic flux of a unit solenoid of infinite length and of a special exciting power of 1 ampere-turn per centimetre is

$$\frac{4\pi}{10} \text{ maxwells,}$$

and the density of the flux is

$$\frac{4\,\pi}{10}\ \textit{webers per square centimetre, or}\ \frac{4\,\pi}{10}\ \textit{gausses.}$$

The reluctance per unit length of the solenoid, the latter being of 1 square centimetre sectional area, is that of 1 cubic centimetre of air, and therefore is unity, or 1 *oersted*, hence the M. M. F. of the coil per ampere-turn of exciting power being the product of magnetic flux and reluctance, is

$$\frac{4\,\pi}{10}$$

C. G. S units of magnetomotive force, or

$$\frac{4\,\pi}{10}\ \textit{gilberts.}$$

A magnetomotive force of

$$\frac{4\,\pi}{10}\ \text{gilberts}$$

being excited by one ampere-turn of magnetizing force, and the magnetomotive force being proportional to the magnetizing force producing the same, it follows that the entire M. M. F. of a circuit, in gilberts, is

$$\frac{4\,\pi}{10}$$

times the total number of ampere-turns; and inversely, in order to express the exciting power necessary to produce a certain M. M. F., the number of gilberts to be multiplied by

$$\frac{10}{4\,\pi} = .796;$$

thus:

$$\text{Number of Ampere-turns} = \frac{10}{4\,\pi} \times \text{Number of Gilberts.} \qquad (225)$$

88. Magnetizing Force Required for any Portion of a Magnetic Circuit.

The magnetizing force required for any circuit is the sum of the magnetizing forces used for its different parts.

From (224) and (225), § 87, follows that the exciting power required for any part of a magnetic circuit is

$$\frac{10}{4\pi}$$

times the product of the specific M. M. F. and the length of that portion of the circuit:

$$\text{Magnetizing Force} = \frac{10}{4\pi} \times \text{Specific M. M. F.} \times \text{Length.}$$

The product of the specific magnetomotive force, for the particular material and density in question, with the constant factor

$$\frac{10}{4\pi},$$

represents the exciting power per unit length of the circuit, or the *Specific Magnetizing Force;* consequently we have:

Magnetizing Force = Specific Magnetizing Force × Length,

or,

Number of Ampere-turns
= Ampere-turns per unit of Length × Length.

Denoting the density of the lines of force in any particular portion of a magnetic circuit by $\mathfrak{B}$, the specific magnetizing force by m, and the length by l, the number of ampere-turns required for that portion of the magnetic circuit can be calculated from the general formula:

$$at = m \times l, \quad \ldots\ldots\ldots\ldots\ldots (226)$$

where m = specific magnetizing force, in ampere-turns per inch, or per centimetre, of length, for the particular material and density employed, see Tables LXXXVIII. and LXXXIX., or Fig. 256;

l = length of the magnetic circuit in the respective material in inches, or centimetres, respectively.

The values of the specific magnetizing forces, m, for various densities, as averaged from a great number of tests by

Ewing,[1] Negbauer,[2] Kennelly,[3] Steinmetz,[4] Thompson,[5] and others, for the various materials are compiled in the following Tables LXXXVIII. and LXXXIX., which give the specific magnetizing force in ampere-turns per inch length, and in ampere-turns per centimetre length, respectively.

The figures in the last column of these tables, referring to air, are obtained by multiplying the magnetic density, $\mathfrak{B}$, by

$$\frac{10}{4\pi}$$

in the metric, and by

$$\frac{10}{4\pi} \times \frac{1}{2.54} = .3133$$

in the English system; for in case of air, the permeability, being unity, does not depend upon the density, and the magnetizing force, in consequence, is directly proportional to the density.

For convenient reference the values of *m* contained in Tables LXXXVIII. and LXXXIX., for the various kinds of *iron*, are plotted in Fig. 256, p. 338.

The said Tables LXXXVIII. and LXXXIX., although carefully averaged with reference to commercial tests of various kinds of iron, cannot be expected to give accurate results in specific cases of actual design, since different samples of one and the same kind of iron often vary as much as 10 per cent. and more in permeability. These tables are, therefore, intended only for the use of the student, while the practical designer is supposed to make up his own table or

[1] J. A. Ewing, "Magnetism in Iron and Other Metals," *The Electrician* (London, 1890-91).

[2] Walter Negbauer, *Electrical Engineer*, vol. ix. p. 56 (February, 1890).

[3] A. E. Kennelly, *Trans. Am. Inst. El. Eng.*, vol. viii. p. 485 (October 27, 1891); *Electrical Engineer*, vol. xii. p. 508 (November 4, 1891); *Electrical World*, vol. xviii. p. 350 (November 7, 1891).

[4] Charles P. Steinmetz, *Trans. Am. Inst. El. Eng.*, vol. ix. p. 3 (January 19, 1892); *Electrical Engineer*, vol. xiii. pp. 91, 121, 143, 167, 261, 282 (January 27, February 3, 10, 17, March 9, 16, 1892); *Electrical World*, vol. xix. pp. 73, 89 (January 30, February 6, 1892).

[5] Milton E. Thompson, Percy H. Knight, and George W. Bacon, *Trans. Am. Inst. El. Eng.*, vol. ix. p. 250 (June 7, 1892); *Electrical Engineer*, vol. xiv. p. 40 (July 13, 1892); *Electrical World*, vol. xix. p. 436 (June 25, 1892).

TABLE LXXXVIII.—SPECIFIC MAGNETIZING FORCES FOR VARIOUS MATERIALS AT DIFFERENT DENSITIES, IN ENGLISH MEASURE.

MAGNETIC DENSITY. Lines of Force per square inch. $\mathfrak{B}''$	UNIT MAGNETIZING FORCE. Ampere-Turns per Inch Length.					
	Annealed Wrought Iron.	Soft Cast Steel.	Mitis Iron.	Cast Iron containing 6.5 % of Aluminum.	Cast Iron (ordinary).	Air, $(=.3133\times\mathfrak{B}'')$.
2,500	1.2	2	2.5	7	9	783
5,000	1.7	2.8	3.4	9.6	13	1,566
7,500	2.1	3.4	4	11.6	16	2,350
10,000	2.2	3.7	4.4	13.5	18.5	3,133
12,500	2.4	4	4.8	15.7	21.3	3,916
15,000	2 7	4.3	5.2	18.2	24.1	4,700
17,500	3.1	4.6	5.6	21	27.1	5,483
20,000	3.5	5	6	24	30.5	6,266
22,500	4	5.4	6.5	27.2	34.5	7,050
25,000	4.5	5.8	7	31	39	7,833
27,500	5	6.2	7.5	35.5	44	8,616
30,000	5.5	6.6	8.1	41.5	50	9,400
32,500	6	7.1	8.7	47.5	57	10,162
35,000	6.5	7.6	9.4	54	65	10,966
37,500	7	8.2	10.1	62	76	11,750
40,000	7.5	8.8	10.9	72	88	12,532
42,500	8	9.4	11.7	83	101	13,315
45,000	8.5	10.1	12.6	95	116	14,100
47,500	9	10.9	13.6	110	136	14,882
50,000	9.6	11.8	14.7	128	160	15,665
52,500	10.3	12.8	15.9	149	189	16,450
55,000	11.1	13.9	17.3	173	222	17,233
57,500	12	15.1	19	200	257	18,016
60 000	13	16.4	21	230	295	18,800
62,500	14.2	17.8	23.2	263	340	..
65,000	15.7	19.3	25.6	300	400	..
67,500	17.5	20.9	28.5	345	470	..
70,000	19.6	22.7	32	400	570	..
72,500	22	24.7	36	460	700	..
75,000	24.7	27	41	525	..	..
77,500	27.7	30	47	600	..	..
80,000	31.2	34	54	700	..	..
82,500	35 2	39	62	..	..	..
85,000	39.7	44	70	..	..	..
87,500	44.7	50	80	..	..	..
90,000	50.7	57	92	..	..	..
92,500	58	65	109	..	..	..
95,000	67	75	131	..	..	..
97,500	78	86	159	..	..	..
100,000	91	100	193	..	..	.
102,500	108	121	245	..	..	..
105,000	137	159	290	..	..	..
107,500	190	227	345	..	..	..
110,000	290	325	410	..	..	..
112,500	395	430	500	..	..	..
115,000	500	550	600	..	..	..
117,500	600	650	700	..	..	..
120,000	700	750	800	..	..	..
122,500	800	850	..	..	..	..
125,000	900	950	..	..	..	..

curve, by actually testing the very iron he is going to use for his machine.

TABLE LXXXIX.—SPECIFIC MAGNETIZING FORCES FOR VARIOUS MATERIALS AT DIFFERENT DENSITIES, IN METRIC MEASURE.

Magnetic Density. Lines of Force per cm² $\mathfrak{B}$	Unit Magnetizing Force. Ampere-Turns per Centimetre Length.					
	Annealed Wrought Iron.	Soft Cast Steel.	Mitis Iron.	Cast Iron containing 6.5 % of Aluminum.	Cast Iron (Ordinary).	Air, $\left(=\frac{10}{4\pi}\times\mathfrak{B}\right)$
500	.5	.9	1.1	3	5	400
1,000	.8	1.25	1.5	4	6	800
1,500	.9	1.45	1.7	5	7	1,200
2,000	.95	1.6	1.9	6.5	8.5	1,600
2,500	1.1	1.75	2.1	8	10	2,000
3,000	1.35	1.95	2.3	9.5	12	2,400
3,500	1.6	2.15	2.6	11	14	2,800
4,000	1.8	2.35	2.8	13	16	3,200
4,500	2.1	2.55	3.1	15	19	3,600
5,000	2.35	2.8	3.4	19	22	4,000
5,500	2.6	3.05	3.7	22.5	26	4,400
6,000	2.85	3.35	4.1	26.5	32	4,800
6,500	3.1	3.65	4.5	31.5	38	5,200
7,000	3.35	4	5	37.5	46	5,600
7,500	3.6	4.4	5.5	45	57	6,000
8,000	3.95	4.9	6.1	56	71	6,400
8,500	4.35	5.5	6.75	68	87	6,800
9,000	4.8	6.0	7.6	81	105	7,200
9,500	5.4	6.7	8.7	99	125	7,600
10,000	6.1	7.5	10.0	118	153	8,000
10,500	7	8.3	11.5	138	190	..
11,000	8	9.2	13.5	163	240	..
11,500	9.4	10.3	16	195	..	..
12,000	10.8	11.7	18.5	235	..	..
12,500	12	14	22	285	..	..
13,000	15	16	26	..	..	..
13,500	17	20	30.5	..	..	..
14,000	20	23	37	..	..	..
14,500	24	27	46	..	..	..
15,000	30	32	58	..	..	..
15,500	36	40	74	..	..	..
16,000	47	52	96	..	..	..
16,500	68	80	124	..	..	..
17,000	108	124	160	..	..	..
17,500	160	176	204	..	..	..
18,000	212	228	250	..	..	..
18,500	264	280	300	..	..	..
19,000	316	333	350	..	..	..
19,500	368	386	400	..	..	..
20,000	420	440	450	..	..	..

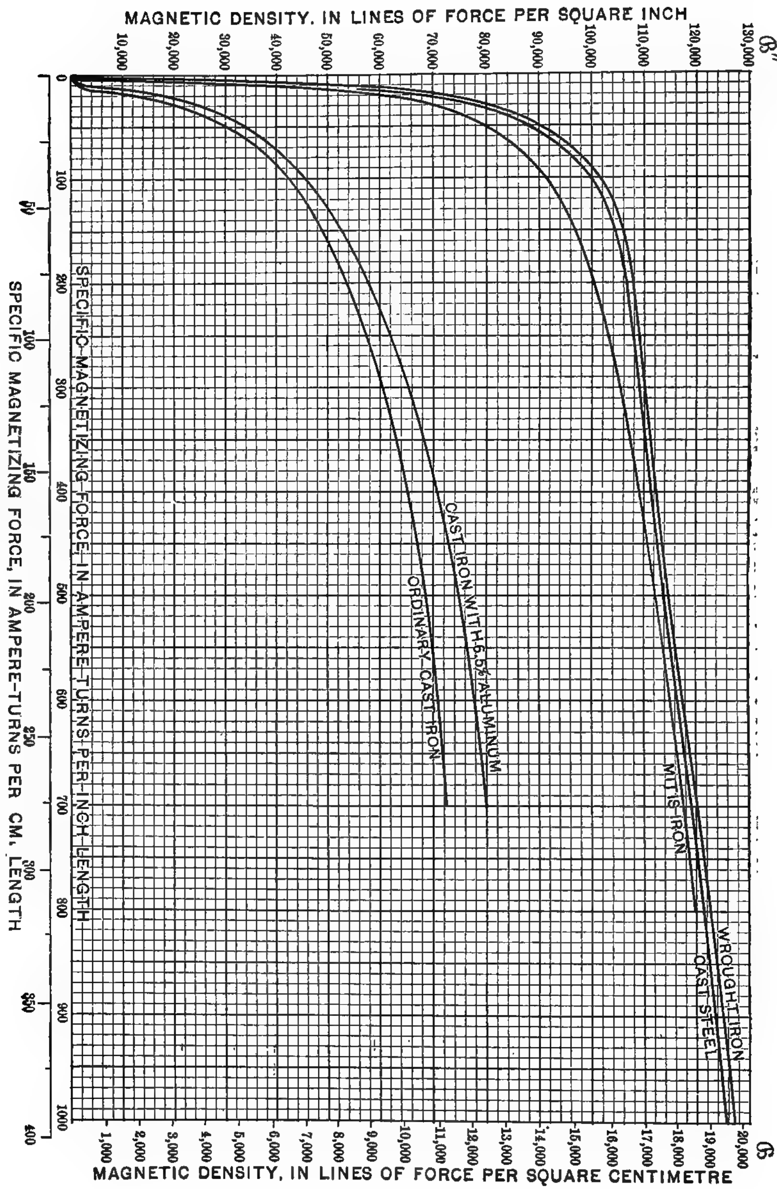


Fig. 256.—Specific Magnetizing Forces for Various Kinds of Iron.

CHAPTER XVIII.

MAGNETIZING FORCES.

89. Total Magnetizing Force of Machine.

The total exciting power required for each magnetic circuit of a dynamo-electric machine is the sum of the magnetizing forces needed to overcome the reluctances due to the gap-spaces, to the armature core, and to the field frame, and of the magnetizing force required to compensate the reaction of the armature winding upon the magnetic field; or, in symbols:

$$AT = at_g + at_a + at_m + at_r, \quad \ldots\ldots(227)$$

where AT = total magnetizing force required for normal output of machine in ampere-turns;

at_g = ampere-turns needed to overcome reluctance of gap-spaces; see formula (228), § 90;

at_a = ampere-turns needed to overcome reluctance of armature core; see formula (230), § 91;

at_m = ampere-turns needed to overcome reluctance of magnet frame; see formula (238), § 92;

at_r = ampere-turns needed to compensate armature reactions; see formula (250), § 93.

In order to keep the angle of field-distortion, upon which depends the amount of *sparking*, below its practical limit, *the ratio of the armature ampere-turns per magnetic circuit to the above number AT of field ampere-turns per magnetic circuit must not exceed the value of the trigonometric tangent of the greatest permissible angle of field distortion* for the type of machine under consideration. (See page 349.) If for some reason this important condition is not fulfilled, although the rules and formulæ given in the previous chapters have been carefully followed, *the machine must be re-designed.*

The angle of field distortion of any dynamo depends upon the number of its poles and upon the type of its armature.

The following table gives the usual limiting values of the distortion angle and the corresponding value of its trigonometric tangent for smooth and toothed armature machines with various numbers of poles:

TABLE LXXXIX*a*.—GREATEST PERMISSIBLE ANGLE OF FIELD DISTORTION AND CORRESPONDING MAXIMUM RATIO OF ARMATURE AMPERE-TURNS TO FIELD AMPERE-TURNS.

NUMBER OF POLES.	GREATEST PERMISSIBLE ANGLE OF FIELD DISTORTION.		MAXIMUM RATIO OF ARMATURE AMPERE-TURNS TO FIELD AMPERE-TURNS.	
	Smooth Armature. α	Toothed or Perforated Armature. α'	Smooth Armature. $\tan \alpha$.	Toothed or Perforated Armature. $\tan \alpha'$.
2	36°	24°	.727	.445
4	18	12	.325	.213
6	12	8	.213	.141
8	9	6	.158	.105
10	7	5	.123	.087
12	6	4	.105	.070
14	5	$3\frac{1}{2}$	.087	.061
16	$4\frac{1}{2}$	3	.075	.052
18	4	$2\frac{2}{3}$	.070	.046
20	$3\frac{1}{2}$	$2\frac{1}{3}$	.061	.040
24	3	2	.052	.035

In dividing the armature ampere-turns per magnetic circuit, $\frac{N_a}{n_z} \times \frac{I'}{2\,n'_p}$, by the field ampere-turns, AT, the actual ratio of armature ampere-turns to field ampere-turns for the machine under consideration is obtained; and by comparing this ratio with the value of the corresponding tangent in the above table it can easily be decided whether or not it will be necessary to alter the design of the machine.

Since, for the purpose of the above check, an approximate value of the field ampere-turns is all that is required, it is not necessary for this preliminary determination of AT to make the detailed calculations treated in §§ 91, 92, and 93, but it will be sufficient to proceed as follows:

To find the gap ampere-turns, at_g, which in all but exceptional cases constitute at least one-half of the total number AT of the field ampere-turns, use formula (228), employing for $\mathcal{H}''$ the value chosen from Table VI. as the basis of the armature calculation.

Next make a rough scale drawing of the magnetic circuit and indicate by lines therein the mean paths of the flux in the various parts, thereby obtaining l''_a, $l''_{w.i.}$, and $l''_{c.i.}$ (or $l''_{c.s.}$, as the case may be) directly by measurement. From Table LXXXVIII., page 336, find the specific magnetizing forces m''_a, $m''_{w.i.}$, etc., corresponding to the flux densities $\mathcal{B}''_a$, $\mathcal{B}''_{w.i.}$, etc., employed in computing the various cross-sections, and form the respective products $l''_a \times m''_a$, $l''_{w.i.} \times m''_{w.i.}$, etc. In this manner the values of at_a and at_m are obtained, the latter by one single multiplication if the field frame is all of the same material, or by adding several products if various portions of the magnet frame are made of different materials.

The compensating ampere-turns at_r, finally, need not be computed at all, it being sufficiently accurate for the purpose on hand to increase the sum of the gap, armature, and frame ampere-turns thus far obtained by *15* or *20* per cent.

90. Ampere-Turns for Air Gaps.

The magnetizing force required to produce a magnetic density of $\mathcal{H}''$ lines of force per square inch in the air spaces, according to § 88, is:

$$at_g = \frac{10}{4\pi} \times \mathcal{H}'' \times \frac{l''_g}{2.54} = .3133 \times \mathcal{H}'' \times l''_g, \quad (228)$$

where $\mathcal{H}''$ = field density, in lines of force per square inch; from formula (142), § 57, for *smooth* armature dynamos, and from $\frac{\Phi}{S'_g}$ for machines with *toothed* or *perforated* armatures, the area of the gap-space, S_g, to be taken, respectively, from the numerators of equations (174), (175), or (176), p. 230; and

l''_g = length of magnetic circuit in air gaps, in inches; the magnetic length of the gaps is obtained by multiplying twice the distance between armature core surface and polepieces by the factor of field-deflection; see Table LXVI., p. 225, for *smooth* armatures, and Table LXVII., p. 230, for *toothed* and *perforated* armatures, respectively.

If the field density is given in lines of force per square centimetre, and the length of the circuit in centimetres, the magnetizing force in ampere-turns is obtained from

$$at_g = \frac{10}{4\pi} \times \mathcal{H} \times l_g = .8 \times \mathcal{H} \times l_g. \quad \ldots(229)$$

91. Ampere-Turns for Armature Core.

For the magnetizing force needed to overcome the reluctance of the armature core we find, according to formula (226):

$$at_a = m''_a \times l''_a, \quad \ldots\ldots\ldots\ldots\ldots(230)$$

where m''_a = average specific magnetizing force, in ampere-turns per inch length, formulæ (231) to (235);

l''_a = mean length of magnetic circuit in armature core in inches, formula (236) or (237), respectively.

Owing to the cylindrical shape of the armature, the area of the surface presented to the lines when entering and leaving the core is much greater than that of the actual cross-section of the armature body. Hence, since every useful line of force, on its way from a north pole to the adjoining south pole, must pass through the smallest core section, it is evident that the magnetizing force required per unit of path length is smallest near the polepieces and greatest opposite the neutral points of the field, while it gradually increases from the minimum to the maximum value as the flux passes from the peripheral surface opposite the north pole to the neutral cross-section, and gradually decreases again to minimum as the flux proceeds from the neutral section to the periphery opposite the south pole.

The average specific magnetizing force, therefore, is obtained by taking the arithmetical mean of the extreme values:

$$m''_a = \frac{1}{2}(m''_{a_1} + m''_{a_2}), \quad \ldots\ldots\ldots\ldots(231)$$

in which m''_{a_1} = maximum specific magnetizing force, for smallest area of magnetic circuit, in armature; see Table LXXXVIII., column for annealed wrought iron;

m''_{a_2} = minimum specific magnetizing force, for largest area of magnetic circuit in armature, Table LXXXVIII.

The maximum specific magnetizing force m''_{a_1} corresponds to the maximum density of $\mathfrak{B}_{a_1} = \frac{\Phi}{S''_{a_1}}$ lines, and the minimum specific magnetizing force m''_{a_2} to a minimum density of $\mathfrak{B}''_{a_2} = \frac{\Phi}{S''_{a_2}}$ lines; Φ being the useful flux of the machine, in maxwells; S''_{a_1} the minimum area of circuit in armature, that is, the net cross-section of the armature core, in square inches; and S''_{a_2} the maximum path-area of armature, in square inches.

The *area* of the magnetic circuit in the armature can be expressed by the product of the net length and the depth of the core, and of the number of poles; hence the *minimum area:*

$$S''_{a_1} = 2\, n_p \times l_a \times b_a \times k_2, \quad \ldots\ldots(232)$$

and the *maximum area:*

$$S''_{a_2} = 2\, n_p \times l_a \times b'_a \times k_2, \quad \ldots\ldots(233)$$

where n_p = number of pairs of magnet poles;

l_a = length of armature core, in inches;

b_a = radial depth of armature core, in inches;

b'_a = maximum depth of armature core, in inches;

k_2 = ratio of net iron section to total cross-section of armature core, see Table XXIII., p. 94.

The *maximum depth*, b'_a, of the armature is determined as follows:

In *multipolar* dynamos the maximum depth, b'_a, of the core is approximately equal to half the circumferential width of one polepiece; for *bipolar* machines b'_a is half the largest chord that can be drawn between the internal and external armature peripheries. In bipolar *smooth* armatures, as seen from Fig. 257,

b'_a is the leg of a right triangle, the hypothenuse of which is the external radius of the armature core, and whose other leg is the internal radius of the armature; hence by the Pythagorean Principle it can be expressed by the core diameter, d_a, and the radial depth, b_a, as follows:

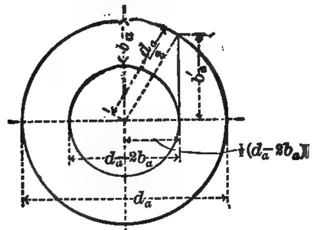


Fig. 257.—Maximum Core-Depth in Bipolar Smooth Armature.

$$b'_a = \sqrt{\frac{d_a^2}{4} - \frac{(d_a - 2\,b_a)^2}{4}} = b_a \times \sqrt{\frac{d_a}{b_a} - 1}. \quad \ldots(234)$$

For bipolar *toothed* and *perforated* armatures we have, similarly, with reference to Fig 258:

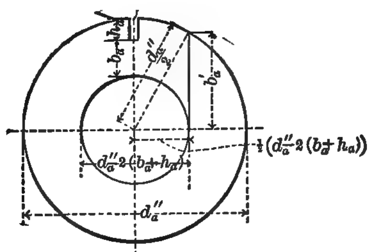


Fig. 258.—Maximum Core-Depth in Bipolar Toothed Armature.

$$b'_a = \sqrt{\frac{d''^2_a}{4} - \frac{\{d''_a - 2\,(b_a + h_a)\}^2}{4}}$$

$$= \sqrt{\frac{d''^2_a - d''^2_a + 4\,d''_a \times (b_a + h_a) - 4\,(b_a + h_a)^2}{4}}$$

$$= (b_a + h_a) \times \sqrt{\frac{d''_a}{b_a + h_a} - 1}. \quad \ldots\ldots\ldots\ldots\ldots(235)$$

The *length* of the magnetic circuit in the armature is obtained from the following formulæ (236) or (237), for smooth and toothed cores, respectively, which are derived from purely geometrical considerations.

The path of the magnetic circuit through a *smooth* armature core is illustrated in Fig. 259, from which it is evident that the length l''_a can be expressed by:

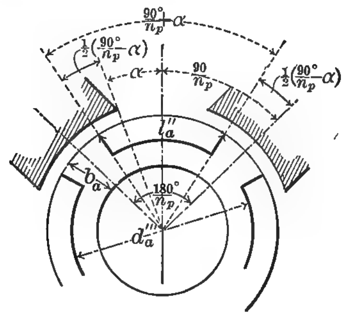


Fig. 259.—Length of Magnetic Path in Smooth Armature Core.

$$l''_a = d'''_a \times \pi \times \frac{\frac{90°}{n_p} + \alpha}{360°} + b_a. \quad \ldots\ldots(236)$$

In case of *toothed* and *perforated* armatures, Fig. 260, the length of the path is:

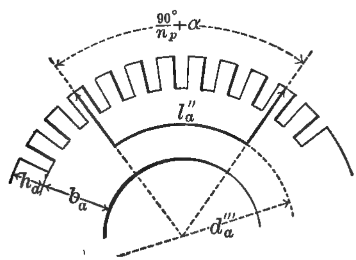


Fig. 260.—Length of Magnetic Path in Toothed Armature Core.

$$l''_a = d'''_a \times \pi \times \frac{\frac{90°}{n_p} + \alpha}{360°} + b_a + 2h_a, \quad \ldots(237)$$

where d'''_a = mean diameter of armature core, in inches;
b_a = radial depth of armature core, in inches;
h_a = depth of armature slots, in inches;
n_p = number of pairs of magnet poles;
α = half angle between adjacent pole corners.

92. Ampere-Turns for Field Magnet Frames.

Taking the most general case of a magnet frame consisting of three different materials—*wrought iron* cores, *cast iron* yokes, and *cast steel* polepieces—then, according to (226) we obtain:

$$at_{m} = m''_{m} \times l''_{m}$$
$$= m''_{w.i.} \times l''_{w.i.} + m''_{c.i.} \times l''_{c.i.} + m''_{c.s.} \times l''_{c.s.}, \quad \ldots (238)$$

where $m''_{w.i.}$, $m''_{c.i.}$, $m''_{c.s.}$ = specific magnetizing forces for wrought iron, cast iron, and cast steel, respectively, from Table LXXXVIII., or Fig. 256; corresponding to the magnetic densities $\mathfrak{B}''_{w.i.}$, $\mathfrak{B}''_{c.i.}$, and $\mathfrak{B}''_{c.s.}$ in the respective materials;

$$m''_{m} = \frac{m''_{w.i.} \times l''_{w.i.} + m''_{c.i.} \times l''_{c.i.} + m''_{c.s.} \times l''_{c.s.}}{l''_{w.i.} + l''_{c.i.} + l''_{c.s.}}$$

= average specific magnetizing force of magnet frame in ampere-turns per inch length;

$l''_{w.i.}$, $l''_{c.i.}$, $l''_{c.s.}$ = lengths of magnetic circuit in wrought iron, in cast iron, and in cast steel, respectively, in inches;

$l''_{m} = l''_{w.i.} + l''_{c.i.} + l''_{c.s.}$ = total length of magnetic circuit in magnet frame, in inches.

The densities $\mathfrak{B}_{w.i.}$, $\mathfrak{B}_{c.i.}$, and $\mathfrak{B}_{c.s.}$ are the quotients of the total magnetic flux, Φ', by the mean total areas, $S''_{w.i.}$, $S''_{c.i.}$, and $S''_{c.s.}$, of the magnetic circuits in the respective materials:

$$\mathfrak{B}''_{w.i.} = \frac{\Phi'}{S''_{w.i.}}; \quad \mathfrak{B}''_{c.i.} = \frac{\Phi'}{S''_{c.i.}}; \quad \mathfrak{B}''_{c.s.} = \frac{\Phi'}{S''_{c.s.}} \quad \ldots .(239)$$

If two or more portions of the frame are made of the same material, but of different cross-sections, either each of these portions has to be treated separately, or their average specific magnetizing force must be found, exactly as in the case of different materials. Thus, if the path in a certain material, for some mechanical or constructive reason, has different sectional areas, S_1, S_2, S_3, . . . in various portions, l_1, l_2, l_3, ... of its length, the total magnetizing force required for that material is:

$$at = m' \times (l_1 + l_2 + l_3 + \ldots)$$
$$= m_1 \times l_1 + m_2 \times l_2 + m_3 \times l_3 + \ldots, \quad \ldots\ldots\ldots\ldots (240)$$

where

$$m' = \frac{m_1 \times l_1 + m_2 \times l_2 + m_3 \times l_3 + \ldots}{l_1 + l_2 + l_3 + \ldots}$$

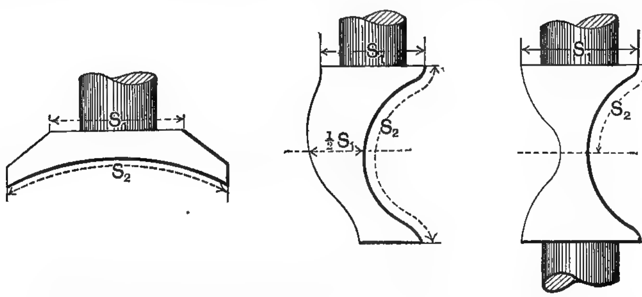
m' = mean specific magnetizing force;
$m_1, m_2, m_3, \ldots$ = specific magnetizing forces in the different sections of the magnetic circuit.

Since the resultant area, S, corresponding to the mean specific magnetizing force m_1 in the above formula, is different from the arithmetical as well as from the geometrical mean of the single areas, $S_1, S_2, S_3, \ldots$, the use of either of these mean areas would lead to an incorrect result. And since, furthermore, the specific magnetizing force does not vary in direct proportion with the flux-density, it would also be a fallacy to use the specific magnetizing force corresponding to the average density computed from the separate densities,

$$m_1 = \frac{\Phi'}{S_1}, \quad m_2 = \frac{\Phi'}{S_2}, \quad m_3 = \frac{\Phi'}{S_3},$$

by taking their arithmetical or even their geometrical mean.

Similarly, if the area presented to the lines of force is not uniform throughout the length of their path in a certain por-



Figs. 261 to 263.—Polepieces with Gradually Increasing Sectional Area.

tion of the field frame, neither the mean area nor the mean flux density, obtained by averaging the respective extreme values, should be used, but the specific magnetizing force for the maximum and minimum cross-section must be calculated, and

either their arithmetical or their geometrical mean be taken, according to whether the variation in cross-section is a continual or a non-gradual one. The sectional areas of the *magnet cores* and of the *yokes* in most cases are *uniform* throughout their respective lengths; but the cross-section of the *polepieces*, owing to their peculiar shape, usually varies along their length. In case the sectional area gradually rises from a minimum near the core to a maximum at the poleface, as in Figs. 261, 262, and 263, the average specific magnetizing force is the *arithmetical mean* of the extreme values:

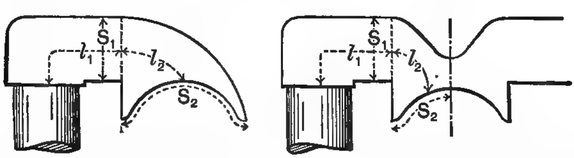
$$m_p = \tfrac{1}{2}(m_1 + m_2), \quad \text{..........} (241)$$

in which $m_p =$ average specific magnetizing force, of polepieces, in ampere-turns per inch;

$m_1 =$ specific magnetizing force, corresponding to cross-section S_1 of polepieces near magnet-core (or to twice the minimum cross-section at center of polepiece, Fig. 262), in square inches;

$m_2 =$ specific magnetizing force, corresponding to pole face area S_2 (maximum cross-section of polepiece), in square inches.

If, on the other hand, the area is partly uniform and partly varying, as in the polepieces shown in Figs. 264 and 265, the *geometrical mean* of the specific magnetizing force of the uniform portion and of the average specific magnetizing force of the varying portion has to be taken as follows:



Figs. 264 and 265.—Polepieces with Partly Uniform and Partly Varying Cross Section.

$$m_p = \frac{m_1 l_1 + \frac{1}{2}(m_1 + m_2) \times l_2}{l_1 + l_2}, \quad \text{..........} (242)$$

where m_1 = specific magnetizing force corresponding to area S_1 of minimum cross-section, in sq. in.;

m_2 = specific magnetizing force corresponding to pole face area S_2 (maximum cross-section), in sq.in.;

l_1 = length of uniform cross-section, in inches;

l_2 = mean length of varying cross-section, in inches.

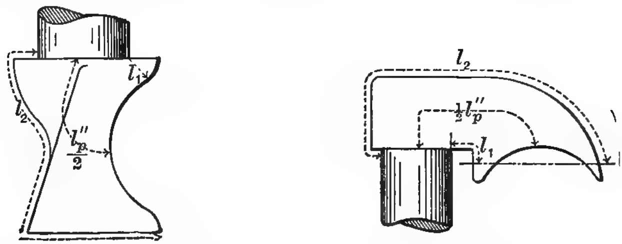
In formulæ (241) and (242) it is assumed that the smallest section of the polepiece is entered by the entire total flux, Φ', and that the pole area only carries the useful flux, Φ. Neither

Figs. 266 and 267.—Mean Length of Magnetic Circuit in Cores and Yokes.

of these assumptions is quite correct (the number of lines entering the polepieces being smaller than Φ', and the flux at the pole face somewhat larger than Φ) but, since their deviations from the facts are in opposite directions, they practically cancel in forming the arithmetical mean of the respective specific magneting forces and give a result as accurate as can be desired.

The *mean length* of the magnetic circuit in portions of the field frame having a homogenous cross-section (*cores* and *yokes*) is measured along the centre line of the frame, as shown in Fig. 266, if there is but one magnetic circuit through that portion. In case of two or more magnetic circuits passing in parallel through any part of the frame, as in Fig. 267, that part is to be correspondingly subdivided parallel with the direction of the magnetic lines, and the mean length of the magnetic circuit, then, is given by the centre-line through a part of the frame thus apportioned to one circuit. In the illustration, Fig. 267, two parallel circuits being shown through each core, the average line of force passes through the cores at a distance from their edges equal to one-quarter of their breadth.

In parts with varying cross-section (*polepieces*) the mean length of the magnetic circuit, depending altogether upon their shape, can only be estimated, one approximation being



Figs. 268 and 269.—Mean Length of Magnetic Circuit in Polepieces.

the arithmetical mean between the shortest and the longest line of force (see Figs. 268 and 269):

$$\tfrac{1}{2}\, l''_p = \tfrac{1}{2}\,(l_1 + l_2), \text{ or } l''_p = l_1 + l_2\,; \quad \ldots . (243)$$

l''_p = mean length of magnetic circuit in polepieces, in inches;

l_1 = shortest line of force in polepiece;

l_2 = longest line of force in polepiece.

93. Ampere-Turns for Compensating Armature Reactions.

The armature current in magnetizing the armature core exerts a double influence upon the magnetic circuit: (1) a direct weakening influence upon the magnetic field, due to the lines of force set up by the armature winding, and (2) an indirect, secondary influence by shifting the magnetic field in the direction of the rotation, thereby causing greater magnetic density to take place in those portions of the polepieces at which the armature leaves the pole than in those at which it enters.

The *direct* effect of the armature current on the field has been studied experimentally by Professor Harris J. Ryan,[1] who, in his paper presented to the American Institute of Electrical Engineers, on September 22, 1891, has shown that the arma-

[1] Harris J. Ryan, *Trans. A. I. E. E.*, vol. viii. p. 451 (September 22, 1891); *Electrical Engineer*, vol. xii. pp. 377, 404 (September 30 and October 7, 1891); *Electrical World*, vol. xvii. p. 252 (October 3, 1891).

ture ampere-turns acting directly against the field ampere turns can be expressed by:

$$at'_r = \frac{N_a \times I'}{2n'_p} \times \frac{k_{13} \times \alpha}{180^\circ}, \quad \ldots\ldots\ldots (244)$$

where at'_r = counter magnetizing force of armature per magnetic circuit, in ampere-turns, to be compensated for by additional windings on field frame;

N_a = total number of turns on armature,

$N_a = N_c$, for ring armatures,

$N_a = \frac{1}{2} N_c$, for drum-wound armatures,

(N_c = total number of armature conductors);

I' = total current-capacity of dynamo, in amperes;

$2n'_p$ = number of armature circuits electrically connected in parallel;

$\frac{N_a \times I'}{2n'_p}$ = total number of ampere-turns on armature;

$k_{13} \times \alpha$ = angle of brush lead.

For *smooth-core* armatures the angle of lead is approximately equal to half the angle between two adjacent pole corners, the constant k_{13} being very nearly = 1, and is accurately expressed by formula (245).

Since the angle of field-distortion depends upon the relative magnitudes of the armature- and field magnetomotive forces acting at right angles to each other, the direction of the distorted field is the resultant of both forces; that is, the diagonal of a rectangle, having the two determining M. M. Fs. as its sides, as shown in Fig. 270, in which *OA* represents the direction and magnitude of the direct M. M. F., and *OB* that of the counter M. M. F. The angle of lead can, consequently, be mathematically expressed by:

$$\tan \alpha = \frac{OB}{OA} = \frac{\text{Total Armature Ampere-Turns}}{\text{Total Field Ampere-Turns}}$$

$$= \frac{\frac{N_a \times I'}{2n'_p}}{n_z \times AT} = \frac{N_a \times I'}{2n'_p \times n_z \times AT},$$

or

$$\alpha = \operatorname{arc} \tan \left(\frac{N_a \times I'}{2n'_p \times n_z \times AT} \right), \quad \ldots\ldots (245)$$

the total number of field ampere-turns being the product of the number, AT, of ampere-turns per magnetic circuit, and of the number, n_z, of magnetic circuits.

In *toothed* and *perforated* machines the weakening effect of the armature magnetomotive force is checked by the presence of iron surrounding the conductors, this checking influence being the stronger the greater the ratio of tooth section to field density, that is, the smaller the tooth density. In a minor degree, the coefficient of brush lead depends upon the ratio of gap length to pitch of slots, and upon the peripheral velocity of the armature. In the following Table XC. averages for this coefficient, k_{13}, for toothed and perforated armatures are given, the upper limits referring to small gaps and high-speed armatures, and the smaller values to large air gaps and to armatures of low circumferential velocity:

TABLE XC.—COEFFICIENT OF BRUSH LEAD IN TOOTHED AND PERFORATED ARMATURES.

MAXIMUM DENSITY OF MAGNETIC LINES IN ARMATURE PROJECTIONS AT NORMAL LOAD.		COEFFICIENT OF BRUSH LEAD, k_{13}		
		Toothed Armatures.		Perforated Armatures.
Lines per sq. in.	Lines per sq. cm.	Straight Teeth.	Projecting Teeth.	
50,000	7,750	0.30 to 0.45	0.25 to 0.35	0.20 to 0.30
75,000	11,600	.35 " .60	.30 " .45	.25 " .35
100,000	15,500	.40 " .80	.40 " .60	.30 " .45
125,000	19,400	.50 " .90	.50 " .70	.40 " .60
150,000	23,250	.70 " 1.00	.60 " .90	.50 " .80

Formula (244) is directly applicable to *single magnetic circuit bipolar* and to the *radial types* of *multipolar* machines. In *double circuit bipolar types*, and for *axial multipolar* dynamos, however, in which the number of magnetic circuits per pole space is twice that of the former machines, respectively, the result of (244), *must be divided by 2 in order to furnish the direct counter magnetizing force per magnetic circuit.*

As to the second, *indirect*, influence of the armature field, the density in the Sections I, I, Fig. 270, of the polepieces, on account of the distortion of the field caused by the action of the armature current, is greater, in the Sections II, II, how-

ever, smaller than the average density obtained by dividing the total flux by the sectional area of the polepieces.

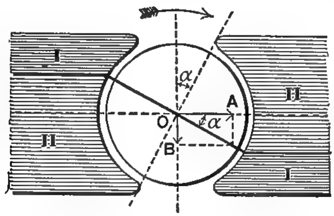


Fig. 270.—Influence of Armature Current upon Magnetic Density in Polepieces.

If the average density in the polepieces, $\Phi' \div S_p$, is denoted by $\mathfrak{B}''_p$, then the distorted densities are

$$\left.\begin{aligned} &\text{in Sections I, I:} \quad \mathfrak{B}''_{pI} = \mathfrak{B}''_p \times \frac{1}{1 - \sin\alpha} \\ &\text{in Sections II, II:} \quad \mathfrak{B}''_{pII} = \mathfrak{B}''_p \times \frac{1}{1 + \sin\alpha} \end{aligned}\right\} \quad \ldots\ldots(246)$$

The magnetizing force required to produce these densities in the polepieces can be found from

$$at'_p = l''_p \times \frac{m''_{pI} + m''_{pII}}{2}, \qquad \ldots\ldots(247)$$

where l''_p = length of magnetic circuit in the polepieces, in inches;

m''_{pI}, m''_{pII} = specific magnetizing forces per inch length for the densities $\mathfrak{B}''_{pI}$ and $\mathfrak{B}''_{pII}$, respectively, formula (245), for the material used; to be taken from Table LXXXVI., or from Fig. 256.

But since the magnetic force necessary to produce the original average density is

$$at_p = l''_p \times m''_p,$$

which is smaller than at'_p, we can find the number of ampere-turns by which the field magnetomotive force is diminished on account of this indirect effect of the armature current, by subtracting at_p from (247). Doing this, we obtain:

$$\begin{aligned} at''_1 &= at'_p - at_p \\ &= l''_p \times \left(\frac{m''_{pI} + m''_{pII}}{2} - m''_p\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(248) \end{aligned}$$

The total weakening effect of the armature winding per magnetic circuit can therefore be found by combining (244) and (248), thus:

$$at_r = at'_r + at''_r$$

$$= \frac{N_a \times I'}{2n'_p} \times \frac{k_{13} \times \alpha}{180^\circ} + l''_p \left(\frac{m''_{pI} + m''_{pII}}{2} - m''_p \right). \quad (249)$$

This is the total number of ampere-turns by the amount of which the exciting power of each magnetic circuit is to be increased in order to compensate for the reactions of the armature current upon the field.

Making the above calculation of at_r, by formula (249), for a great number of practical machines, the author has found that with sufficient accuracy the complex formula (249) can be replaced by the simple equation:

$$at_r = k_{14} \times \frac{N_a \times I'}{2n'_p} \times \frac{k_{13} \times \alpha}{180^\circ}, \quad \ldots\ldots (250)$$

if the following values of the coefficient k_{14} are employed:

TABLE XCI.—COEFFICIENT OF ARMATURE REACTION FOR VARIOUS DENSITIES AND DIFFERENT MATERIALS.

AVERAGE MAGNETIC DENSITY IN POLEPIECES.						Coefficient of Armature Reaction
Wrought Iron and Cast Steel.		Mitis Iron.		Cast Iron.		
Lines per sq. in. $\mathfrak{B}''_p$	Lines per sq. cm. $\mathfrak{B}_p$	Lines per sq. in. $\mathfrak{B}''_p$	Lines per sq. cm. $\mathfrak{B}_p$	Lines per sq. in. $\mathfrak{B}''_p$	Lines per sq. cm. $\mathfrak{B}_p$	k_{14}
80,000*	12,400*	....	....	....	....	1.25
90,000	13,950	70,000*	10,850*	....	....	1.30
100,000	15,500	80,000	12,400	....	....	1.40
105,000	16,250	90,000	13,950	20,000*	3,100*	1.50
110,000	17,000	100,000	15,500	30,000	4,650	1.60
115,000	17,800	105,000	16,250	40,000	6,200	1.70
120,000	18,600	110,000	17,000	50,000	7,750	1.80
....	....	115,000	17,800	55,000	8,500	1.90
....	....	120,000	18,600	60,000	9,300	2.00
....	....	....	....	65,000	10,100	2.10
....	....	....	....	70,000	10,850	2.25

* Or less.

94. Grouping of Magnetic Circuits in Various Types of Dynamos.

In applying formula (227), § 89, for the total magnetizing power of a dynamo, the number of the magnetic circuits and their grouping has to be taken into account.

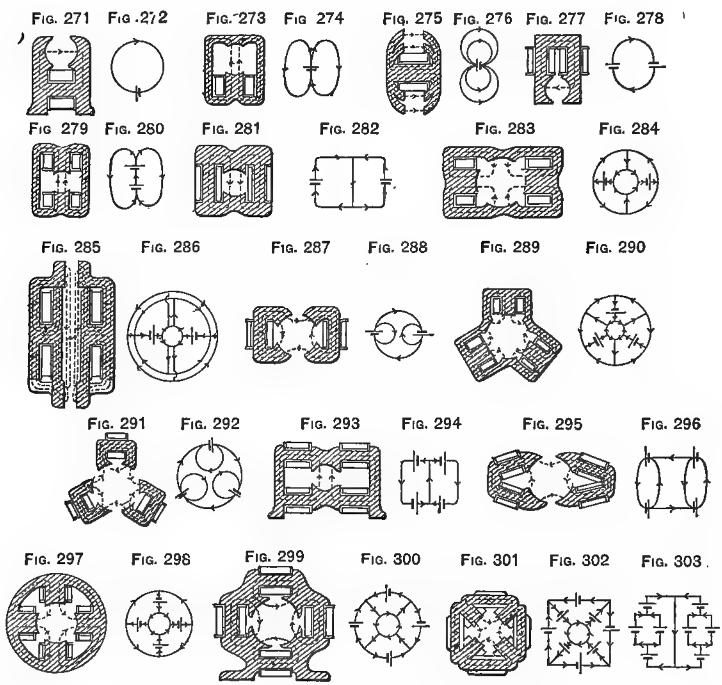
Considering each magnet, or each group of magnet coils wound upon the same core, as a separate source of M. M. F., we can classify the various types of dynamos according to the number of sources of magnetomotive force, and according to their grouping, as follows:

(1) One source of M. M. F., single circuit, Figs. 271 and 272;
(2) One source of M. M. F., double circuit, Figs. 273 and 274;
(3) One source of M. M. F., multiple circuit, Figs. 275 and 276;
(4) Two sources of M. M. F. in series, single circuit, Figs. 277 and 278;
(5) Two sources of M. M. F. in series, double circuit, Figs. 279 and 280;
(6) Two sources of M. M. F. in parallel, single circuit, Figs. 281 and 282;
(7) Two sources of M. M. F. in parallel, double circuit, Figs. 283 and 284;
(8) Two sources of M. M. F. in parallel, multiple circuit, Figs. 285 and 286;
(9) Two sources of M. M. F. in series, each also supplying a shunt circuit, Figs. 287 and 288;
(10) Three or more sources of M. M. F. in parallel, multiple circuit, Figs. 289 and 290;
(11) Three or more sources of M. M. F. in series, each having a shunt circuit, Figs. 291 and 292;
(12) Four sources of M. M. F., two in series and two in parallel, single circuit, Figs. 293 and 294;
(13) Four sources of M. M. F. in series, each pair also supplying a shunt circuit, Figs. 295 and 296;
(14) Four or more sources of M. M. F. in series, parallel, two sources in series in each circuit, Figs. 297 and 298;

(15) Four or more sources of M. M. F., all in parallel, multiple circuit, Figs. 299 and 300;

(16) Four or more sources of M. M. F., arranged in one or more parallel branches in each of which two separate sources are placed in series with a group of two in parallel, Figs. 301, 302 and 303.

In order to facilitate the conception of the grouping of the magnetomotive forces, to the following illustrations of the 16 classes enumerated above the electrical analogues of corresponding grouping of E. M. Fs. have been added:



Figs. 271 to 303.—Grouping of Magnetic Circuits in Various Types of Dynamos, and Electrical Analogues.

Of the *first* class, Fig. 271, which has but one magnetic circuit, are the bipolar single magnet types shown in Figs. 191, 192, 193 and 194.

In the *second* class, Fig. 273, there are two parallel magnetic

circuits, each containing the entire magnetizing force; of this class are the single magnet bipolar iron-clad types, illustrated in Figs. 204, 205 and 206.

The *third* class, Fig. 275, has as many magnetic circuits as there are pairs of magnet poles, and each circuit contains the entire magnetizing force; the single magnet multipolar types, Figs. 214 and 215, belong to this class.

The *fourth* class, Fig. 277, has but one magnetic circuit, and is represented by the single horseshoe types, Figs. 187 to 190, and by the bipolar double magnet types, Figs. 195, 196 and 198.

In the *fifth* class, Fig. 279, there are two magnetic circuits, each of which contains both magnets; the bipolar double magnet iron-clad types shown in Figs. 203 and 207 belong to this class.

The *sixth* class, Fig. 281, has also two magnetic circuits, but each one contains only one magnet; of this class are the bipolar double magnet types illustrated in Figs. 197, 199 and 200.

In the *seventh* class, Fig. 283, there are four parallel magnetic circuits, each of which contains but one magnet; the fourpolar iron-clad types, Figs. 218, 219 and 220, and the fourpolar double magnet type, Fig. 223, belong to this class.

In the *eighth* class, Fig. 285, the number of magnetic circuits is equal to twice the number of poles, opposite pole faces of same polarity considered as one pole, and each circuit contains one magnet; this class is represented by the double magnet multipolar type, Fig. 216.

The *ninth* class, Fig. 287, has three magnetic circuits, two of which contain one magnet each, while the third one contains both the magnets.

In the *tenth* class, Fig. 289, there are as many magnetic circuits as there are poles, two circuits passing through each magnet; the multipolar iron-clad type, Fig. 217, is of this class.

The *eleventh* class, Fig. 291, has one more circuit than there are pairs of poles, one circuit containing all the magnets, while all the rest contain but one magnet each; to this class belongs the multiple horseshoe type, Fig. 222.

In the *twelfth* class, Fig. 293, there are two magnetic cir-

cuits, each containing two magnets; it is represented by the double horseshoe types, Figs. 201 and 202.

Class *thirteen*, Fig. 295, has three circuits, two containing two magnets each and the third one all four magnets; to this class belongs the fourpolar horseshoe type, Fig. 221.

In class *fourteen*, Fig. 297, there are as many circuits as there are poles, each circuit containing two magnetomotive forces in series; this class of grouping is common to the radial multipolar types, Figs. 208 and 209, and to the axial multipolar type, Fig. 212.

In class *fifteen*, Fig. 299, the number of magnetic circuits is equal to the number of poles, and each circuit contains one magnet; the tangential multipolar types, Figs. 210 and 211, and the quadruple magnet type, Fig. 224, are the varieties of this class.

The *sixteenth* class, Fig. 301, finally, has as many magnetic circuits as there are poles, and each circuit contains three magnets; the raditangent multipolar type which is shown in Fig. 213, represents this class of grouping.

Similarly as the total joint E. M. F. of a number of sources of electricity connected in series-parallel is the sum of the E. M. Fs. placed in series in any of the parallel branches, so the total M. M. F. of a dynamo-electric machine is the sum of the M. M. Fs. in series in any of its magnetic circuits.

In considering, therefore, one single magnetic circuit for the computation of the magnetizing forces required for overcoming the reluctances of the air gaps, armature core and field frame, the result obtained by formula (227) represents the exciting force to be distributed over all the magnets in that one circuit, and, consequently, the same magnetizing force is to be applied to all the remaining magnetic circuits, provided all circuits contain the same number of magnets.

In case of several magnetic circuits with a different number of M. M. Fs. in series, as in classes 9, 11 and 13, which have one long circuit containing all the magnets, and several small circuits with but one or two magnets, respectively, the total M. M. F. of the machine is either the sum of all M. M. Fs. or the joint M. M. F. of one of the small circuits, according to whether the long, or one of the small circuits has been used in calculating the magnetizing force required for the machine.

PART VI.

CALCULATION OF MAGNET WINDING.

CHAPTER XIX.

COIL WINDING CALCULATIONS.

95. General Formulæ for Coil Windings.

In practice it frequently is desired to make calculations concerning the arrangement, etc., of magnet windings, without reference to their magnetizing forces; and it is for the simplification of such computations that the following general formulæ for coil windings are compiled.

In Fig. 304 a coil bobbin is represented, and the following symbols are used:

D_m = external diameter of coil space, in inches;
d_m = internal diameter of coil space, in inches;
l_m = length of coil space, in inches;
h_m = height of coil space, in inches;
V_m = volume of coil space, in cubic inches;
δ_m = diameter of magnet wire, bare, in inches;
δ'_m = diameter of magnet wire, insulated, in inches;
N_m = total number of convolutions;
L_m = total length of magnet wire, in feet;
wt_m = total weight of magnet wire, in pounds;
r_m = resistance of magnet wire, in ohms;
ρ_m = resistivity of magnet wire, in ohms per foot;
$\lambda_m = \frac{1}{\rho_m}$ = specific length of magnet wire, in feet per ohm;
λ'_m = specific length of magnet wire, in feet per pound.

The *total number of convolutions* filling a coil space of given dimensions with a wire of given size is:

$$N_m = \frac{l_m}{\delta'_m} \times \frac{h_m}{\delta'_m} = \frac{l_m \times h_m}{{\delta'_m}^2}. \quad \ldots\ldots(251)$$

The *diameter* (insulated) of wire required to fill a bobbin of given size with a given number of convolutions, irrespective of resistance, is:

$$\delta'_m = \sqrt{\frac{l_m \times h_m}{N_m}}. \quad \ldots\ldots\ldots (252)$$

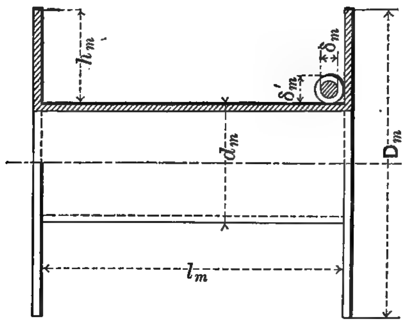


Fig. 304.—Dimensions of Coil Bobbin.

The *total length* of wire of given diameter which can be wound on a bobbin of given dimensions, is:

$$L_m = \frac{1}{12} \times \frac{D_m + d_m}{2} \times \pi \times \frac{l_m \times h_m}{\delta'^2_m}$$

$$= .131 \times (D_m + d_m) \times \frac{l_m \times h_m}{\delta'^2_m}, \quad \ldots\ldots\ldots\ldots\ldots (253)$$

or:

$$L_m = \frac{1}{12} \times (d_m + h_m) \times \pi \times \frac{l_m \times h_m}{\delta'^2_m}$$

$$= .262 \times (d_m + h_m) \times \frac{l_m \times h_m}{\delta'^2_m}. \quad \ldots\ldots\ldots\ldots\ldots (254).$$

From (254) the dimensions of a coil can be calculated on which a certain length of wire of given diameter can be wound.

If the internal diameter and the height of the coil space are given, the length can be computed from:

$$l_m = \frac{12 \times L_m \times \delta'^2_m}{(d_m + h_m) \times \pi \times h_m}$$

$$= 3.82 \times \frac{L_m \times \delta'^2_m}{(d_m + h_m) \times h_m}. \quad \ldots\ldots (255)$$

When length and winding depth are known, the internal coil diameter is found from:

$$d_m = \frac{12 \times L_m \times \delta'^2_m - h_m^2 \times l_m \times \pi}{l_m \times h_m \times \pi}$$
$$= 3.82 \times \frac{L_m \times \delta'^2_m}{l_m \times h_m} - h_m. \quad \ldots\ldots\ldots\ldots(256)$$

And if the length and the internal diameter of the coil are given, its winding depth can be obtained from:

$$h_m = \sqrt{\frac{12 \times L_m \times \delta'^2_m}{l_m \times \pi} + \frac{d_m^2}{4}} - \frac{d_m}{2}. \quad \ldots(257)$$

Since the length of a wire is the product of its weight and its specific length, formula (257) can be modified so as to give the height of the coil space required to wind a given weight of wire of given diameter upon a core of known dimensions:

$$h_m = \sqrt{\frac{wt_m \times \delta'^2_m \times \lambda'_m}{l_m \times \pi} + \frac{d_m^2}{4}} - \frac{d_m}{2}. \quad (258)$$

The *resistance* which a coil of wire of known resistivity will offer when wound on a given bobbin, is:

$$r_m = L_m \times \rho_m \text{ ohms}, \quad \ldots\ldots\ldots\ldots(259)$$

or, by inserting the value of L_m from (254):

$$r_m = .262 \times \rho_m \times (d_m + h_m) \times \frac{l_m \times h_m}{\delta'^2_m}. \quad (260)$$

The *diameter* of a wire which shall fill a bobbin of given dimensions and offer a given resistance can be found as follows: The coil space occupied by L_m feet of wire having a diameter (insulated) of δ'_m inch, is:

$$V_m = 12\, L_m \times \delta'^2_m, \quad \ldots\ldots\ldots\ldots(261)$$

while, expressed in the dimensions of the bobbin, the same volume is:

$$V_m = \frac{l_m \times \pi}{4}(D_m^2 - d_m^2)$$
$$= \frac{l_m \times \pi}{4}\left[(d_m + 2\, h_m)^2 - d_m^2\right]$$
$$= l_m \times \pi (h_m\, d_m + h_m^2)\,; \quad \ldots\ldots\ldots\ldots(262)$$

consequently we have:

$$\delta'^2_m = \frac{l_m \times \pi}{12\ L_m} \times (h_m\ d_m + h_m^2)$$

$$= .262 \times \frac{l_m}{L_m} \times (h_m\ d_m + h_m^2). \quad \ldots(263)$$

In order to replace in this formula the unknown length L_m, by the given resistance, r_m, we express the latter by the dimensions of the wire. The resistance of a copper rod of one square inch area and one inch length being .000000675 ohm at 15.5° Cent. (= 60° Fahr.), that of a copper wire of length L_m feet and diameter δ_m inch, is:

$$r_m = .000000675 \times \frac{12\ L_m}{\delta_m^2 \times \frac{\pi}{4}}. \quad \ldots\ldots.(264)$$

Inserting the value of L_m from (264) into (263), we have:

$$\delta'^2_m = \frac{.000000675 \times 12 \times l_m \times \frac{\pi}{12}}{r_m \times \delta_m^2 \times \frac{\pi}{4}}$$

$$= .0000027 \times \frac{l_m}{r_m \times \delta_m^2} \times (h_m\ d_m + h_m^2),$$

whence:

$$\delta_m \times \delta'_m = \sqrt{.0000027 \times \frac{l_m}{r_m} \times (h_m\ d_m + h_m^2)}$$

$$= .00164 \sqrt{\frac{l_m}{r_m} \times (h_m\ d_m + h_m^2)}. \quad \ldots\ldots\ldots(265)$$

In practical cases the diameters δ_m and δ'_m are usually very little different from each other, so that with sufficient accuracy we can put $\delta_m \times \delta'_m = \delta_m^2$, and, consequently, from (265):

$$\delta_m = \sqrt{.00164 \times \sqrt{\frac{l_m}{r_m} \times (h_m\ d_m + h_m^2)}}$$

$$= .04 \times \sqrt[4]{\frac{l_m}{r_m} (h_m\ d_m + h_m^2)}. \quad \ldots\ldots\ldots.(266)$$

In order to allow for the difference between δ_m and δ'_m, for irregularities in the winding, and, eventually, for insulation between the layers, the next *smaller* gauge wire should be taken. The selection of the next *smaller* size of wire is particularly necessary in case of winding an existing spool, since a length of this, effecting the given resistance, will not quite fill the coil space, while for the next larger size of wire the spool would not hold enough of the wire to produce the required resistance.

96. Size of Wire Producing Given Magnetizing Force at Given Voltage Between Field Terminals. Current Density in Magnet Wire.

Designating the E. M. F. between the terminals of the magnet winding by E_m, the current flowing through the field circuit by I_m, and the magnet resistance by r_m, Ohm's Law furnishes the relation:

$$I_m = \frac{E_m}{r_m}.$$

Multiplying both sides of this equation by the length of the wire, L_m, we obtain:

$$L_m \times I_m = L_m \times \frac{E_m}{r_m}. \quad \ldots\ldots\ldots(267)$$

But, the total length, in feet, of the magnet wire is the product of the number of convolutions, N_m, and of the mean length of one turn, l_t, in feet, thus:

$$L_m = N_m \times l_t;$$

furthermore, by (259):

$$L_m = \frac{r_m}{\rho_m};$$

hence,

$$N_m \times I_m \times l_t = \frac{E_m}{\rho_m},$$

from which follows the specific length of the wire which gives the desired magnetizing force at the specified voltage between the field terminals, viz.:

$$\lambda_m = \frac{1}{\rho_m} = \frac{(N_m \times I_m) \times l_t}{E_m} = \frac{AT \times l_t}{E_m}, \quad \ldots(268)$$

that is to say, the specific length (feet per ohm) of the required wire is the quotient of the number of ampere-feet by the given voltage. In taking from the gauge table the standard size of wire whose feet per ohm are nearest to the figure found by (268), the size of magnet wire that furnishes the required number of ampere-turns, AT, at the given potential difference E_m, can directly be determined by the length, l_t, of the mean turn. Since the value of $\frac{1}{\rho}$, from (268), gives the specific length of the *hot* magnet wire, the next *smaller* gauge number should be chosen.

Inserting (259) into (267) we obtain:

$$I_m \times L_m = \frac{E_m}{\rho_m}, \text{ or } \frac{1}{I_m} = \frac{L_m \times \rho_m}{E_m},$$

which, multiplied by the sectional area of the wire, δ_m^2, gives the cross-section of the wire per unit of current strength, that is, its current density:

$$i_m = \frac{\delta_m^2}{I_m} = \frac{L_m}{E_m} \times (\delta_m^2 \times \rho_m).$$

The product $(\delta_m^2 \times \rho_m)$ of the sectional area (in circular mils) of a wire into its specific resistance (in ohms per foot) gives the resistance of one mil-foot of wire of the given material, *i. e.*, in the case of copper:

$\delta_m^2 \times \rho_m = 12$ ohms, at about 60° Cent. (= 140° Fahr.);

consequently the current density in the magnet wire:

$$i_m = \frac{\delta_m^2}{I_m} = 12 \times \frac{L_m}{E_m}. \quad \ldots\ldots\ldots\ldots(269)$$

For a given machine, therefore, (E_m being constant) the current density only depends upon the total length of the wire, and is independent of its size.

Formula (269) may be used to determine the practical limits of L_m, by limiting the value of the current density, i_m. From (269) follows directly:

$$L_m = \frac{i_m}{12} \times E_m; \quad \ldots\ldots\ldots\ldots(270)$$

and since the practical value of i_m ranges between 240 and 1800 Circular Mils per ampere (= 5300 to 700 amperes per square inch, or 8.2 to 1.1 amperes per square millimetre), we have:

$$\left.\begin{array}{ll} \text{for } (i_m)_{min} = 240 \text{ C.M. per amp.} \ldots (L_m)_{min} = 20\, E_m; \\ \text{for } (i_m)_{max} = 1,800 \text{ C.M. per amp.} \ldots (L_m)_{max} = 150\, E_m. \end{array}\right\} \quad (271)$$

The total length of magnet wire, in feet, should therefore be from 20 to 150 times the difference of potential between the field terminals, in volts.

From (269) we can also derive the following formula, which gives, directly in Circular Mils, the area of a magnet wire effecting a certain magnetizing force at given potential between field terminals, viz.:

$$\delta_m{}^2 = \frac{12 \times L_m \times I_m}{E_m} = \frac{12 \times N_m \times I_m \times l_t}{E_m}$$

$$= \frac{(N_m \times I_m) \times (12 \times l_t)}{E_m} = \frac{AT \times l_T}{E_m}, \quad \ldots\ldots\ldots (272)$$

that is to say, the area of the requisite magnet wire is the quotient of the number of ampere-inches (l_T being the length of the mean turn in inches) to be wound upon the cores, by the potential between the field terminals. Assuming an approximate value for the mean turn, l_T, the minimal limit of which is always given by the circumference of the magnet core, a preliminary value of δ_m can be quickly determined, and from this the value of l_T is easily adjusted if necessary; a recalculation with the correct value of l_T will then furnish the final value of the area of the magnet wire.

A set of valuable curves which show the relation between ampere-turns and mean length of turn, and between current and total length of wire, respectively, and which can be used for graphically obtaining the results of formula (272) as well as other data concerning the magnet winding, has been devised by Harrison H. Wood.[1]

Formula (272) is only approximate, being based upon the assumption that the final temperature of the magnet coils is

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[1] "Curves for Winding Magnets," by H. H. Wood; *Electrical World*, vol. xxv. pp. 503 and 529 (April 27 and May 4, 1895).

about 60° C. If the actual rise above 15.5° C. of the magnet temperature is denoted by θ_m, the accurate formula for the area of the wire would be:

$$\delta_m^2 = \frac{10.5 \times AT \times l_t}{E_m} \times (1 + .004 \times \theta_m), \quad (273)$$

10.5 being the resistance, in ohms, of a copper wire, one foot long and one mil in diameter, at a temperature of 15.5° C. (= 60° F.).

From (273) a very useful formula for the weight of the magnet winding can be derived. By Ohm's Law we have:

$$E_m = I_m \times r_m = \frac{P_m}{E_m} \times r_m,$$

in which P_m = energy absorbed in magnet winding, in watts (see § 98); consequently:

$$\delta_m^2 = \frac{10.5 \times AT \times l_t \times E_m}{P_m \times r_m} \times (1 + .004 \times \theta_m).$$

But the resistance of the magnet winding can be expressed by:

$$r_m = \frac{10.5 \times 10^6}{12} \times (1 + .004 \times \theta_m) \times \frac{wt'_m}{k_{15}} \times \left(\frac{1}{\delta_m^2}\right)^2,$$

where wt'_m = weight of magnet winding, including insulation, in pounds;

k_{15} = specific weight of magnet winding, in pounds per cubic inch, depending upon size of wire and thickness of insulation; see Table XCII.

Hence:

$$\delta_m^2 \times \frac{12 \times 10^{-6} \times k_{15} \times AT \times l_t \times E_m \times \delta_m^4}{P_m \times wt'_m},$$

or,

$$wt'_m = \frac{12 \times 10^{-6} \times k_{15} \times AT \times l_t \times E_m \times \delta_m^2}{P_m},$$

and since by (272) we have, approximately:

$$E_m \times \delta_m^2 = 12 \times AT \times l_t,$$

we finally obtain:

$$wt'_{\mathrm{m}} = 144 \times 10^{-6} \times k_{15} \times \frac{(AT' \times l_{\mathrm{t}})^2}{P_{\mathrm{m}}},$$

or,

$$wt'_{\mathrm{m}} = k_{16} \times \frac{\left(\frac{AT \times l_{\mathrm{t}}}{1000}\right)^2}{P_{\mathrm{m}}}. \quad \ldots\ldots\ldots\ldots\ldots (274)$$

The constant k_{16} is $= 144 \times k_{15}$, and can be taken from the following Table XCII.:

TABLE XCII.—SPECIFIC WEIGHTS OF COPPER WIRE COILS, SINGLE COTTON INSULATION.

GAUGE OF WIRE.		Diameter, Bare, Inch.	Insulation S. C. C. Inch.	Total Space Occupied by Wire. Cir. Mils.	Area of Copper. Square Mils.	Ratio of Copper to Total Volume of Coil.	Specific Weight of Winding. lbs. per cu. inch.	Value of Constant in Formula (274).
B. W. G.	B. & S.	δ_{m}	$\delta'_{\mathrm{m}} - \delta_{\mathrm{m}}$	δ'^2_{m}	$\delta_{\mathrm{m}}^2 \times \frac{\pi}{4}$		k_{15}	k_{16}
(6)	4	.204	.012	46,656	32,685	.702	.225	32.5
(7)	5	.182	.012	37,637	26,016	.69	.221	31.8
8	..	.165	.012	31,329	21,383	.683	.218	31.4
..	6	.162	.010	29,584	20,612	.697	.223	32.2
9	(7)	.148	.010	24,964	17,203	.688	.220	31.7
10	(8)	.134	.010	20,736	14,103	.682	.218	31.4
11	(9)	.120	.010	16,900	11,310	.669	.214	30.8
12	..	.109	.010	14,161	9,331	.66	.211	30.4
..	10	.102	.010	12,544	8,171	.65	.208	30.0
13	..	.095	.010	11,025	7,088	.644	.206	29.7
..	11	.091	.010	10,209	6,504	.637	.204	29.4
(14)	12	.081	.007	7,744	5,153	.665	.213	30.7
15	13	.072	.007	6,241	4.072	.65	.208	30.4
16	(14)	.065	.007	5,184	3,318	.64	.205	29.5
17	(15)	.058	.007	4,225	2,642	.625	.200	28.8
(18)	16	.051	.007	3,364	2,043	.607	.194	27.9
..	17	.045	.005	2,500	1,590	.637	.204	29.5
19	..	.042	.005	2,209	1,385	.627	.201	29.0
..	18	.040	.005	2.025	1,257	.628	.201	29.0
..	19	.036	.005	1,681	1,018	.607	.194	27.9
20	..	.035	.005	1,600	962	.601	.1925	27.7
21	20	.032	.005	1 369	804	.587	.187	27.0
22	21	.028	.005	1,089	616	.546	.175	25.2
23	22	.025	.005	900	491	.565	.181	26.1
24	23	.022	.005	729	380	.521	.167	24.1
25	24	.020	.005	625	314	.503	.161	23.2
26	25	.018	.005	529	254.5	.48	.1535	22.1
27	26	.016	.005	441	201	.457	.146	21.0
28	27	.014	.005	361	154	.428	.137	19.8
29	28	.013	.005	324	133	.41	.131	18.9
30	..	.012	.005	289	113	.391	.125	18.0
..	29	.011	.005	256	95	.371	.119	17.2

From the above Table it is found that for the most usual sizes of magnet wire (No. 6 B. W. G. to No. 20 B. W. G.) the

average value of k_{15} is $= .21$, and that of k_{16} is $= 30$, and therefore approximately:

$$wt'_m = \frac{30 \times \left(\frac{AT \times l_t}{1000}\right)^2}{P_m}, \quad \ldots\ldots\ldots\ldots (275)$$

that is to say:

$$\text{Weight of winding} = \frac{30 \times \left(\frac{\text{Ampere-feet}}{1000}\right)^2}{\text{Watts absorbed by Magnet Winding.}}$$

By means of (275) the weight of wire can be found that supplies a given magnetizing force at a fixed loss of energy in the field winding.

97. Heating of Magnet Coils.

The conditions of heat radiation from an electro-magnet being similar to those of an armature at rest, with polepieces removed, the unit temperature increase of magnet coils can be obtained by extending Table XXXVI., § 35, for the specific increase of armatures, to conform with the above conditions. Plotting for this purpose the temperatures given in the first horizontal row for zero peripheral velocity, as functions of the ratio of pole-area to total radiating surface, and prolonging the temperature curve so obtained until it intersects the zero ordinate, the specific temperature rise $\theta'_m = 75°$ C. ($= 135°$ F.) for 1 watt of energy loss per square inch of radiating surface, is found. The actual temperature increase of any magnet coil can, therefore, be obtained by the formula:

$$\theta_m = \theta'_m \times \frac{P_m}{S_M} = 75° \times \frac{P_m}{S_M}, \quad \ldots\ldots (276)$$

where θ_m = rise of temperature in magnets, in Centigrade degrees;

P_m = energy absorbed in magnet-winding, in watts;

$= I_m^2 \times r_m$; $= \frac{E_m^2}{r_m}$;

$\left\{\begin{array}{l} I_m = \text{current in magnet winding, in amperes;} \\ E_m = \text{E. M. F. between field terminals, in volts;} \\ r_m = \text{resistance of magnet winding, in ohms;} \end{array}\right.$

S_M = radiating surface of magnet coils, in square inches.

The *radiating surface of the magnets* depends upon the shape and size of the cores as well as the upon the arrangement of the field frame, and can be readily deduced geometrically from the dimensions of the coil. If the polepieces, or yokes, completely overlap the end flanges of the magnet coils, air has access to the prismatical surface only, and the radiating surface is—

for *cylindrical* magnets:

$$S_M = D_m \times \pi \times l'_m = (d_m + 2h_m) \times \pi \times l'_m ; \quad (277)$$

for *rectangular* magnets:

$$S_M = 2 \times l'_m \times (l + b + h_m \times \pi); \quad \ldots.(278)$$

and for magnets of *oval* cross-section (rectangle between two semicircles):

$$S_M = 2 \times l'_m \times \left[(l - b) + \left(\frac{b}{2} + h_m\right) \times \pi\right]. \quad ..(279)$$

In case that also one of the end surfaces of each coil is exposed to the air, or that one-half of each coil flange helps the prismatical surface to liberate the heat developed by the field current, the radiating surface becomes:

$$S_{M_1} = S_M + n_m \times l_T \times h_m. \quad \ldots\ldots..(280)$$

If there is a clearance between the magnet coils and the yokes and polepieces such as to make both the entire end surfaces of each magnet coil active in giving off heat, the radiating surface is:

$$S_{M_2} = S_M + 2n_m \times l_T \times h_m. \quad \ldots\ldots..(281)$$

And when, finally, the yokes and polepieces touch the end flanges of the coils, but the latter project over the former so that heat can radiate from the projecting portions, the radiating surface will be:

$$S_{M_3} = S_M + 2n_m \times h_m \times (l_T - b_y). \quad \ldots.(282)$$

In the above formulæ (277) to (282):

S_M = radiating surface of prismatic surface of magnet coil;

S_{M_1} = radiating surface of prismatic surface plus one end surface per coil;

S_{M_2} = radiating surface of prismatic surface plus two end flanges per coil;
S_{M_3} = radiating surface of prismatic surface plus projecting portions of coil flanges;
d_m = diameter of circular core-section;
D_m = external diameter of cylindrical magnet coil;
h_m = height of magnet winding, see Table LXXX., § 83;
l'_m = total length of magnet coils per magnetic circuit;
l = length of rectangular or oval core-section;
b = breadth of rectangular or oval core-section;
l_T = length of mean turn of magnet wire;
b_y = breadth of yoke, or polepiece;
n_m = number of separate magnet coils in each magnetic circuit.

If the surface, S'_M, of the magnet cores is given instead of the radiating surface, S_M, of the coils, the value of θ'_m in (276), instead of being constant at 75° C., ranges between 75° and 4° C. (or 135° and 7° F., respectively), according to the ratio of depth of magnet winding to thickness of core; that is, according to the ratio of radiating surface to core surface. In the following, Table XCIII., the specific temperature rise, θ'_m, is given for *round* magnets, varying in winding depth from .01 to 2 core diameters, and for *rectangular* and *oval* cores ranging in radiating surface from 1.02 to 15 times the surface of the cores.

If, for a given type of machine, the approximate ratio of radiating surface to core surface is known, the calculation of the magnet winding can, by means of Table XCIII., directly be based upon the given surface of the magnet cores.

98. Allowable Energy Dissipation for Given Rise of Temperature in Magnet Winding.

From formula (276), § 97, it is evident that for a given coil the temperature rise depends solely upon the amount of energy consumed, and conversely it follows that by limiting the tem-

TABLE XCIII.—SPECIFIC TEMPERATURE INCREASE IN MAGNET COILS OF VARIOUS PROPORTIONS AT UNIT ENERGY LOSS PER SQUARE INCH OF CORE SURFACE.

Cylindrical Magnets.								Rectangular and Oval Magnets.	
					Increase of Magnet Temperature for Each Watt per Square Inch of Core Surface.				
					Radiating Surface Consisting in				
Ratio of Winding Depth to Core Diameter.	Winding Depth in Parts of Core Diameter.	Ratio of External Coil Surface to Core Surface.	Ratio of Cylindrical Surface Plus One End Surface of Coil to Core Surface.	Ratio of Cylindrical Surfaces and Both End Surface of Coil to Core Surface.	Cylindrical Coil Surface.	Cylindrical Surface Plus One End Surface.	Cylindrical Surface and Both End Flanges.	Ratio of Radiating Surface to Core Surface.	Temperature Increase for Unit Energy Dissipation.
1 . 100	.01	1.02	1.03 to 1.05	1.04 to 1.07	75°	74° to 72°	73° to 71°	1.02	75°
1 : 50	.02	1.04	1.05 " 1.10	1.07 " 1.12	73	72 " 69	71 " 68	1.05	72
1 : 30	.033	1.067	1.07 " 1.15	1.10 " 1.20	71	70 " 65	69 " 63	1.1	69
1 : 20	.05	1.1	1.12 " 1.20	1.15 " 1.30	69	68 " 63	65 " 58	1.25	60
1 : 15	.067	1.133	1.15 " 1.25	1.20 " 1.40	67	65 " 60	63 " 53.5	1.5	50
1 : 12	.083	1.167	1.20 " 1.30	1.25 " 1.45	65	63 " 58	60 " 51.5	1.75	43
1 : 10	.1	1.2	1.25 " 1.40	1.30 " 1.55	63	60 " 53 5	58 " 48	2	37.5
1 : 9	.11	1.22	1.28 " 1.45	1.35 " 1.60	61.5	59 " 51.5	55.5 " 47	2.25	33
1 : 8	.125	1.25	1.30 " 1.50	1.40 " 1.70	60	58 " 50	53.5 " 44	2.5	30
1 : 7	.143	1.286	1.35 " 1.55	1.45 " 1.80	58.5	55.5 " 48.5	51.5 " 41.5	3	25
1 : 6	.167	1.33	1.40 " 1.60	1.50 " 1.90	56.5	53.5 " 39.5	50 " 39 5	3.5	21.5
1 : 5	.2	1.4	1.50 " 1.75	1.60 " 2	53.5	50 " 43	47 " 37.5	4	19
1 : 4	.25	1.5	1.65 " 2	1.80 " 2.5	50	45.5 " 37.5	41.5 " 30	5	15
1 : 3	.33	1.67	1.85 " 2.5	2 " 3	45	40.5 " 30	37.5 " 25	6	12.5
1 : 2	.5	2	2.25 " 3	2.5 " 4	37.5	33 " 19	30 " 19	8	9.5
1 : 1	1	3	3.5 " 5.5	4 " 8	25	21.5 " 13.5	19 " 9.5	10	7.5
3 : 2	1.5	4	5 " 8.5	6 " 12	19	15 " 9	12.5 " 6	12	6
2 : 1	2	5	6.5 " 12	8 " 18	15	11.5 " 6	9.5 " 4	15	5

perature increase of the coil, the maximum of its energy dissipation is also fixed. By transposition of (276) we obtain:

$$P_m = \frac{\theta_m}{75^\circ} \times S_M, \quad \ldots\ldots\ldots\ldots (283)$$

and

$$P_m = \frac{\theta_m}{\theta'_m} \times S'_M; \quad \ldots\ldots\ldots\ldots (284)$$

where P_m = energy dissipation in magnet winding, in watts;
θ_m = temperature increase of magnet coils, in degrees Centigrade;
θ'_m = specific temperature rise of magnet coils, for one watt per square inch of core-surface;
S_M = radiating surface of magnet coils, in square inches; see formulæ (277) to (282);
S'_M = surface of magnet cores, in square inches.

The temperature rise of magnet coils in practice varies between 10° and 50° C., and in exceptional cases reaches 75° C., the latter increase causing, in summer, a final temperature of the magnets of about 100° C., which is the limit of safe heating of coils of insulated wire. For ordinary cases, therefore, the allowable energy dissipation in the field magnets ranges between

$$P_m = \frac{10}{75} \times S_M = .133 S_M$$

and

$$P_m = \frac{50}{75} \times S_M = .667 S_M,$$

that is, between .133 and .667 watt per square inch (= .02 to .10 watt per square centimetre), or radiating surface is to be provided at the rate of from 7½ to 1½ square inches per watt (= 50 to 10 square centimetres per watt). The arithmetical mean of these limits, .4 watt per square inch (= .062 watt per square centimetre), or 2½ square inches (= 16 square centimetres) per watt, is a good practical average for medium-sized machines, and corresponds to a rise of magnet temperature of 30° C. (= 54° F.).

The energy dissipation, P_m, thus being fixed by the temper-

ature increase specified, the working resistance of the magnet winding can be obtained by means of Ohm's Law, thus:

$$r'_m = \frac{E_m}{I_m} = \frac{E_m \times I_m}{I_m^2} = \frac{P_m}{I_m^2}, \quad \ldots\ldots(285)$$

or,

$$r'_m = \frac{E_m}{I_m} = \frac{E_m^2}{I_m \times E_m} = \frac{E_m^2}{P_m}, \quad \ldots\ldots(286)$$

according to whether the intensity of the current flowing through the field circuit, or the E. M. F. between the field terminals, respectively, is given, the former being the case in *series-wound* machines and the latter in *shunt-wound* dynamos. In a series machine the field current is equal to the given current output, $I_m = I$; while in a shunt dynamo the potential between the field terminals is identical with the known E. M. F. output of the machine, $E_m = E$; see § 14, Chapter II.

CHAPTER XX.

SERIES WINDING.

99. Calculation of Series Winding for Given Temperature Increase.

The number of ampere-turns, AT, being found by the formulæ given in Chapter XVIII., and the field current in a series dynamo being equal to the given current output, I, of the machine, the number of series turns, N_{se}, can readily be obtained by dividing the former by the latter:

$$N_{se} = \frac{AT}{I}. \quad \ldots\ldots\ldots\ldots\ldots (287)$$

The number of turns multiplied by the mean length of one convolution, in feet, gives the total length of the series field wire:

$$L_{se} = \frac{N_{se} \times l_T}{12}, \quad \ldots\ldots\ldots\ldots (288)$$

in which the length of the mean turn, in inches, is—
for *cylindrical* magnets:

$$l_T = (d_m + h_m) \times \pi ; \ldots\ldots\ldots\ldots (289)$$

for *rectangular* magnets:

$$l_T = 2 \times (l + b) + h_m \times \pi ; \ldots\ldots\ldots (290)$$

and for *oval* magnets (rectangle between two semicircles):

$$l_T = 2 \times (l - b) + (b + h_m) \times \pi ; \ldots\ldots (291)$$

where d_m = diameter of circular core-section;
l = length of rectangular or oval section;
b = breadth of rectangular or oval section;
h_m = height of magnet winding, from Table LXXX., § 83.

An approximate value for the length of the average turn for *cylindrical* magnets can be obtained from

$$l_T = k_{17} \times d_m, \quad \ldots\ldots\ldots\ldots\ldots(292)$$

where k_{17} = ratio of length of mean turn to core diameter, see Table XCIV.

The ratio k_{17} depends upon the size of the magnet, and ranges as follows:

TABLE XCIV.—LENGTH OF MEAN TURN FOR CYLINDRICAL MAGNETS.

DIAMETER OF MAGNET CORE, d_m INCHES.	HEIGHT OF WINDING SPACE, h_m INCHES.		RATIO OF MEAN TURN TO CORE DIAMETER, $k_{17} = \frac{(d_m + h_m) \times \pi}{d_m}$.	
	Bipolar Types.	Multipolar Types.	Bipolar Types.	Multipolar Types.
1	$\frac{1}{2}$	$\frac{3}{4}$	4.71	5.50
2	$\frac{3}{4}$	$1\frac{1}{4}$	4.32	5.11
3	1	$1\frac{3}{4}$	4.19	4.97
4	$1\frac{1}{4}$	2	4.12	4.71
6	$1\frac{1}{2}$	$2\frac{1}{4}$	3.93	4.32
8	$1\frac{3}{4}$	$2\frac{1}{2}$	3.83	4.12
10	$1\frac{7}{8}$	$2\frac{3}{4}$	3.73	4.01
12	2	3	3.66	3.93
15	$2\frac{1}{8}$	$3\frac{1}{4}$	3.59	3.82
18	$2\frac{1}{4}$	$3\frac{1}{2}$	3.54	3.75
21	$2\frac{3}{8}$	$3\frac{3}{4}$	3 50	3.70
24	$2\frac{1}{2}$	4	3.47	3.67
27	$2\frac{5}{8}$	$4\frac{1}{4}$	3.45	3.64
30	$2\frac{3}{4}$	$4\frac{1}{2}$	3.43	3.62
33	$2\frac{7}{8}$	$4\frac{3}{4}$	3 41	3.60
36	3	5	3.40	3.58

The averages given for the height of the winding space h_m, in Tables LXXX. and XCIV., enable an approximate value of the radiating surface, S_M, to be found by formulæ (277) to (282), respectively, and the latter, together with the specified temperature increase, θ_m, furnishes the allowable energy dissipation, P_{se}, by virtue of equation (283). From formula (285), then, the required series field resistance can be obtained thus:

$$r'_{se} = \frac{P_{se}}{I_{se}^2} = \frac{\theta_m}{75} \times \frac{S_M}{I^2}, \ at \ (\theta_m + 15.5 \ ^\circ\text{C.}), \quad \ldots\ldots(293)$$

or:

$$r_{se} = \frac{\theta_m}{75} \times \frac{S_M}{I^2} \times \frac{1}{1 + .004 \times \theta_m}, \text{ at } 15.5^\circ \text{ C.} \quad ..(294)$$

In dividing (288) by (294), finally, the specific length λ_{se}, in feet per ohm, of the series winding giving a magnetizing force of AT ampere-turns at a rise of the magnet temperature of θ_m degrees Centigrade, is received, viz.:

$$\lambda_{se} = \frac{L_{se}}{r_{se}} = \frac{\frac{AT}{I} \times \frac{l_T}{12}}{\frac{\theta_m}{75} \times \frac{S_M}{I^2} \times \frac{1}{1 + .004 \times \theta_m}}$$

$$= 6.25 \times \frac{AT \times l_T \times I}{\theta_m \times S_M} \times (1 + .004 \times \theta_m), \quad ....(295)$$

where AT = ampere-turns required for field excitation, formula (227);

l_T = length of mean turn, in inches, formulæ (289) to (292), respectively;

I = current output of dynamo, in amperes;

θ_m = specified temperature increase of magnet winding, in Centigrade degrees;

S_M = radiating surface of magnet coils, in square inches, formulæ (277) to (282).

The conclusion of the series field calculation, now, consists in selecting, from the standard wire gauge tables, a wire whose "feet per ohm" most nearly correspond to the result of formula (295). If no one single wire will satisfactorily answer, either n wires of a specific length of

$$\frac{\lambda_{se}}{n}$$

feet per ohm each may be suitable stranded into a cable, or a copper ribbon may be employed for winding the series coil. In the latter case it is desirable to have an expression for the sectional area of the series field conductor. Such an expression is easily obtained by multiplying the specific length, λ_{se}, by the specific resistance, for, since

$$\text{ohms} = \text{specific resistance} \times \frac{\text{feet}}{\text{circular mils}},$$

we have:

circular mils = specific resistance × feet per ohm;

the specific resistance of copper is 10.5 ohms per mil-foot, at 15.5° C., and the area of the series field conductor, consequently, is:

$$\delta_{se}^2 = 10.5 \times \lambda_{se}$$
$$= 65 \times \frac{AT \times l_T \times I}{\theta_m \times S_M} \times (1 + .004 \times \theta_m). \quad \ldots (296)$$

In formulæ (293) to (296), it is supposed that all the magnet coils of the machine are connected in series. If this, however, is not the case, the main current must be divided by the number of parallel series-circuits, in order to obtain the proper value of I for these formulæ.

Having found the size of the conductor, the number of turns, N_{se}, from (287), will render the effective height, h'_m, of the winding space for given total length, l'_m, of coil, by transposition of formula (252), § 95, thus:

$$h'_m = N_{se} \times \frac{(\delta'_{se})^2}{l'_m}. \quad \ldots\ldots\ldots\ldots (297)$$

$(\delta'_{se})^2$ being the area, in square inches, of the square, or rectangle, that contains one *insulated* series field conductor (wire, cable, or ribbon).

If h'_m, from (297), should prove materially different from the average winding depth taken from Table LXXX., the actual values of l_T and S_M should be calculated, and the size of the series field conductor checked by inserting these actual values into formula (295) or (296).

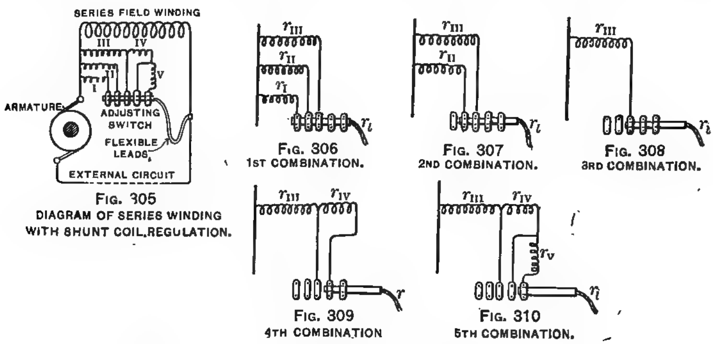
The product of the number of turns by the actual mean length of one convolution will give the actual length, L_{se}, of the series field winding, and from the latter the real resistance and the weight of the winding can be calculated. (See § 102.)

100. Series Winding with Shunt Coil Regulation.

For some purposes it is desired to employ a series dynamo whose voltage can be readily adjusted between given limits. Such adjustment can best be attained by connecting across the terminals of the series field winding a shunt of variable

resistance which is opened if the maximum voltage is desired, while its least resistance is offered for obtaining the minimum voltage of the machine, intermediate grades of resistance being used for regulating the voltage of the machine between the maximum and the minimum limits. The series winding in this case is calculated, according to § 99, for the maximum voltage of the machine, and then the various combinations of the shunt-coils are so figured as to produce the desired regulation, and to safely carry the proper amount of current.

As an example let us take five coils arranged, as shown in Fig. 305, so as to permit of being grouped, by moving the



Figs. 305 to 310.—Shunt Coil Combinations.

slider of the adjusting switch into five different combinations, illustrated by Figs. 306 to 310.

The resistances and sectional areas of these coils are to be so determined as to enable 60, $66\frac{2}{3}$, 75, $83\frac{1}{3}$, and 90 per cent. of the maximum voltage to be taken from the machine. It is evident that in this case 40, $33\frac{1}{3}$, 25, $16\frac{2}{3}$, and 10 per cent., respectively, of the maximum field current will have to be absorbed by the respective combinations of the shunt coils, and their resistance, therefore, must be:

Resistance first combination

$$= \frac{60}{40} \times \text{resistance of series field} = 1.5\, r'_{se}.$$

Resistance second combination

$$= \frac{66\frac{2}{3}}{33\frac{1}{3}} \times \text{resistance of series field} = 2\, r'_{se}.$$

Resistance third combination

$$= \frac{75}{25} \times \text{resistance of series field} = 3\, r'_{se}.$$

Resistance fourth combination

$$= \frac{83\frac{1}{3}}{16\frac{2}{3}} \times \text{resistance of series field} = 5\, r'_{se}.$$

Resistance fifth combination

$$= \frac{90}{10} \times \text{resistance of series field} = 9\, r'_{se}.$$

For the arrangement shown in Figs. 305 to 310, the first combination consists of coils I, II, and III, in parallel, the second combination of coils II and III in parallel, in the third combination only coil III is in circuit, in the fourth combination coils III and IV are in series, and the fifth combination has coils III, IV, and V in series. In all combinations there are, furthermore, the flexible leads carrying the current from the field terminal to the adjusting slider; these are in series to the group of coils in every case, and their resistance, r_1, consequently is to be deducted from the resistance of the combination in order to obtain the resistance of the group of coils alone. Expressing the resistances of the various groups by the resistances of the single shunt-coils, we therefore obtain:

First group:

$$\frac{1}{\frac{1}{r_{\text{I}}} + \frac{1}{r_{\text{II}}} + \frac{1}{r_{\text{III}}}} = 1.5\, r'_{se} - r_1; \quad \ldots\ldots(298)$$

Second group:

$$\frac{1}{\frac{1}{r_{\text{II}}} + \frac{1}{r_{\text{III}}}} = 2\, r'_{se} - r_1; \quad \ldots\ldots..(299)$$

Third group:

$$r_{\text{III}} = 3\, r'_{se} - r_1; \quad \ldots\ldots..(300)$$

Fourth group:

$$r_{\text{III}} + r_{\text{IV}} = 5\, r'_{se} - r_1; \quad \ldots\ldots..(301)$$

Fifth group:

$$r_{\text{III}} + r_{\text{IV}} + r_{\text{V}} = 9\, r'_{se} - r_1. \quad \ldots\ldots.(302)$$

From this set of equations the resistances of the separate shunt-coils can be derived as follows:

Inserting (299) into (298):

$$\frac{1}{\frac{1}{r_{\text{I}}} + \frac{1}{2\,r'_{se} - r_1}} = 1.5\,r'_{se} - r_1,$$

whence:

$$r_{\text{I}} = \frac{(2\,r'_{se} - r_1) \times (1.5\,r'_{se} - r_1)}{(2\,r'_{se} - r_1) - (1.5\,r'_{se} - r_1)}$$

$$= \frac{3\,r'^{2}_{se} - 3.5\,r'_{se}\,r_1 + r_1^2}{.5\,r'_{se}} = 6\,r'_{se} - 7\,r_1 + \frac{2\,r_1^2}{r'_{se}}.$$

The resistance of the leads being very small, r_1^2 can be neglected, hence the resistance of coil I:

$$r_{\text{I}} = 6\,r'_{se} - 7\,r_1. \quad \ldots\ldots\ldots\ldots (303)$$

(300) into (299) gives:

$$\frac{1}{\frac{1}{r_{\text{II}}} + \frac{1}{3\,r'_{se} - r_1}} = 2\,r'_{se} - r_1,$$

or:

$$r_{\text{II}} = \frac{(3\,r'_{se} - r_1) \times (2\,r'_{se} - r_1)}{(3\,r'_{se} - r_1) - (2\,r'_{se} - r_1)}$$

$$= \frac{6\,r'^{2}_{se} - 5\,r'_{se}\,r_1 + r_1^2}{r'_{se}} = 6\,r'_{se} - 5\,r_1 + \frac{r_1^2}{r'_{se}}.$$

Neglecting again r_1^2, the resistance of coil II is obtained:

$$r_{\text{II}} = 6\,r'_{se} - 5\,r_1. \quad \ldots\ldots\ldots\ldots (304)$$

From (300) we have, directly:

$$r_{\text{III}} = 3\,r'_{se} - r_1. \quad \ldots\ldots\ldots\ldots (305)$$

By subtracting (300) from (301):

$$r_{\text{IV}} = 2\,r'_{se}. \quad \ldots\ldots\ldots\ldots (306)$$

By subtracting (301) from (302):

$$r_{\text{V}} = 4\,r'_{se}. \quad \ldots\ldots\ldots\ldots (307)$$

In the above formulæ, r'_{se} is the resistance of the series field, hot, at maximum E. M. F. output of machine; and r_1 the resistance of the current-leads at the temperature of the

room. The resistance r_1 is determined by finding the length and the sectional area of the leads, the former being dependent upon the distance of the adjusting switch from the field terminal, and the latter upon the maximum current to be carried, which in the present case is 40 per cent. of the current output of the machine.

The currents flowing through the shunt coils in the various combinations can be obtained by the well-known law of the divided circuit, by virtue of which the relative strengths of the currents in the different branches are directly proportional to their conductances, or in inverse proportion to their resistances.

The first combination consists in three parallel branches having the resistances r_{I}, r_{II}, and r_{III}, respectively, and carries a total current of $.4\,I$ amperes, hence the currents in the branches:

$$I_{\mathrm{I}} = \frac{r_{\mathrm{II}}\, r_{\mathrm{III}}}{r_{\mathrm{II}}\, r_{\mathrm{III}} + r_{\mathrm{I}}\, r_{\mathrm{III}} + r_{\mathrm{I}}\, r_{\mathrm{II}}} \times .4\,I,$$

$$I_{\mathrm{II}} = \frac{r_{\mathrm{I}}\, r_{\mathrm{III}}}{r_{\mathrm{II}}\, r_{\mathrm{III}} + r_{\mathrm{I}}\, r_{\mathrm{III}} + r_{\mathrm{I}}\, r_{\mathrm{II}}} \times .4\,I,$$

and

$$I_{\mathrm{III}} = \frac{r_{\mathrm{I}}\, r_{\mathrm{II}}}{r_{\mathrm{II}}\, r_{\mathrm{III}} + r_{\mathrm{I}}\, r_{\mathrm{III}} + r_{\mathrm{I}}\, r_{\mathrm{II}}} \times .4\,I.$$

Inserting into these equations the values of the resistances from (303) to (307), respectively, we obtain:

$$I_{\mathrm{I}} = \frac{(6r'_{se} - 5r_1)(3r'_{se} - r_1)}{(6r'_{se} - 5r_1)(3r'_{se} - r_1) + (6r'_{se} - 7r_1)(3r'_{se} - r_1) + (6r'_{se} - 7r_1)(6r'_{se} - 5r_1)} \times .4\,I$$

$$= \frac{18r'^2_{se} - 21r'_{se}\, r_1 + 5r_1^2}{72r'^2_{se} - 121r'_{se}\, r_1 + 47r_1^2} \times .4\,I = \frac{1}{4} \times .4\,I = .1\,I,$$

$$I_{\mathrm{II}} = \frac{18r'^2_{se} - 28r'_{se}\, r_1 + 7r_1^2}{72r'^2_{se} - 121r'_{se}\, r_1 + 47r_1^2} \times .4\,I = \frac{1}{4} \times .4\,I = .1\,I,$$

and

$$I_{\mathrm{III}} = \frac{36r'^2_{se} - 72r'_{se}\, r_1 + 35r_1^2}{72r'^2_{se} - 121r'_{se}\, r_1 + 47r_1^2} \times .4\,I = \frac{1}{2} \times .4\,I = .2\,I.$$

In the second combination there are but two parallel

branches, having the resistances r_{II} and r_{III}, and the total current carried is $.333\ I$ amperes; therefore:

$$I_{II} = \frac{r_{III}}{r_{II} + r_{III}} \times .333\ I = \frac{3r'_{se} - r_1}{9r'_{se} - 6\ r_1} \times .333\ I$$

$$= \frac{1}{3} \times .333\ I = .111\ I,$$

and

$$I_{III} = \frac{r_{II}}{r_{II} + r_{III}} \times .333\ I = \frac{6r'_{se} - 5r_1}{9r'_{se} - 6r_1} \times .333\ I$$

$$= \frac{2}{3} \times .333\ I = .222\ I.$$

The third, fourth, and fifth combinations are simple circuits only, the current through the coils therefore is identical with the total current flowing through the combination, viz.: $.25\ I$, $.167\ I$ and $.1\ I$ amperes respectively; the first named current, consequently, flows through coil III when in the third combination, the second current through coils III and IV, when in the fourth combination, and the last figure given is the current intensity in coils III, IV, and V, when in the fifth combination. Taking the maximum value for the current flowing in each coil, the following must be their current capacities:

Coil I and V: $I_I = I_V = .1\ I = \frac{I}{10}$, ...(**308**)

" II: $I_{II} = .111\ I = \frac{I}{9}$, ...(**309**)

" III: $I_{III} = .25\ I = \frac{I}{4}$, ...(**310**)

" IV: $I_{IV} = .167\ I = \frac{I}{6}$, ...(**311**)

By allowing 1000 circular mils per ampere current intensity, the proper size of wire for the different shunt coils can then readily be determined from formulæ (308) to (311).

The preceding formulæ (298) to (311) of course only apply to the special arrangement and to the particular regulation selected as an example, but can easily be modified for any given case [see formulæ (457) to (466), § 134], the method of their derivation being thoroughly explained.

CHAPTER XXI.

SHUNT WINDING.

101. Calculation of Shunt Winding for Given Temperature Increase.

The problem here to be considered is to find the data for a shunt winding which will furnish the requisite magnetizing force at the specified rise of the magnet temperature, and with a given regulating resistance in series to the shunt coils, at normal output.

The shunt regulating resistance, or as it is sometimes called, the *extra-resistance*, admits of an adjustment of the resistance of the shunt-circuit within the limits prescribed, thereby inversely varying the strength of the shunt-current, which in turn correspondingly influences the magnetizing force and, ultimately, regulates the E. M. F. of the dynamo. In cutting out this regulating resistance, the maximum E. M. F. at the given speed is obtained while the minimum E. M. F. obtainable is limited by the total resistance of the regulating coil. By specifying the percentage of extra-resistance in circuit at normal load, and the total resistance of the coil, any desired range may be obtained; see § 103.

Designating the given percentage of extra-resistance by r_x, the total energy absorbed in the shunt-circuit, consisting of magnet winding and regulating coil, can be expressed by:

$$P'_{sh} = P_{sh} \times \left(1 + \frac{r_x}{100}\right) = \frac{\theta_m}{75} \times S_M \times \left(1 + \frac{r_x}{100}\right), \quad \textbf{(312)}$$

where

$$P_{sh} = \frac{\theta_m}{75} S_M = \text{energy absorbed in the magnet winding alone.}$$

The potential between the field terminals of a shunt dynamo being equal to the E. M. F. output, E, of the machine, the current flowing through the shunt-circuit is:

$$I_{sh} = \frac{P'_{sh}}{E}, \quad \ldots\ldots\ldots\ldots\ldots\textbf{(313)}$$

and the number of shunt turns, therefore:

$$N_{sh} = \frac{AT}{I_{sh}} = \frac{AT \times E}{P'_{sh}}, \quad \ldots\ldots\ldots (314)$$

By means of formulæ (289) to (292), which apply equally well to shunt as to series windings, the approximate mean length of one turn is found, and the latter multiplied by the number of turns gives the total length of the shunt wire:

$$L_{sh} = \frac{N_{sh} \times l_T}{12} = \frac{AT \times E \times \frac{l_T}{12}}{\frac{\theta_m}{75} \times S_M \times \left(1 + \frac{r_x}{100}\right)}. \quad \ldots (315)$$

By Ohm's Law we next find the total resistance of the shunt-circuit at normal load, viz.:

$$r''_{sh} = \frac{E}{I_{sh}} = \frac{E^2}{P'_{sh}} = \frac{E^2}{\frac{\theta_m}{75} \times S_M \times \left(1 + \frac{r_x}{100}\right)}. \quad \ldots (316)$$

This contains the r_x per cent. of extra resistance; in order to obtain the resistance of the shunt winding alone, r''_{sh} must be decreased in the ratio of

$$1 : \left(1 + \frac{r_x}{100}\right),$$

and we have:

$$r'_{sh} = r''_{sh} \times \frac{1}{1 + \frac{r_x}{100}}$$

$$= \frac{E^2}{\frac{\theta_m}{75} \times S_M \times \left(1 + \frac{r_x}{100}\right)} \times \frac{1}{1 + \frac{r_x}{100}}, \quad \ldots\ldots\ldots (317)$$

which is the resistance of the magnet winding when hot, at a temperature of $(15.5 + \theta_m)$ degrees Centigrade; the magnet resistance, cold, at 15.5° C., consequently, is:

$$r_{sh} = r'_{sh} \times \frac{1}{1 + .004 \times \theta_m}.$$

$$= \frac{E^2}{\frac{\theta_m}{75} \times S_M \times \left(1 + \frac{r_x}{100}\right)} \times \frac{1}{1 + \frac{r_x}{100}} \times \frac{1}{1 + .004 \times \theta_m}. \quad (318)$$

The division of (315) by (318), then, furnishes the specific length of the required shunt wire:

$$\lambda_{sh} = \frac{L_{sh}}{r_{sh}}$$

$$= \frac{AT}{E} \times \frac{l_T}{12} \times \left(1 + \frac{r_x}{100}\right) \times (1 + .004 \times \theta_m). \quad ..(319)$$

The size of the shunt wire can then be readily taken from a wire-gauge table; if a wire of exactly this specific length is not a standard gauge wire, either a length of L_{sh} feet of the next larger size is to be taken, and the difference in resistance made up by additional extra-resistance, or such quantities of the next larger and the next smaller gauge wires are to be combined as to produce the required resistance, r_{sh}, by the correct length, L_{sh}. To fulfill the latter condition, the geometrical mean of the specific lengths of the two sizes must correspond to the result obtained by formula (319); thus, if λ'_{sh} is the specific length of one size of wire and λ''_{sh} that of the other, such proportions, L'_{sh} and L''_{sh}, of the total length, $L_{sh} = L'_{sh} + L''_{sh}$, are to be taken of each that:

$$\frac{\lambda'_{sh} \times L'_{sh} + \lambda''_{sh} \times L''_{sh}}{L'_{sh} + L''_{sh}} = \lambda_{sh}, \quad ......(320)$$

Since in this equation every term contains a length as a factor, any length, for instance L'_{sh}, may be unity, and we have:

$$\frac{\lambda'_{sh} + \lambda''_{sh}\left(\frac{L''_{sh}}{L'_{sh}}\right)}{1 + \left(\frac{L''_{sh}}{L'_{sh}}\right)} = \lambda_{sh},$$

from which follows the proper ratio of the lengths of the two wires:

$$\left(\frac{L''_{sh}}{L'_{sh}}\right) = \frac{\lambda_{sh} - \lambda'_{sh}}{\lambda''_{sh} - \lambda_{sh}}, \quad ....(321)$$

If the two sizes are combined by their weight, the specific weights, in pound per ohm, are to be substituted for the specific lengths in the above equations.

The sectional area of the shunt wire which exactly furnishes the requisite magnetizing power at the given voltage between field terminals, with the prescribed percentage of extra-resistance in circuit, and at the specified increase of magnet temperature, may be directly obtained by the formula:

$$\delta_{sh}^2 = 10.5 \times \lambda_{sh}$$

$$= .875 \times \frac{AT}{E} \times l_T \times \left(1 + \frac{r_x}{100}\right) \times (1 + .004 \times \theta_m). \quad \textbf{(322)}$$

In the above formulæ, E is the E. M. F. supplying the shunt coils of *one* magnetic circuit, and is identical with the terminal voltage of the machine, if the shunt coils are grouped in as many parallel rows as there are magnetic circuits. But if the number of parallel shunt-circuits differs from the number of magnetic circuits, the output E. M. F. of the machine, in order to obtain the proper value of E for calculating the shunt winding, must be multiplied by the ratio of the former to the latter number.

The size, or sizes, of the shunt wire thus being decided upon, by means of formulæ (319) or (322), the actual value of h_m, and therefrom the real length of the mean turn is to be computed (see formulæ (289) to (291)), and to be inserted into formulæ (319), or (322), respectively.

In case of two sizes of wire being used, the winding depth can with sufficient accuracy in most cases be found by means of the formula:

$$h_m = \frac{N_{sh}}{l_m} \times \frac{(\delta'_{sh})^2 + \left(\frac{L''_{sh}}{L'_{sh}}\right) \times (\delta''_{sh})^2}{1 + \left(\frac{L''_{sh}}{L'_{sh}}\right)}, \quad \ldots \textbf{(323)}$$

which, however, on account of the fact that the mean length of a turn of the one size of wire is different from that of the other, and that, therefore, the ratio of the number of turns of the two sizes differs from the ratio of their length, is only approximately correct and gives accurate results in case of

comparatively long and shallow coils only. For short and deep coils, Fig. 311, the heights of the winding spaces for the

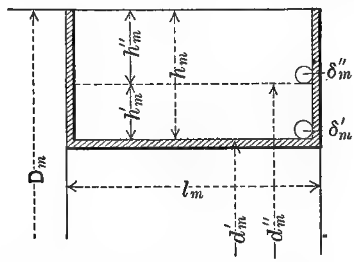


Fig. 311.—Dimensions of Shunt Coil.

two sizes are to be separately determined by formula (257), thus:

$$h_m = h'_m + h''_m = \sqrt{\frac{12\, L'_{sh} \times \delta'^{\,2}_{sh}}{l_m \times \pi} + \frac{d'^{\,2}_m}{4}}$$

$$+ \sqrt{\frac{12\, L''_{sh} \times \delta''^{\,2}_{sh}}{l_m \times \pi} + \frac{d''^{\,2}_m}{4}} - \frac{d'_m + d''_m}{2}, \quad \text{..(324)}$$

where h_m = total height of winding space, in inches;

h'_m and h''_m = partial heights of winding space occupied by wire of first and second size, respectively;

δ'_{sh} and δ''_{sh} = diameters of insulated shunt wires, inch;

L'_{sh} and L''_{sh} = total length of the two sizes of wire, in feet;

d'_m = internal diameter of coil formed by first size of wire (= core-diameter plus insulation), in inches;

$d''_m = d'_m + 2\, h'_m = D_m - 2\, h''_m$;

= external diameter of coil of first size of wire, identical with internal diameter of coil of second size, in inches;

l_m = length of coil, in inches; if there is more than one coil in each magnetic circuit, l_m is the total length of all the coils in one circuit.

102. Computation of Resistance and Weight of Magnet Winding.

To complete the calculation of the magnet winding, it is necessary to find its actual resistance and its weight.

For the *resistance* in ohms we have:

$$\begin{aligned} r_{\mathrm{m}} &= \text{Length} \times \text{ohms per foot} \\ &= L_{\mathrm{m}} \times \left(10.5 \times \frac{1}{\delta_{\mathrm{m}}^{2}}\right) = 10.5 \times \frac{L_{\mathrm{m}}}{\delta_{\mathrm{m}}^{2}} \\ &= 10.5 \times \frac{N_{\mathrm{m}} \times l_{\mathrm{t}}}{\delta_{\mathrm{m}}^{2}} = .875 \times \frac{N_{\mathrm{m}} \times l_{\mathrm{T}}}{\delta_{\mathrm{m}}^{2}}, \qquad \text{..(325)} \end{aligned}$$

in which N_{m} = total number of series, or shunt turns on magnets, formula (287) or (314);

l_{t} = mean length of one turn, in feet;

l_{T} = mean length of one turn, in inches, formulæ (289) to (292);

L_{m} = total length of magnet wire, in feet, formula (288) or (315);

δ_{m}^{2} = sectional area of magnet wire, in circular mils, formula (296) or (322).

The *weight* of the magnet winding is the product of the total length of wire by its specific weight, in pounds per foot, the former being the product of number of turns and mean length of turn, and the latter being obtainable from the wire-gauge table. In order to express the weight of the winding by the data known from previous calculations, we proceed as follows:

$$\begin{aligned} \text{Weight} &= \text{length in feet} \times \text{weight per foot} \\ &= \text{constant} \times \text{length} \times \text{specific length}, \end{aligned}$$

in which:

$$\begin{aligned} \text{Constant} &= \frac{\text{specific weight}}{\text{specific length}} = \frac{\text{pounds per foot}}{\text{feet per ohm}} \\ &= \frac{\text{ohms} \times \text{pounds}}{(\text{feet})^2} \end{aligned}$$

$$= \frac{\left(\text{ohms per mil-ft.} \times \dfrac{\text{feet}}{\text{cir. mils}}\right) \times (\text{lbs. p. cu. in.} \times \text{in.} \times \text{sq. ins.})}{(\text{feet})^2},$$

and particularly for *copper:*

Constant

$$= \frac{\left(10.5 \times \dfrac{\text{feet}}{\text{cir. mils}}\right) \times \left(.316 \times \text{feet} \times 12 \times \dfrac{\text{cir. mils} \times \frac{\pi}{4}}{1{,}000{,}000}\right)}{(\text{feet})^2}$$

$$= \frac{10.5 \times .316 \times 12 \times \frac{\pi}{4}}{1{,}000{,}000} \times \frac{(\text{feet})^2 \times \text{cir. mils}}{(\text{feet})^2 \times \text{cir. mils}} = 31.3 \times 10^{-6}.$$

The desired formula for the bare weight, in pounds, of any magnet winding, therefore, is:

$$wt_m = 31.3 \times 10^{-6} \times N_m \times l_t \times \lambda_m, \quad \ldots\ldots(326)$$

where N_m = total number of series, or shunt turns on magnets;

l_t = mean length of turn, in feet;

λ_m = specific length of magnet wire, in feet per ohm, formula (295) or (319).

Writing (326) in the form

$$wt_m = 31.3 \times 10^{-6} \times \frac{(\text{feet})^2}{\text{ohms}},$$

and multiplying both numerator and denominator by the square of the current flowing in the magnet wire, we obtain:

$$wt_m = 31.3 \times 10^{-6} \times \frac{\text{amp.}^2 \times \text{feet}^2}{\text{amp.}^2 \times \text{ohms}},$$

or:

$$wt_m = \frac{31.3 \times \left(\dfrac{\text{ampere-feet}}{1000}\right)^2}{\text{watts}}, \quad \ldots\ldots(327)$$

which agrees, substantially, with formula (275), § 96. The denominator of equation (327), since the specific length of the magnet wire in (326) is given at 15.5° C., represents the energy lost in the magnets at that temperature, that is, the actual energy consumption, at the final temperature $(15.5 + \theta_m)$, of the magnet winding, divided by $(1 + .004 \times \theta_m)$; hence the weight of bare magnet wire necessary to produce a given mag-

netizing force, AT, at a specified rise, θ_m, of the magnet temperature:

$$wt_m = 31.3 \times \frac{\left(\frac{AT \times l_t}{1000}\right)^2}{\frac{\theta_m}{75} \times S_M} \times (1 + .004 \times \theta_m), \quad \textbf{(328)}$$

in which AT = number of ampere-turns required;
l_t = mean length of one turn, in feet;
θ_m = specified rise of temperature, in Centigrade degrees;
S_M = radiating surface of magnets, in square inches.

In case of a compound winding, (328) will give the weights of the series and shunt wires, respectively, if AT is replaced by AT_{se} and AT_{sh}, and if the energies consumed by each of the two windings individually are substituted for the total energy loss in the magnets.

By transformation, the above formula (328) can be employed to calculate the temperature increase θ_m, caused in exciting a magnetizing force of AT ampere-turns by a given weight, wt_m pounds, of bare wire filling a coil of known radiating surface, S_M square inches. Solving (328) for θ_m, we obtain:

$$\theta_m = \frac{\left[31.3 \times \left(\frac{AT \times l_t}{1000}\right)^2 \times \frac{75}{S_M}\right]}{wt_m - .004 \times \left[31.3 \times \left(\frac{AT \times l_t}{1000}\right)^2 \times \frac{75}{S_M}\right]}. \quad \textbf{(329)}$$

The weight of copper contained in a coil of given dimensions is:

$$wt_m = l_T \times l'_m \times h_m \times .21, \quad \ldots\ldots\ldots \textbf{(330)}$$

where l_T = mean length of one turn, in inches;
l'_m = length of coil, in inches;
h_m = height of winding space, in inches;
.21 = average specific weight, in pounds per cubic inch, of insulated copper wire, see Table XCII., § 96.

103. Calculation of Shunt Field Regulator.

The voltage of a shunt-wound machine is regulated by means of a variable rheostat inserted into the shunt-circuit.

The total resistance of this shunt regulator must be the sum of the resistances that are to be cut out of, and added to, the shunt-circuit in order to effect, respectively, an increase and a decrease of the exciting current sufficient to cause the normal E. M. F. to rise and fall to the desired limits. The amount of regulating resistance required to produce a given maximum or minimum E. M. F. is obtained, in per cent. of the magnet resistance, by determining the additional ampere-turns needed for maximum voltage, or the difference between the magnetizing forces for normal and for minimum voltage respectively, for, the magnetic flux, and with it the magnetic densities in the various portions of the magnetic circuit, must be varied in direct proportion with the E. M. F. to be generated.

If the dynamo is to be regulated between a maximum E. M. F., E'_{max}, and a minimum E. M. F., E'_{min}, the magnetizing forces required for the resulting maximum and minimum flux are found as follows:

The exciting power required for the air gaps varies directly with the field density, hence the maximum magnetizing force, by (228):

$$at'_{g} = .3133 \times \left(\mathcal{H}'' \times \frac{E'_{\text{max}}}{E'}\right) \times l''_{g}$$

and the minimum magnetizing force:

$$at''_{g} = .3133 \times \left(\mathcal{H}'' \times \frac{E_{\text{min}}}{E'}\right) \times l''_{g}.$$

The values of l''_{g} in these formulæ may differ from each other, and also from that for normal voltage, owing to the fact that the product of field density and conductor velocity may have increased or decreased sufficiently to influence the constant k_{13} in formula (166). For each value of $\mathcal{H}''$, therefore, Table LXVI., § 64, must be consulted.

For the iron portions of the magnetic circuit the specific magnetizing forces for the new densities are to be found from Table LXXXVIII., § 88, and to be multiplied by the length of the path in the frame; thus, for maximum voltage:

$$at'_{m} = m''_{\text{max}} \times l''_{m},$$

m''_{max} corresponding to a density of $\mathcal{B}''_{m} \times \frac{E'_{\text{max}}}{E'}$,

and for minimum voltage:

$$at''_{\text{m}} = m''_{\text{min}} \times l''_{\text{m}},$$

m''_{min} corresponding to a density of $\mathfrak{B}''_{\text{m}} \times \frac{E'_{\text{min}}}{E'}$.

The magnetizing force required to compensate the armature reactions, finally, is affected by the change of density in the polepieces, the latter determining the constant k_{15} in formula (250); in calculating the compensating ampere-turns for the maximum voltage, the value of k_{15} from Table XCI. is to be taken for a density of

$$\mathfrak{B}''_{\text{p}} \times \frac{E'_{\text{max}}}{E'},$$

and in case of the minimum voltage, for a density of

$$\mathfrak{B}''_{\text{p}} \times \frac{E'_{\text{min}}}{E'}$$

lines per square inch.

Having determined the maximum and minimum magnetizing forces for the various portions of the circuit, their respective sums are the excitations, AT_{max} and AT_{min}, needed for the maximum and minimum voltage. The number of turns being constant, the magnetizing force is varied by proportionally adjusting the exciting current, and this in turn is effected by inversely altering the resistance of the field circuit. The excitation for maximum voltage is

$$\frac{AT_{\text{max}}}{AT}$$

times that for normal load, hence the corresponding minimum shunt resistance, that is, the resistance of the magnet winding alone, must be

$$\frac{AT}{AT_{\text{max}}}$$

times the normal resistance of the shunt-circuit, or, the extra-resistance in circuit at normal load is:

$$r_x = 100 \times \frac{AT_{\text{max}} - AT}{AT}$$

per cent. of the magnet resistance. The magnetizing force for minimum voltage, similarly being

$$\frac{AT_{\min}}{AT}$$

times that for normal output, the maximum shunt resistance is

$$\frac{AT}{AT_{\min}}$$

times the normal, or, regulating resistance amounting to

$$100 \times \frac{AT - AT_{\min}}{AT}$$

per cent. of the normal resistance, which is

$$\left(100 \times \frac{AT - AT_{\min}}{AT}\right) \times \frac{AT_{\max}}{AT}$$

per cent. of the magnet resistance, is to be added to the normal shunt resistance in order to reduce the E. M. F. to the required limit. Expressing the sum of these percentages in terms of the magnet resistance, we obtain the total resistance of the shunt regulator:

$$r_r = \left(\frac{AT_{\max} - AT}{AT} + \frac{AT_{\max}}{AT} \times \frac{AT - AT_{\min}}{AT}\right) \times r'_{sh}. \quad (331)$$

This resistance is to be divided into a number of subdivisions, or "steps," said number to be greater the finer the degree of regulation desired. Since the shunt-current decreases with the number of steps included into the circuit, material can be saved by winding the coils last in circuit with finer wires than the first ones. At the maximum voltage the shunt-current, by virtue of Ohm's Law, is:

$$(I_{sh})_{\max} = \frac{E_{\max}}{r'_{sh}}, \quad \ldots\ldots\ldots\ldots (332)$$

and at minimum voltage we have:

$$(I_{sh})_{\min} = \frac{E_{\min}}{r'_{sh} + r_r}, \quad \ldots\ldots\ldots\ldots (333)$$

the current capacity of any coil of the regulator, therefore, can with sufficient accuracy be determined by proper interpolation

between the values obtained by formula (332) and (333). Thus, the current passing through the shunt-circuit when n_x coils of the regulator are contained in the same, is found:

$$(I_{sh})_x = (I_{sh})_{max} - n_x \times \frac{(I_{sh})_{max} - (I_{sh})_{min}}{n_r}, \quad (334)$$

where n_r is the total number of the coils, or steps, of the regulator. From (334) we obtain by transposition:

$$n_x = \frac{(I_{sh})_{max} - (I_{sh})_x}{(I_{sh})_{max} - (I_{sh})_{min}} \times n_r, \quad \ldots\ldots\ldots (335)$$

the latter formula giving the number of coils which must be added to the magnet winding in order to cause any given current, $(I_{sh})_x$, to flow through the shunt-circuit.

CHAPTER XXII.

COMPOUND WINDING.

104. Determination of Number of Shunt and Series Ampere-Turns.

Since in a compound dynamo the series winding is to supply the excitation necessary to produce a potential equal to that lost by armature and series field resistance, and by armature reaction, the number of shunt ampere-turns for a compound-wound machine is the magnetizing force needed on open circuit, and the number of series ampere-turns required for perfect regulation is the difference between the excitation needed for normal load and that on open circuit. The proper number of shunt and series ampere-turns can, therefore, be computed as follows:

The useful flux required on open circuit is that number of lines of force which will produce the output E. M. F., E, of the dynamo, viz.:

$$\Phi_o = \frac{6 \times n'_p \times E \times 10^9}{N_c \times N};$$

hence the ampere-turns needed to overcome, *on open circuit*, the reluctances of air gaps, armature core, and magnet frame, respectively, are:

$$at_{g_o} = .3133 \times \frac{\Phi_o}{S_g} \times l''_g,$$

$$at_{a_o} = m''_{a_o} \times l''_a,$$

and $at_{m_o} = m''_{w.i._o} \times l''_{w.i.} + m''_{c.i._o} \times l''_{c.i} + m''_{c.s._o} \times l''_{c.s.}$,

in which m''_{a_o}, $m''_{w.i._o}$, $m''_{c.i._o}$ and $m''_{c.s._o}$ are the specific magnetizing forces corresponding to densities $\frac{\Phi_o}{S''_a}$, $\frac{\lambda_o \Phi_o}{S''_{w.i.}}$, $\frac{\lambda_o \Phi_o}{S''_{c.i.}}$, and $\frac{\lambda_o \Phi_o}{S''_{c.s.}}$, respectively, λ_o being the leakage factor on open circuit.

No current flowing in the armature, there is no armature reaction on open circuit, and no compensating ampere-turns are

therefore needed; consequently the total number of ampere-turns on open circuit, to be supplied by shunt winding, is:

$$AT_{sh} = AT_o = at_{g_o} + at_{a_o} + at_{m_o} \quad \ldots\ldots (336)$$

Next a similar set of calculations is made for the *normal output.* The useful flux in this case is:

$$\Phi = \frac{6 \times n'_p \times E' \times 10^9}{N_c \times N},$$

where $E' = E + I'\,r'_a + I\,r'_{se}$, for ordinary compound winding; see formula (19), § 14;

and $E' = E + I' \times (r'_a + r'_{se})$, for long-shunt compound winding; see formula (22), § 14.

Since, however, I and I' are very nearly alike, E' is practically the same in either case. Besides, E' can only be approximately determined at this stage of the calculation, since the series field resistance is not yet known. Taking the latter as .25 of the armature resistance, we therefore have for either kind of a compound winding:

$$E' = E + 1.25\, I'\, r'_a. \quad \ldots\ldots\ldots\ldots (337)$$

In case the machine is to be *overcompounded* for loss in the line, the percentage of drop—usually 5 per cent.—is to be included into the output E. M. F., hence the total E. M. F. generated at normal load, for 5 per cent. overcompounding:

$$E' = 1.05\, E + 1.25\, I'\, r'_a \quad \ldots\ldots\ldots\ldots (338)$$

The magnetizing forces required at normal load, then, are:

$$at_g = .3133 \times \frac{\Phi}{S_g} \times l''_g\,;$$

$$at_a = m''_a \times l''_a;$$

$$at_m = m''_{w.i.} \times l''_{w.i.} + m''_{c.i.} \times l''_{c.i.} + m''_{c.s.} \times l''_{c.s.}\,;$$

and $$at_r = k_{14} \times \frac{N_a \times I'}{n'_p} \times \frac{k_{13} \times \alpha}{180}.$$

m''_a, $m''_{w.i.}$, $m''_{c.i.}$, and $m''_{c.s.}$ are the specific magnetizing forces corresponding to the densities $\frac{\Phi}{S''_a}$, $\frac{\lambda\,\Phi}{S''_{w.i.}}$, $\frac{\lambda\,\Phi}{S''_{c.i.}}$, and $\frac{\lambda\,\Phi}{S''_{c.s.}}$, respectively, λ being the leakage factor at normal output.

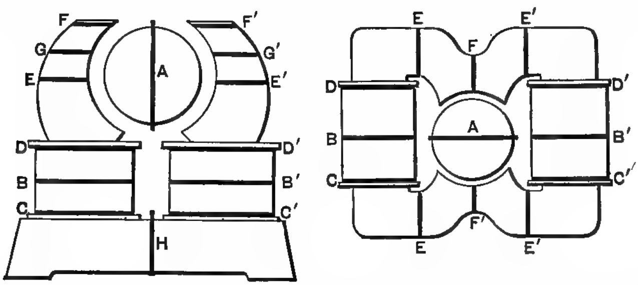
Their sum is the total number of ampere-turns needed for excitation at normal output:

$$AT = at_g + at_a + at_m + at_r;$$

this is supplied by shunt and series winding combined, consequently the compounding number of series ampere-turns:

$$AT_{se} = AT - AT_{sh} = AT - AT_o. \quad \ldots . (339)$$

In the above formulæ for at_{m_o} and at_m, the factors λ_o and λ are the leakage coefficients of the machine on open circuit and



Figs. 312 and 313.—Positions of Exploring Coils for Determining Distribution of Flux in Dynamos.

at normal load, respectively. Although the effect of the armature current upon the distribution of the magnetic flux in the different parts of the machine is very marked, as shown by tests made by H. D. Frisbee and A. Stratton,[1] the ratio of the total leakage factors in the two cases, especially in compound-wound machines, is so small that the factor λ, as obtained from formulæ (157), can be used for the calculation of both the shunt and the total ampere-turns. Since, however, it is very instructive to note the actual difference between the distribution of the magnetic flux at normal output and that on open circuit, the results of the tests mentioned above are compiled in the following Table XCV., in which all the flux intensities in the various parts of the different machines experimented upon are given in per cent. of the useful flux through the

[1] "The Effect of Armature Current on Magnetic Leakage in Dynamos and Motors," graduation thesis by Harry D. Frisbee and Alex. Stratton, Columbia College; *Electrical World*, vol. xxv. p. 200 (February 16, 1895).

TABLE XCV.—INFLUENCE OF ARMATURE CURRENT ON RELATIVE DISTRIBUTION OF MAGNETIC FLUX.

Position of Exploring Coil. (See Figs. 312 and 313.)	Part of Machine.	Flux, in Per Cent. of Useful Lines.							
		5 HP. Crocker-Wheeler Shunt Motor. (Toothed Ring.)		5 HP. Crocker-Wheeler Compound Dynamo. (Toothed Ring.)		1.5 KW. Edison Shunt Dynamo. (Smooth Drum.)		2 HP. Sprague Shunt Motor. (Smooth Drum.)	
		Open Circuit.	Normal Output.	Open Circuit.	Normal Output.	Open Circuit.	Normal Output.	Open Circuit.	Normal Output.
A	Armature	100	100	100	100	100	100	2 × 100	2 × 100
B	Center of magnet, right	115	120	122	129.5	156.5	157	179	170
B′	Center of magnet, left	117	121.5	115	125.5	157	158	172.5	165.5
C	Joint of magnet and yoke, right	...	...	124	130	156	155	172	154.5
C′	Joint of magnet and yoke, left	116.5	121.5	117	127.5	160	158.5	164	162.5
D	Joint of magnet and polepiece, right	108	110	118	121	143	143	170	164
D′	Joint of magnet and polepiece, left	112	119	110.7	121	146	145	160	147
E	Center of polepiece, right	58.5	34	63	45	74	67	{ 119 / 154 }	{ 129 / 134.5 }
E′	Center of polepiece, left	63	93	61	85	75	81.5	{ 114.5 / 141.5 }	{ 103 / 138 }
F	End of polepiece, right	2	2.3	5.2	5.7	17.5	16.5	4.3	24
F′	End of polepiece, left	2.3	4	4.6	6.6	20	21	4.5	17
G	Midway on polepiece, right	43.5	18.5	38	23	...	...	...	...
G′	Midway on polepiece, left	38	65	41	62	...	...	...	...
H	Center of yoke	...	...	...	...	154	153	...	...
	Leakage factor, on open circuit, λ_0	1.17	...	1.24	...	1.60	...	1.79	...
	Leakage factor, at normal load, λ	....	1.215	...	1.30	...	1.585	...	1.70
	Ratio of leakage factors, $\lambda : \lambda_0$	1.04		1.05		.99		.95	

armature, the various positions of the exploring coils being shown in the accompanying Figs. 312 and 313. Appended to this table are the respective leakage factors, obtained in dividing for each case the maximum percentage of flux by 100, and also the ratios of the leakage factor at normal output to that on open circuit.

105. Calculation of Compound Winding for Given Temperature Increase.

After having determined the number of shunt and series ampere-turns giving the desired regulation, the calculation of the compound winding itself merely consists in a combination of the methods treated in Chapters XX. and XXI.

The total energy dissipation, P_m, allowable in the magnet winding for the given rise of θ_m degrees being obtained from formula (283), this energy loss is to be suitably apportioned to the two windings, preferably in the ratio of their respective magnetizing forces, so that the amount to be absorbed by the series winding is:

$$P_{se} = \frac{AT_{se}}{AT} \times P_m = \frac{AT_{se}}{AT} \times \frac{\theta_m}{75} \times S_M \text{ watts}; \quad \textbf{(340)}$$

hence, by (294), the resistance of the series winding, at 15.5° C.:

$$r_{se} = \frac{P_{se}}{I^2} \times \frac{1}{1 + .004 \times \theta_m}$$

$$= \frac{AT_{se}}{AT} \times \frac{\theta_m}{75} \times \frac{S_M}{I^2} \times \frac{1}{1 + .004 \times \theta_m}. \quad \ldots\ldots\ldots\ldots\textbf{(341)}$$

The number of series turns being readily found from

$$N_{se} = \frac{AT_{se}}{I},$$

the total length of the series field conductor is:

$$L_{se} = N_{se} \times \frac{l'_T}{12} = \frac{AT_{se}}{I} \times \frac{l'_T}{12} \text{ feet},$$

and this, divided by the series field resistance, furnishes the specific length of the required series field conductor, thus:

$$\lambda_{se} = \frac{L_{se}}{r_{se}} = \frac{AT_{se}}{I} \times \frac{l'_T}{12} \times \frac{AT}{AT_{se}} \times \frac{75}{\theta_m} \times \frac{I^2}{S_M} \times (1 + .004 \times \theta_m)$$

$$= 6.25 \times \frac{AT \times I \times l'_T}{S_M \times \theta_m} \times (1 + .004 \times \theta_m), \quad \ldots\ldots (342)$$

where l'_T = mean length of one series turn, in inches.

The sectional area of the series field conductor, therefore, analogous to (296), is:

$$\delta_{se}^2 = 10.5 \times \lambda_{se}$$

$$= 65 \times \frac{AT \times I \times l'_T}{S_M \times \theta_m} \times (1 + .004 \times \theta_m). \quad \ldots\ldots (343)$$

If one single wire of this cross-section would be impractical, one or more cables stranded of n_{se} wires, each of

$$(\delta'_{se})^2 = \frac{\delta_{se}^2}{n_{se}}$$

circular mils, may be used, or a copper ribbon may be employed.

The actual series field resistance, at 15.5° C., then being:

$$r_{se} = 10.5 \times \frac{L_{se}}{\delta_{se}^2} = 10.5 \times \frac{N_{se} \times \frac{l'_T}{12}}{\delta_{se}^2}$$

$$= .875 \times \frac{N_{se} \times l'_T}{\delta_{se}^2} = .875 \times \frac{N_{se} \times l'_T}{n_{se} \times (\delta'_{se})^2}, \quad \ldots\ldots\ldots (344)$$

the actual energy consumption in the series winding is:

$$P_{se} = I^2 \times r'_{se}$$

$$= .875 \times \frac{I^2 \times N_{se} \times l'_T}{n_{se} \times (\delta'_{se})^2} \times (1 + .004 \times \theta_m), \quad \ldots\ldots (345)$$

and, consequently, the energy loss permissible in the shunt winding:

$$P_{sh} = P_m - P_{se}$$

$$= \frac{\theta_m}{75} \times S_M - I^2 \times r_{se} \times (1 + .004 \times \theta_m). \quad \ldots\ldots (346)$$

If the extra-resistance at normal load is to be r_x per cent. of the shunt resistance, the total watts consumed by the entire

shunt-circuit can be obtained by (312); formulæ (313) to (317) then furnish the number of shunt turns, the total length, and the resistance of the shunt wire, and from (318) and (319) the specific length and the sectional area are finally received:

$$\lambda_{sh} = \frac{AT_{sh}}{E} \times \frac{l''_T}{12} \times \left(1 + \frac{r_x}{100}\right) \times (1 + .004 \times \theta_m) \quad \ldots .(347)$$

$$\delta_{sh}^2 = .875 \times \frac{AT_{sh}}{E} \times l''_T \times \left(1 + \frac{r_x}{100}\right) \times (1 + .004\,\theta_m) \quad (348)$$

In estimating the mean lengths of series and shunt turns, l'_T and l''_T, respectively, all depends upon the manner of placing the field winding upon the cores. If the winding is performed by means of two or more bobbins upon each core, the series winding filling one spool, preferably that nearest to the brush cable terminals, and the shunt winding occupying the remaining ones, then the approximate mean length, l'_T, of one series-turn is equal to that of one shunt turn, l''_T, and also identical with the average turn, l_T, given by Tables LXXX., § 83, and XCIV., § 99. But, if the field coils are wound directly upon the cores—the series winding usually being wound on first—the lengths l'_T and l''_T differ from each other, and can be approximately determined by apportioning from $\frac{1}{4}$ to $\frac{1}{3}$ of the average winding height, given in Table LXXX., to the series winding, and the remainder to the shunt winding.

PART VII.

EFFICIENCY OF GENERATORS AND MOTORS.

DESIGNING OF A NUMBER OF DYNAMOS OF SAME TYPE.

CALCULATION OF ELECTRIC MOTORS, UNIPOLAR DYNAMOS, MOTOR-GENERATORS, ETC.

DYNAMO-GRAPHICS.

CHAPTER XXIII.

EFFICIENCY OF GENERATORS AND MOTORS.[1]

106. Electrical Efficiency.

The *electrical efficiency*, or the *economic coefficient* of a dynamo, is the ratio of its useful to the total electrical energy in its armature, the latter being the sum of the former and of the energy losses due to the armature and field resistances; hence the electrical efficiency of a *generator*:

$$\eta_e = \frac{P}{P'} = \frac{P}{P + P_a + P_M}, \quad \ldots\ldots (349)$$

and that of a *motor*:

$$\eta_e = \frac{P'}{P} = \frac{P - (P_a + P_M)}{P}, \quad \ldots\ldots (350)$$

where η_e = electrical efficiency of machine;
P = electrical energy, at terminals of machine;
P' = electrical activity in armature, or total energy engaged in electromagnetic induction;
P_a = energy absorbed by armature winding;
P_M = energy used for field excitation.

In case of a generator, P is the *output* available at the brushes, while in a motor it is the total energy delivered to the terminals, that is, the *intake* of the motor.

Inserting into (349) and (350) the expressions for P, P_a, and P_M in terms of E. M. F. current-strength and resistance, the following formulæ for the electrical efficiency are obtained:

Series-wound generator:

$$\eta_e = \frac{EI}{EI + I^2(r'_a + r'_{se})}; \quad \ldots\ldots\ldots\ldots\ldots (351)$$

[1] See "Efficiency of Dynamo-Electric Machinery," by Alfred E. Wiener; *American Electrician*, vol. ix. p. 259 (July, 1897).

Shunt-wound generator:

$$\eta_e = \frac{E\,I}{E\,I + I'^2\,r'_a + I_{sh}^2\,r''_{sh}}; \quad \ldots\ldots\ldots\ldots(352)$$

Compound-wound generator:

$$\eta_e = \frac{E\,I}{E\,I + I'^2\,(r'_a + r'_{se}) + I_{sh}^2\,r''_{sh}}; \quad \ldots\ldots(353)$$

Series-wound motor:

$$\eta_e = \frac{E\,I - I^2\,(r'_a + r'_{se})}{E\,I}; \quad \ldots\ldots\ldots\ldots(354)$$

Shunt-wound motor:

$$\eta_e = \frac{E\,I - (I'^2\,r'_a + I_{sh}^2\,r''_{sh})}{E\,I}; \quad \ldots\ldots\ldots\ldots(355)$$

Compound-wound motor:

$$\eta_e = \frac{E\,I - [I'^2\,(r'_a + r'_{se}) + I_{sh}^2\,r''_{sh}]}{E\,I}. \quad \ldots(356)$$

Since the electrical efficiency does not include waste by hysteresis, eddy currents, and friction, but is depending upon the energy losses due to heating by the current only, it may be adjusted to any desired value by properly proportioning the resistances of the machine; see formulæ (10), (13), (20), and (23), § 14. The electrical efficiency of modern dynamos is very high, ranging from $\eta_e = .85$, or 85 per cent., for small machines, to as high as $\eta_e = .99$, or 99 per cent., for very large generators.

107. Commercial Efficiency.

By the *commercial* or *net efficiency* of a dynamo-electric machine is meant the ratio of its output to its intake. The intake of a generator is the mechanical energy required to drive it, and is the sum of the total energy generated in the armature and of the energy losses due to hysteresis, eddy currents, and friction; the intake of a motor is the electrical energy delivered to its terminals. The output of a generator is the electrical energy disposable at its terminals; the output of a motor is the mechanical energy disposable at its shaft, and

consists in the useful energy of the armature diminished by hysteresis, eddy current, and friction losses. The commercial efficiency of a *generator*, therefore, is:

$$\eta_c = \frac{P}{P''} = \frac{P}{P' + P'_o} = \frac{P}{P' + P_h + P_e + P_o}$$

$$= \frac{P}{P + P_a + P_M + P_h + P_e + P_o}, \quad \ldots\ldots\ldots\ldots (357)$$

and that of a *motor*:

$$\eta_c = \frac{P''}{P} = \frac{P' - P'_o}{P} = \frac{P' - (P_h + P_e + P_o)}{P}$$

$$= \frac{P - (P_a + P_M + P_h + P_e + P_o)}{P}; \quad \ldots\ldots\ldots (358)$$

in which η_c = commercial or net efficiency of dynamo;
P = electrical energy at terminals, *i. e.*, output of generator, or intake of motor;
P' = electrical activity in armature;
P'' = mechanical energy at dynamo shaft, *i. e.*, driving power of generator, or mechanical output of motor, respectively;
P_a = energy absorbed by armature winding;
P_M = energy used for field excitation;
P_h = energy consumed by hysteresis;
P_e = energy consumed by eddy currents;
P_o = energy loss due to air resistance, brush friction, journal friction, etc.;
P'_o = energy required to run machine at normal speed on open circuit.

Substituting in the above formulæ the values of P, P_a, and P_M, the following set of formulæ, resembling (351) to (356), is obtained:

Series-wound generator:

$$\eta_c = \frac{EI}{EI + I^2 (r'_a + r'_{se}) + P_o'}; \quad \ldots\ldots\ldots\ldots (359)$$

Shunt-wound generator:

$$\eta_c = \frac{EI}{EI + I'^2 r'_a + I_{sh}{}^2 r''_{sh} + P'_o}; \quad \ldots\ldots\ldots (360)$$

Compound-wound generator:

$$\eta_c = \frac{E\,I}{E\,I + I'^2\,(r'_a + r'_{se}) + I_{sh}^2\,r''_{sh} + P'_o}; \qquad (361)$$

Series-wound motor:

$$\eta_c = \frac{E\,I - [I^2\,(r'_a + r'_{se}) + P'_o]}{E\,I}; \quad \ldots\ldots\ldots\ldots(362)$$

Shunt-wound motor:

$$\eta_c = \frac{E\,I - [I'^2\,r'_a + I_{sh}^2\,r''_{sh} + P'_o]}{E\,I}; \quad \ldots\ldots(363)$$

Compound-wound motor:

$$\eta_c = \frac{E\,I - [I'^2\,(r'_a + r'_{se}) + I_{sh}^2\,r''_{sh} + P'_o]}{E\,I}. \qquad (364)$$

In case of belt-driving, the mechanical energy at the dynamo shaft, in foot-pounds per second, can also be expressed by the product of the belt-speed, in feet per second, and of the effective driving power of the belt, in pounds, or, converted into watts:

$$P'' = \frac{746}{550} \times \frac{v_B}{60} \times (F_B - f_B)$$
$$= 1.3564 \times v'_B \times (F_B - f_B), \quad \ldots\ldots\ldots\ldots\ldots\ldots(365)$$

where v_B = belt velocity, in feet per minute;
v'_B = belt velocity, in feet per second;
F_B = tension on tight side of belt, in pounds;
f_B = tension on slack side of belt, in pounds.

The commercial efficiency of a *generator*, therefore, may be expressed by:

$$\eta_c = \frac{P}{P''} = \frac{E\,I}{1.3564 \times v'_B \times (F_B - f_B)}, \quad \ldots(366)$$

and the commercial efficiency of a *motor*, by:

$$\eta_c = \frac{P''}{P} = \frac{1.3564 \times v'_B \times (F_B - f_B)}{E\,I}, \quad \ldots(367)$$

The commercial efficiency, η_c, of a dynamo is always smaller than its electrical efficiency, η_e, since the former, besides the electrical energy-dissipation, includes all mechanical and mag-

netic energy losses, such as are due to journal bearing friction, to hysteresis, to eddy currents, and to magnetic leakage. The commercial efficiency, therefore, depends upon the amount of the electrical efficiency, upon the shape of the armature, upon the design, workmanship, and alignment of the bearings, upon the pressure of the brushes, upon the quality of the iron employed in its armature and field magnets, and upon the degree of lamination of the armature core; while the electrical efficiency is a function of the electrical resistances only. The mechanical and magnetical losses vary very nearly proportional to the speed; the no load energy consumption for any speed, consequently, is approximately equal to the open circuit loss at normal speed multiplied by the ratio of the given to the normal speed.

The commercial efficiency of well-designed machines ranges from $\eta_c = .70$, or 70 per cent., for small dynamos, to $\eta_c = .96$, or 96 per cent., for large ones.

Since in a direct-driven generator the commercial efficiency is the ratio of the mechanical power available at the engine shaft to the electrical energy at the machine terminals, for comparisons between direct and belt-driven dynamos the loss in belting should also be included into the commercial efficiency of the belt-driven generator. The following Table XCVI. contains averages of these losses for various arrangements of belts:

Table XCVI.—Losses in Dynamo Belting.

Arrangement of Belts.	Loss in Belting in Per Cent. of Power Transmitted.
Horizontal Belt	5 to 10 per cent.
Vertical Belt	7 " 12 "
Countershaft and Horizontal Belt	10 " 15 "
Countershaft and Vertical Belt	12 " 20 "
Main and Countershaft with Belts	20 " 30 "

108. Efficiency of Conversion.

The *efficiency of conversion*, or the *gross-efficiency*, is the ratio of the electrical activity in the armature to the mechanical energy at the shaft, or vice versa; that is to say, in a *generator*

it is the ratio between the total electrical energy generated and the gross mechanical power delivered to the shaft, and in a *motor* is the ratio of the mechanical output to the useful electrical energy in the armature. Or, in symbols, for a *generator:*

$$\left.\begin{aligned}\eta_g &= \frac{P'}{P''} = \frac{P'}{P' + P'_o} = \frac{P + P_a + P_M}{P + P_a + P_M + P'_o} \\ &= \frac{E\,I + I'^2\,(r_a + r_{se}) + I_{sh}^2\, r''_{sh}}{E\,I + I'^2\,(r_a + r_{se}) + I_{sh}^2\, r''_{sh} + P'_o} \\ &= \frac{E'\,I'}{746\ hp} = \frac{E'\,I'}{1.3564 \times v'_B \times (F_B - f_B)},\end{aligned}\right\} \quad \ldots(368)$$

and for a *motor :*

$$\left.\begin{aligned}\eta_g &= \frac{P''}{P'} = \frac{P' - P'_o}{P'} = \frac{P - (P_a + P_M + P'_o)}{P - (P_a + P_M)} \\ &= \frac{E\,I - [I'^2\,(r_a + r_{se}) + I_{sh}^2\, r''_{sh} + P'_o]}{E\,I - [I'^2\,(r_a + r_{se}) + I_{sh}^2\, r''_{sh}]} \\ &= \frac{746\ hp}{E'\,I'} = \frac{1.3564 \times v'_B \times (F_B - f_B)}{E'\,I'}.\end{aligned}\right\} \quad \ldots(369)$$

The efficiency of conversion, η_g, is the quotient of the commercial and electrical efficiencies, and therefore varies between

$$\eta_g = \frac{\eta_c}{\eta_e} = \frac{.7}{.85} = .82, \text{ or } 82 \text{ per cent.},$$

for small dynamos, and

$$\eta_g = \frac{\eta_c}{\eta_e} = \frac{.96}{.99} = .97, \text{ or } 97 \text{ per cent.},$$

for large machines.

109. Weight-Efficiency and Cost of Dynamos.

As the commercial efficiency increases with the size of the machine, so the *weight-efficiency*—that is, the output per unit weight of the machine—in general is greater for a large than for a small dynamo, and the cost of the machine per unit output, therefore, gradually decreases as the output increases.

If all the different sized machines of a firm were made of the

same type, all having the same linear proportions, and if all had the same, or a gradually increasing circumferential velocity, and were all figured for the same temperature increase in their windings, then the weight-efficiency would gradually increase according to a certain definite law, and the cost per KW would decrease by a similar law. In practice, however, such definite laws do not exist for the following reasons: (1) Up to a certain output a bipolar type is usually employed, while for the larger capacities the multipolar types are more economical; this change in the type causes a sudden jump to take place, both in the weight-efficiency and in the specific cost, between the largest bipolar and the smallest multipolar sizes. (2) The machines of the different capacities are not all built in linear proportion to each other, but, in order to economize material, tools, and patterns the outputs of two or three consecutive sizes are often varied by simply increasing the length of armature and polepieces; in this case a small machine with a long armature may be of greater weight-efficiency and of a smaller specific price than the next larger size with a short armature. (3) The conductor-velocity is not the same in all sizes; as a general rule, it is higher in the bigger machines, but often the increase from size to size is very irregular, causing deviation in the gradual increase of the weight-efficiency. (4) Certain sizes of machines being more popular than others, a greater number of these can be manufactured simultaneously, and therefore these sizes can be turned out cheaper than others, and the specific cost of such sizes will likely be smaller than that of the next larger ones. (5) Large generators frequently are fitted with special parts, such as devices for the simultaneous adjustment and raising of the brushes, arrangements for operating the switches, brackets for supporting the heavy main and cross-connecting cables, platforms, stairways, etc., the additional weight and cost of these extra parts often lowering the weight efficiency and increasing the specific cost beyond those of smaller sizes not possessing such complications. These various considerations, then, show why prices differ so widely, and why the ratio of weight to output is so varied: and they offer a reason for the fact that the data derived from different makers' price-lists are at such a great variance from each other.

In the following Table XCVII. the author has compiled the average weights and weight-efficiencies (watts per pound), for all sizes of high-, medium-, and low-speed dynamos as averaged from the catalogues of numerous representative American manufacturers of high-grade electrical machinery:

TABLE XCVII.—AVERAGE WEIGHT AND WEIGHT-EFFICIENCY OF DYNAMOS.

Capacity of Dynamo, in Kilowatts.	High Speed.			Medium Speed.			Low Speed.		
	Average Weight (Total, Net). Lbs.	Weight per Kilowatt. Lbs.	Output per Pound Watts.	Average Weight (Total, Net). Lbs.	Weight per Kilowatt. Lbs.	Output per Pound. Watts.	Average Weight (Total, Net). Lbs.	Weight per Kilowatt. Lbs.	Output per Pound. Watts.
.1	25	250	4	35	350	2.9	50	500	2
.25	55	220	4.5	80	320	3.1	112	450	2.2
.5	100	200	5	150	300	3.3	210	420	2.4
1	190	190	5.3	280	280	3.6	400	400	2.5
2	350	175	5.7	500	250	4	720	360	2.8
5	775	155	6.5	1,150	230	4.4	1,650	330	3
10	1,400	140	7.2	2,150	215	4.7	3,000	300	3.3
25	3,000	120	8.3	4,750	190	5.3	7,000	280	3.6
50	5,500	110	9.1	8,500	170	5.9	12,500	250	4
100	10,500	105	9.5	16,000	160	6.3	23,000	230	4.4
200	20 000	100	10	30,000	150	6.7	41,000	205	4.9
300	29,000	97	10.3	42,000	140	7.2	57,000	190	5.3
400	37,500	94	10.7	53,000	133	7.5	72,000	180	5.6
600	54,000	90	11.1	74,500	124	8.1	99,000	165	6.1
800	70,000	87	11.5	93,000	116	8.6	120,000	150	6.7
1,000	85,000	85	11.8	110,000	110	9.1	140,000	140	7.2
1,500	123,000	82	12.2	150,000	100	10	180,000	120	8.3
2,000	160,000	80	12.5	190,000	95	10.5	220,000	110	9.1

Since the speeds for the same outputs vary greatly in machines of different manufacturers, there exist considerable deviations from the averages given above. When bearing this in mind, the above table may be effectively employed to check the general proportions and design of the calculated machine.

CHAPTER XXIV.

DESIGNING OF A NUMBER OF DYNAMOS OF SAME TYPE.

110. Simplified Method of Armature Calculation.

In case a number of different sizes of machines are to be designed of the same type, the method of calculating may be materially simplified by subdividing the fundamental formulæ in two parts, the one containing those quantities which remain constant for the type in question, while the other embodies all

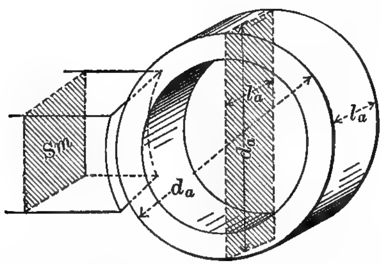


Fig. 314.—Cross-Section of Field Magnet and Rectangle Inclosing Armature Core.

factors that vary with the output of the machine. By adopting a fixed ratio between the cross-section of the field magnet and the area of the rectangle containing the longitudinal section through the armature core, which is perfectly proper for any particular type of dynamo, and by basing all calculations upon the density of the magnetic lines in the magnet frame, Cecil P. Poole [1] has obtained a set of simple formulæ which admit of ready separation into a "preliminary" and a "working" part.

Representing the area of the magnet frame by the quotient

[1] "A Simplified Method of Calculating Dynamo-Output and Proportions," by Cecil P. Poole, *Electrical Engineer*, vol. xvi. p. 483 (December 6, 1893).

of the longitudinal armature section and the number of pairs of poles, Fig. 314, thus:

$$S_m = \frac{d_a \times l_a}{n_p}, \quad \ldots\ldots\ldots\ldots\ldots (370)$$

the E. M. F. generated in the armature can be expressed by:

$$E' = \mathcal{B}''_m \times (S_m \times n_p) \times \frac{1}{\lambda} \times \frac{N_c}{n'_p} \times \frac{N}{60} \times 10^{-8}$$

$$= \mathcal{B}''_m \times \frac{1}{\lambda} \times d_a \times l_a \times \frac{N_c}{n'_p} \times \frac{v_c \times 12}{d_a \times \pi} \times 10^{-8}$$

$$= 3.82 \times \frac{1}{\lambda} \times \mathcal{B}''_m \times l_a \times v_c \times \frac{N_c}{n'_p} \times 10^{-8}, \quad \ldots . (371)$$

where $\mathcal{B}''_m$ = magnetic density in magnet cores, in lines per square inch;
S_m = area of magnet-core, in square inches;
d_a = diameter of armature core, in inches;
l_a = length of armature core, in inches;
N_c = total number of armature conductors;
n_p = number of pairs of magnet poles;
n'_p = number of bifurcations of current in armature;
N = speed, in revolutions per minute;
v_c = conductor-velocity, in feet per second;
λ = factor of magnetic leakage.

Expressing the length of the armature core as a multiple of its diameter:

$$l_a = k_{18} \times d_a,$$

and writing for the number of conductors on the armature:

$$N_c = \frac{d_a \pi}{\delta'''_a} \times n_1,$$

where d_a = diameter of armature core, in inches;
δ'''_a = pitch of conductors on armature circumference, in inches;
n_1 = number of layers of armature conductors;

formula (371) becomes:

$$E' = 3.82 \times \frac{1}{\lambda} \times \mathcal{B}''_m \times v_c \times k_{19} \times d_a \times \frac{d_a \pi \times n_1}{n'_p \times \delta'''_a} \times 10^{-8},$$

and by transformation we obtain:

$$d_a = 2887 \sqrt{\frac{E' \times \lambda \times \delta'''_a \times n'_p}{\mathfrak{B}''_m \times k_{18} \times v_c \times n_1}}. \quad \ldots(372)$$

The pitch, δ'''_a, in this formula depends upon the size and shape of the armature conductor and its arrangement upon the armature core; it is, therefore, convenient to put:

$$\delta'''_a = k_{19} \times \delta'_a,$$

in which δ'_a = *insulated* width of armature conductor, in inches; and

k_{19} = pitch-factor, depending upon type of armature.

The factor k_{19}, for *smooth drum* armatures, varies between 1.05 and 1.2, according to the spacing of the conductors on the circumference, see Table XVII., § 23; in *smooth rings* the limits of k_{19} are 1.01 and 1.25; for *toothed* armatures it ranges from 2.05 to 2.2, if the width of the slot is equal to the top breadth of the tooth; if the latter is not the case, these limits must be multiplied by the ratio of the pitch of the slots to twice the slot width; and for *perforated* armatures the value of k_{19} lies between 1.5 and 2, according to the ratio of the width of the perforations to their distance apart.

Uniting in (372) all the constant factors into one quantity we can write for the working formula:

$$d_a = K \times \sqrt{\frac{E' \times \lambda \times \delta'''_a \times n'_p}{n_1}}, \quad \ldots(373)$$

in which the constant has the value:

$$K = \frac{2887}{\sqrt{\mathfrak{B}''_m \times k_{18} \times v_c}}. \quad \ldots\ldots\ldots(374)$$

If all the dynamos of the type under consideration are to have the same voltage, the same pitch-factor and number of layers of armature conductor, and are to have their armatures connected in the same manner, then E', k_{19}, n_1 and n'_p are constant, and may be transferred from (373) to (374), still more simplifying the working formula, which under these conditions becomes:

$$d_a = K' \times \sqrt{\lambda \times \delta'_a}, \quad \ldots\ldots\ldots(375)$$

while the corresponding preliminary formula is:

$$K' = 2887 \times \sqrt{\frac{E' \times n'_p \times k_{19}}{\mathfrak{B}''_m \times k_{18} \times v_c \times n_1}} \quad \ldots (376)$$

Having found the armature diameters for the various sizes, their lengths can then be readily obtained by multiplication with k_{18}; and diameter and length of the armature determine the principal dimensions of the field frame.

The calculation of the total magnetizing force and of the field winding, for the number of dynamos of the same type, by similarly extracting from the respective formulæ all the fixed quantities, may also be somewhat simplified, but the direct methods given for the field calculation are already so simple that not much can be gained by so doing, and it is therefore preferable to separately consider every single case.

111. Output as a Function of Size.

If the ratio of the dimensions of two dynamos of the same type is $1 : m$, the ratio of their respective outputs can be expressed as an exponential function of this ratio of size, as follows:

$$\frac{P_1}{P_2} = \left(\frac{1}{m}\right)^x, \text{ or } \frac{P_2}{P_1} = m^x.$$

If the exponent x is given for the various practical conditions, the dimensions of any dynamo for a required output can, therefore, be calculated from the dimensions, and the known output of one machine of the type in question, from the formula:

$$m = \left(\frac{P_2}{P_1}\right)^{\frac{1}{x}}, \quad \ldots\ldots\ldots\ldots\ldots (377)$$

which gives the *multiplier*, by which the linear dimensions of the known machine are to be altered in order to obtain the required output.

The author, by a mathematical deduction,[1] has found the theoretical value of the required exponent to be:

$$x = 2.5.$$

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[1] "Relation Between Increase of Dimensions and Rise of Output of Dynamos," by Alfred E. Wiener, *Electrical World*, vol. xxii. pp. 395 and 409 (November 18 and 25, 1893); *Elektrotech. Zeitschr.*, vol. xv. p. 57 (February 1, 1894).

In the mathematical determination of x, however, the thickness of the insulation around the armature conductor has, for convenience, been neglected. The theoretical value found, therefore, holds good only for the imaginary case that the entire winding space is filled with copper. Since the percentage of the winding space occupied by insulating material is the larger the smaller the armature, the difference between the actual and the theoretical output will be the greater, comparatively, the smaller the dynamo, and it follows that the exponent, x, varies with the sizes of the machines to be compared. Furthermore, the area of the armature conductor decreases with the voltage of the machine; in a high-voltage dynamo, therefore, a larger portion of the winding space is occupied by the insulation than would be the case if the same machine were wound for low tension. From this it follows that the output of any dynamo, if wound for low voltage, is greater than if wound for high potential, and the value of the exponent x, consequently, also depends upon the voltages of the machines to be compared. Taking up by actual calculation the influence of size and of voltage upon the value of x, the general law was found that the exponent of the ratio of outputs of two dynamos of the same type increases with decreasing ratio of their linear dimensions as well as with decreasing ratio of their voltages; the theoretical value being correct only for the case that the dynamo to be newly designed is to have 10 or more times the voltage, and at least the 8-fold size of the given one. This law is observed to really hold in practice, as can be derived from the following Table XCVIII., which gives average values of the exponent x for all the different ratios of size and voltage:

TABLE XCVIII.—EXPONENT OF OUTPUT-RATIO IN FORMULA FOR SIZE-RATIO FOR VARIOUS COMBINATIONS OF POTENTIALS AND SIZES.

RATIO OF POTENTIALS, $E_2 : E_1$	VALUE OF EXPONENT x, FOR RATIO OF LINEAR DIMENSIONS, m =		
	1 to 2	3 to 8	8 and over.
Up to $\frac{1}{8}$	3.00	2.85	2.70
$\frac{1}{4}$ to 4	2.80	2.70	2.60
10 and over	2.60	2.55	2.50

The values given in the above table, besides for the comparison of machines of the same type, are found to hold good also for the comparison of the outputs of similar armatures in frames of different types. But the figures contained in Table XCVIII. are based upon the assumption that the field-densities and the conductor-velocities of the two machines to be compared are identical, a condition which is very seldom fulfilled in practice, particularly not in dynamos of different type, as, for instance, when comparing a bipolar with a multipolar machine. Hence, any difference in the field-densities and in the peripheral speeds of the two machines to be compared must be properly considered, that is to say, the exponent x given in the preceding table for the voltage-ratio and the size-ratio in question must be multiplied by the ratio of their products of field-density and conductor-velocity, for, the E. M. F., and therefore the output, of a dynamo is directly proportional to the flux-density of its magnetic field and to the cutting-speed of its armature conductors.

CHAPTER XXV.

CALCULATION OF ELECTRIC MOTORS.

112. Application of Generator Formulæ to Motor Calculation.

All the formulæ previously given for generators apply equally well to the case of an electric motor; for, in general, a well-designed generator will also be a good motor. Hence the first step in calculating an electric motor is to determine the electrical capacity and E. M. F. of this motor when driven as a generator, at the specified speed.[1]

Considering a given dynamo as a generator, its output, P_1, in watts, at the terminals, is the total energy, P', generated in its armature by electromagnetic induction, diminished by the amount of energy absorbed between the armature conductors and the machine terminals; that is, by the loss due to internal electrical resistances. In other words, the output is the total electrical energy produced in the armature multiplied by the electrical efficiency of the dynamo. The output, P''_2, of the same machine, when run with the same speed as a motor, is the useful electrical energy, P', active within its armature in setting up electromagnetic induction, less the energy lost between armature and pulley; that is, less the loss caused by hysteresis, eddy currents, and friction, or is the product of electrical activity and gross efficiency. Conversely, the power, P''_1, to be supplied to the generator pulley, must be the total energy, P', produced in the armature, increased by an amount equal to the magnetic and frictional losses, or must be P' divided by the gross efficiency. And the energy, P_2, finally, required at the motor terminals in order to set up in the armature an electrical activity of P' watts, is found by adding to P' the energy needed to overcome the internal resistances of

[1] "Calculation of Electric Motors," by Alfred E. Wiener, *Electrical World*, vol. xxviii., pp. 693 and 725 (December 5 and 12, 1896).

the motor, or by dividing P' by the electrical efficiency. Designating the electrical efficiency of the machine, *i. e.*, the ratio of its useful to the total electrical energy in its armature, by η_e, and its gross efficiency, or efficiency of conversion, *i. e.*, the ratio between the electrical activity in the armature and the mechanical power at the pulley, by η_g, we therefore have:

Output of machine as *generator:*

$$P_1 = \eta_e \times P'; \quad \ldots\ldots\ldots\ldots (378)$$

Output of machine as *motor:*

$$P''_2 = \eta_g \times P'; \quad \ldots\ldots\ldots\ldots (379)$$

Power to be supplied to machine when run as *generator* (driving power):

$$P''_1 = \frac{P'}{\eta_g}; \quad \ldots\ldots\ldots\ldots (380)$$

Energy to be supplied to machine when run as *motor* (intake of motor):

$$P_2 = \frac{P'}{\eta_e}, \quad \ldots\ldots\ldots\ldots (381)..$$

Where P_1, P_2 = electrical energy at terminals of machine, as generator and motor, respectively;
P' = electric energy active in armature conductors, being the same in both cases;
P''_1, P''_2 = mechanical energy at dynamo pulley, for generator and motor, respectively.

By transposition of (379) the electrical capacity of the machine can be expressed by the motor output, thus:

$$P' = \frac{P''_2}{\eta_g},$$

which is to say that, in order to find the dimensions and windings for a motor of

$$hp = \frac{P''_2}{746} \text{ horse-power},$$

it is necessary to figure a generator which at the given speed has a total capacity of

$$P' = \frac{P''_2}{\eta_g} = \frac{746 \times hp}{\eta_g} \text{ watts.} \quad \ldots\ldots (382)$$

The E. M. F. for which the generator is to be calculated, or the *Counter E. M. F.* of the motor, is the voltage at the motor terminals diminished by the drop of potential within the machine, or:

$$E' = E - I \times (r'_a + r'_{se}), \quad \ldots\ldots\ldots (383)$$

in which E' = E. M. F. active in armature, in volts;
E = voltage supplied to motor terminals;
I = current intensity at motor terminals;
r'_a = armature resistance, at working temperature, in ohms;
r'_{se} = resistance of series field, warm, in ohms, for series and compound machines; in case of shunt dynamo $r'_{se} = 0$.

Formula (383), though theoretically accurate, is not practically so, since for the same excitation, armature current and speed, the counter E. M. F. of a motor is greater than the E. M. F. when used as a generator, for the following reason: While in a *generator* a forward displacement, or a *lead*, of the brushes has the effect of *weakening*, and a backward displacement, or a *lag*, that of *strengthening* the field magnet, in a *motor* a *lead* tends to *magnetize*, and a *lag* to *demagnetize* the field. Sparkless running, however, requires a *lead* of the brushes in a *generator* and a *lag* of the same in a *motor*, so that in *both* cases the armature reactions *weaken* the field. Since hysteresis as well as eddy currents have the effect of shifting the magnetic field in the direction of the rotation, thereby increasing the lead in a generator and diminishing the lag in a motor, it follows that—for equal magnetizing force, equal current intensity, and equal speed—the lag in a motor is *less* than the lead in a corresponding generator. For the purpose at hand, however, formula (383) gives the required counter E. M. F. with sufficient accuracy, particularly because neither the current strength nor the resistances usually being prescribed, the drop must be estimated by means of Table VIII., § 19.

By dividing the electrical activity, P', as obtained from formula (382), by the E. M. F., E', the current-capacity of the corresponding generator is found:

$$I' = \frac{P'}{E'}. \quad \ldots\ldots\ldots\ldots\ldots (384)$$

For the purpose of simplifying this conversion of a motor into a generator of equal electrical activity, the following Table XCIX. is given, which contains the average efficiencies, and the active energy for motors of various sizes:

TABLE XCIX.—AVERAGE EFFICIENCIES AND ELECTRICAL ACTIVITY OF ELECTRIC MOTORS OF VARIOUS SIZES.

OUTPUT OF MOTOR IN HORSE-POWER. hp	ELECTRICAL EFFICIENCY. η_e	GROSS EFFICIENCY. η_g	COMMERCIAL EFFICIENCY. $\eta_c = \eta_e \times \eta_g$	ELECTRICAL ACTIVITY IN ARMATURE, IN KILOWATTS. $P' = \frac{.746 \times hp}{\eta_g}$
$\frac{1}{12}$	.85	.82	.70	.08
$\frac{1}{8}$	.87	.83	.72	.13
$\frac{1}{4}$	.89	.84	.75	.22
$\frac{1}{2}$	.90	.87	.78	.43
1	.91	.88	.80	.85
2	.92	.89	.82	1.7
5	.93	.90	.84	4.1
10	.94	.92	.86	8.1
20	.95	.93	.88	16
30	.96	.935	.90	24
50	.97	94	.91	40
100	.975	.945	.92	79
200	.98	.95	.93	157
500	.985	.955	.94	390
1000	.985	.96	.95	780
2000	.99	.97	.96	1540

If a dynamo which has been connected for working as a *generator* is supplied with current from the mains instead, it will run as a *motor*, the *direction of rotation* depending upon the manner of field excitation. A *series* dynamo, since both the armature and field currents are then reversed, will run in the *opposite* direction from that which it was driven as generator, and must therefore have its brushes reversed and given a lead in the opposite direction; or, if direction in the original generator direction is desired, must have either its armature or its field connections reversed. A *shunt* dynamo will turn in the *same* direction when run as a motor, for, while the armature

current is reversed, the exciting current will have the same direction as when worked as a generator. A *compound* dynamo, finally, will run as a motor in the *opposite* direction, if the series winding is more powerful than the shunt, and in the *same* sense, if the shunt is the more powerful; and while the field excitation as a *generator* is the *sum* of the series and shunt windings as a *motor* it is their *difference.*

113. Counter E. M. F.

Whereas in a generator there is but one E. M. F., in a motor there must always be two. If I = current at machine terminals, E = direct E. M. F., E' = counter E. M. F., R = total resistance of circuit, and r = internal resistance of machine, this difference between a generator and a motor can be best expressed [1] by the formulæ for the current in the two cases, thus—

for *generator:*
$$I = \frac{E}{R};$$

for *motor:*
$$I = \frac{E - E'}{r}, \text{ or } E' = E - Ir.$$

The current and direct E. M. F. are the same in both cases, but the resistance is much less in case of a motor, the difference being replaced by the counter E. M. F., which acts like a resistance to reduce the current.

Upon the amount of this counter E. M. F. depend the speed and the current, and therefore the power of an electric motor. For, since the E. M. F. generated by electromagnetic induction is proportional to the peripheral velocity of the armature, it follows that, other factors remaining unchanged, the speed conversely depends upon the counter E. M. F. only. The latter is the case in a series motor run from constant current supply, since in this the magnetizing force is constant at all loads. In a shunt motor, however, the field current varies with the load, and the speed, therefore, depends upon the field magnetism as well as upon the counter E. M. F. If the exciting current in a constant potential shunt motor is decreased, the E. M. F. decreases correspondingly, and a rise of

[1] "The Electric Motor," by Francis B. Crocker, *Electrical World*, vol. xxiii. p. 673 (May 19, 1894).

the current flowing in the motor is the consequence, as follows directly from the above equation for the motor current. The speed in this case, therefore, rises until the counter E. M. F. reaches a sufficient value to shut off the excess of current.

If the counter E. M. F. is low, which is the case when the motor is starting or running slowly, resistance has to take its place in order to govern the current of the motor. The introduction of resistance in series with the armature, the so-called *starting resistance*, is usually resorted to for this regulation, but this is very wasteful of energy and involves the use of a large and clumsy rheostat, while the counter E. M. F. itself affords a means to easily design a motor to run at the same, or at a higher, speed at full load than when lightly loaded. This may be done by slightly exaggerating the effect of armature reaction, so that the field magnetism will be considerably reduced by the large armature current which flows at full load, thus diminishing the counter E. M. F. and increasing the speed in the manner explained above. In this way the remarkable effect of greater speed with heavier load is obtained without any special device or construction; all that is necessary being a slight modification in design, involving no increase in cost or complication.

114. Speed Calculation of Electric Motors.

If a generator, which at a speed of N_1 revolutions per minute produces a total E. M. F. of

$$E'_1 = E + I' \times (r'_a + r'_{se}) \text{ volts,}$$

is run as a motor having same current strength in armature, the motor armature, in order that no more nor less than this current, I', its full load as a generator, shall flow, must generate a counter E. M. F. of

$$E'_2 = E - I' \times (r'_a + r'_{se}) \text{ volts.}$$

The speed necessary to generate this back voltage, speed being proportional to voltage, is:

$$N_2 = \frac{E'_2}{E'_1} \times N_1 = \frac{E - I' \times (r'_a + r'_{se})}{E + I' \times (r'_a + r'_{se})} \times N_1, \quad (385)$$

which is the speed of the motor at full load, provided the E. M. F., E, supplied to its terminals is equal to the voltage when run as generator.

The speed of the motor for any given E. M. F., applied to its armature terminals, depends (1) upon the load impressed upon the motor armature, or the torque τ, it has to exert; (2) on the electrical resistance $(r'_a + r'_{se})$, of the armature and the series field; and (3) upon its specific generating power, or its capability of producing counter E. M. F.; *i. e.*, the number of volts, e'', it produces at a speed of one revolution per second.

The specific generating power of the motor being

$$e'' = \Phi \times \frac{N_c}{n'_p} \times 10^{-8} \text{ volts at 1 rev. per sec.}, \quad \textbf{(386)}$$

where Φ = useful flux, in maxwells;
N_c = number of conductors on armature;
n'_p = number of pairs of armature circuits electrically in parallel;

the total counter E. M. F. at the required speed of N_2 revolutions per minute, will be

$$E'_2 = e'' \times \frac{N_2}{60} = \frac{\Phi \times N_c \times N_2}{60 \times 10^8 \times n'_p}, \quad \ldots\textbf{(387)}$$

and the current flowing in the armature, therefore, is:

$$I' = \frac{E - E'_2}{r'_a + r'_{se}} = \frac{E - e'' \times \dfrac{N_2}{60}}{r'_a + r'_{se}}. \quad \ldots\ldots\textbf{(388)}$$

The activity of this current expended upon the counter E. M. F. will be their product, $E'_2 \times I'$ watts, and this must be equal to the total rate of working, which is the product of circumferential speed and turning moment, or torque; that is, it must be equal to

$$2\pi \times N_2 \times \tau \times \frac{746}{33,000} \text{ watts},$$

where the torque, τ, is calculated from formula (93), § 40; hence we have:

$$\left(e'' \times \frac{N_2}{60}\right) \times \frac{E - e'' \times \dfrac{N_2}{60}}{r_a + r_{se}} = 2\pi \times \frac{N_2}{60} \times \tau \times \frac{746}{550},$$

from which

$$N_2 = 60 \times \left(\frac{E}{e''} - 8.52 \times \frac{(r_a + r_{se}) \times \tau}{e''^2} \right) \quad (389)$$

From (389) follows that, if either the internal resistance or the torque is zero, since the second term in the parenthesis then disappears, the speed of the motor is:

$$N_2 = 60 \times \frac{E}{e''} = \frac{60 \times E \times n'_p \times 10^8}{\Phi \times N_c} \quad ..(390)$$

This reduced formula (390), indeed, holds very nearly in practice for very large motors (in which the internal resistance is very small), and also is approximately followed in case of motors running free (the torque then being only that necessary to overcome the frictions).

The important requirement of *constant speed* under variable load may be almost perfectly met by the compound-wound motor, is nearly met by the shunt-wound motor, and is not met without the aid of special mechanism by the series-wound motor. A compound-wound motor will maintain its speed perfectly constant under all loads, if the series winding is so adjusted that the increase of current strength through the series coils and armature shall diminish the M. M. F. of the field magnets to the degree necessary to compensate for the drop of pressure in the armature winding. (See § 148.)

If constant speed is required, such as is the case in operating silk mills and textile machinery, the compound motor will therefore be found to give the best satisfaction, since in shunt motors, although running with "practically constant" speed, the variation may be too great to be without influence upon the product of manufacture.

When started without load the speed of a shunt motor gradually increases and reaches a maximum, from which it falls down again as soon as the load is put on. The rise at no load is due to the fact that since the potential at the field terminals is constant, the field current decreases as the resistance of the field coils increases, owing to their heating, thereby decreasing the magnetizing power, and in consequence the counter E. M. F. of the motor. The subsequent decrease of the speed is caused by the increase of the armature current with increas-

ing load, and by the heating of the armature due to the passing current, the counter E. M. F. decreasing with increasing drop of voltage in the armature. Tests made by Thomas J. Fay [1] with shunt motors of various sizes gave the results compiled in the following Table C.:

TABLE C.—TESTS ON SPEED-VARIATION OF SHUNT MOTORS.

CAPACITY OF MOTOR, HP.	Normal Speed, at No Load, Cold, Revs. per Min.	Increase of Speed from No Load, Cold, to No Load, Hot, Due to Heating of Field Coils.	Decrease of Speed from No Load, Hot, to Full Load, Hot, Due to Heating of Armature.	Final Change in Speed. + = Increase. − = Decrease.
3	1400	20% of normal speed	12% of normal speed	+ 8% of normal speed.
5	1200	8⅓ " "	5 " "	+ 3⅓ " "
7½	1360	5¼ " "	4 " "	+ 1¼ " "
10	1200	2 " "	8⅔ " "	− 6⅔ " "
15	1180	2½ " "	3½ " "	− 1 " "
20	860	½ " "	4 " "	− 3½ " "

From this table it will be seen that the resistance of the field and of the armature can be so proportioned with relation to each other that the final speed at full load hot is equal to the normal speed at no load cold. But in order to reduce to a minimum the variation of the speed during the period of heating up of the motor, it is necessary that both the increase due to the heating of the magnet coils and the decrease due to the heating of the armature should be reduced as much as possible. For this purpose the field winding should be so proportioned as not to heat very much above the temperature of the surrounding air, and the armature resistance should be as low as possible.

115. Calculation of Current for Electric Motors.

a. Current for Any Given Load.

The current in the armature of a motor for any load, P''_x watts $= 746 \times hp_x$ horse power, at the pulley, since at any instant the entire energy supplied to the motor must be equal to the sum of the expenditures, can be found from the equation:

$$E \times (I'_x + I_{sh}) = P''_x + I'^2_x \times (r'_a + r'_{se}) + E \times I_{sh} + P_o,$$

[1] "Constant Speed Motors," by Thomas J. Fay, *Electrical Age*, vol. xv. p. 38 (January 19, 1895).

which gives:

$$I'_x = \frac{E - \sqrt{E^2 - 4(r'_a + r'_{se}) \times (P''_x + P_o)}}{2(r'_a + r'_{se})}, \quad (391)$$

where E = line potential supplied to motor terminals, in volts;
I'_x = current in armature of motor, in amperes, for any given load;
I_{sh} = current in shunt field of motor, in amperes;
P''_x = useful load of motor, in watts;
P_o = energy required for no load, in watts;
r'_a = armature resistance, in ohm;
r'_{se} = series field resistance, in ohm.

Formula (391) directly applies to series- and compound-wound motors; in case of shunt-wound motors, r'_{se} being $= 0$, it reduces it to:

$$I'_x = \frac{E - \sqrt{E^2 - 4\,r'_a \times (P''_x + P_o)}}{2\,r'_a}. \quad ..(392)$$

b. Current for Maximum Commercial and Electrical Efficiency.[1]

As the energy commercially utilized in a motor is:

$$P''_2 = E \times I' - I'^2 \times (r'_a + r'_{se}) - P_o$$

and the entire energy supplied is:

$$P''_2 = E \times I' + P_{sh};$$

the *commercial* efficiency can be expressed by

$$\eta_c = \frac{E \times I' - I'^2 \times (r'_a + r'_{se}) - P_o}{E \times I' + P_{sh}},$$

and similarly the *electrical* efficiency, by:

$$\eta_e = \frac{E \times I' - I'^2 (r'_a + r'_{se})}{E\,I' + P_{sh}},$$

P_{sh} being the energy absorbed in the shunt.

[1] "Shunt Motors," by W. D. Weaver, *Electrical World*, vol. xxi. p. 137, (February 25, 1893).

These efficiencies are maxima for:

$$I = \sqrt{\frac{P_{sh} + P_o}{r'_a + r'_{se}} + \left(\frac{P_{sh}}{E}\right)^2} - \left(\frac{P_{sh}}{E}\right), \qquad (393)$$

and

$$I = \sqrt{\frac{P_{sh}}{r'_a + r'_{se}} + \left(\frac{P_{sh}}{E}\right)^2} - \left(\frac{P_{sh}}{E}\right), \qquad (394)$$

respectively. Formula (393), therefore, gives the current that must be supplied to the armature of a motor in order to have the maximum commercial efficiency, and formula (394) the current for maximum electrical efficiency.

116. Designing of Motors for Different Purposes.

According to the purpose a motor has to serve, its efficiency is desired to either be high and nearly constant over a wide range of its load, or to increase in proportion with the output and be highest at the maximal load the motor can carry.

The shape of the efficiency curve of a motor depends upon the proportioning of its various losses. The losses in a motor are of two kinds, fixed and variable. The fixed losses are those due to the shunt field current, hysteresis, and eddy currents, brush friction, bearing friction, and air resistance. The variable losses are those due to armature and series resistance, and to commutation, and increase with the load. If the fixed losses are small compared with the variable ones, the efficiency at light loads will be high and will rapidly drop as the load, and with it the variable loss, increases. If, on the other hand, the fixed losses are very large, and the variable losses small, the efficiency with small loads will be low, but will increase as the load becomes greater, for the reason that the total energy increases proportional to the load while the losses in this case remain nearly constant, increasing but very little with the load.

In order to have the fixed losses in a motor small and the variable losses great, it is necessary to employ a massive magnetic circuit with few shunt ampere-turns, an ample cross-section of iron in the armature core, and a large number of turns on armature and series field; hence the energy lost in shunt field excitation, in hysteresis, and eddy currents is small, but that lost by armature and series field resistance and by com-

mutation is great. The reverse of these conditions insures an increase in the fixed, or a decrease in the variable, losses.

Curves I. and II., Fig. 315, show the variation of the commercial efficiency with the load in two motors of different design, both having the same efficiency, η_c = 80 per cent. at

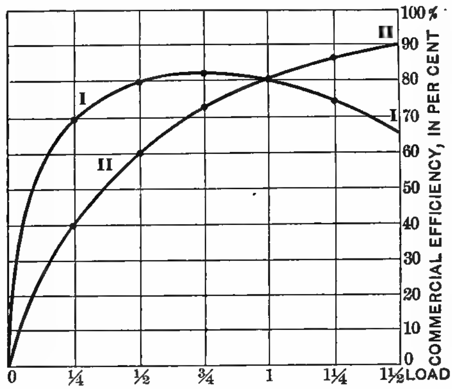

Fig. 315.—Efficiency Curves of Two Motors of Different Design.

normal load, but I. having very high efficiencies at light loads, while II. has very low efficiencies at small loads, but even greater than normal efficiency with overloads:

TABLE CI.—COMPARISON OF EFFICIENCIES OF TWO MOTORS BUILT FOR DIFFERENT PURPOSES.

	Efficiency at Various Loads.					
	¼ Load.	½ Load.	¾ Load.	Normal Load.	25 Per cent. Overload.	50 Per cent. Overload.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
I.	70	80	83	80	73	65
II.	40	60	72	80	86	90

An efficiency curve similar to I. is desired in constant power work where the greatest load is put on the motor but once in starting, and where, after the friction of rest has been overcome, the motor is called upon to work on half to three-fourths its normal output continually; motors, consequently, which are to be employed for running printing presses, machine-

shop tools, power pumps, etc., must be designed with a heavy frame of low magnetic density, a weak field, small excitation, and a powerful armature. In order to obtain an efficiency curve similar to II., which is preferable in all cases where the motor is not doing steady work, but is called upon to give more than its normal power at frequent intervals, as, for instance, in operating electric railways, elevators, cranes, hoists, etc., the motor must be provided with a light frame of high magnetic density, a strong field, powerful excitation, and a weak armature.

117. Railway Motors.

a. Railway Motor Construction.[1]

The construction of motors used for railway propulsion deviates in many respects, electrically as well as mechanically, from that of ordinary motors. The principal conditions that must be fulfilled in the design of a railway motor are the following:

(1) The motor should be extremely compact, so that it may be easily placed in the space available within the truck; yet it must be easily accessible, and all its parts subject to wear must be easily exchangeable. All parts of the machine must furthermore be so designed and the winding so executed that the continual vibrations due to the motion of the car are unable to loosen the same, or to get them out of working order.

(2) A railway motor must be so designed that with minimum weight a maximum output is obtained.

(3) The speed of the armature must be properly chosen with regard to the minimum and maximum load, to the speed of the car, the diameter of the car wheels, and the ratio of speed reduction.

(4) The regulation of the speed should be simple, reliable, and perfectly adapted to all grades and curvatures of the track.

(5) The type of the motor should be so chosen, and the design so carried out, that there is no external magnetic leakage,

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[1] See "Praktische Gesichtspunkte für die Konstruction von Motoren für Strassenbahnbetrieb," by Emil Kolben, *Elektrotechn. Zeitschrift,* vol. xiii. No. 34 and 35 (August 19 and 26, 1892).

that at the same time all the vital parts of the motor are protected from mechanical injuries, and that it can be so supported from the truck that, if possible, none of its weight is resting directly upon the car axle. Particular care must also be bestowed upon the selection of insulating materials and the manner of insulation, in order to guard the machine against the influence of dampness, mud, and water.

(1) *Compact Design and Accessibility.*

Since it is usual to equip each car with two motors which are directly suspended from the car axles and the frame of the truck, the extreme dimensions of the motor are limited by the diameter of the wheels, their distance apart longitudinally, and by the gauge of the track. The trucks most commonly used have 30 or 33-inch wheels, a wheel base of 6 to 7 feet, and the standard gauge of 4 feet 8½ inches. The height of the motor is further limited by the condition that a space of at least 3 inches should be left between the lowest point of the motor and the top of the rails in order to enable the motor to pass over stones or other small obstructions upon the track. The arrangement should be such that the working parts can be easily inspected during the trip from a trapdoor in the flooring of the car. If it is impracticable to provide the car-barn with pits below the tracks, the motor should be so arranged that the armature, the field coils, and the brushes can be taken out through the same trapdoor. In order to facilitate the quick replacing of a disabled armature, it is advisable to split the motor frame horizontally, and to make one part revolvable by means of strong hinges.

(2) *Maximum Output with Minimum Weight.*

The energy required for propelling a car being proportional to its weight, it must be the aim to make the entire equipment as light as is consistent with strength and durability. In order to reduce the weight of the motor to a minimum, it is of the utmost importance to use only the best materials suitable for the respective parts, namely, the softest annealed sheet iron for the armature core, silicon bronze or drop-forged copper

for the commutator segments, and softest cast steel for the field frame. If reduction gears are used, the pinions should be of hard bronze or of good tool steel, and the gear wheels of cast steel, or of fine grain cast-iron. In order to obtain the maximum possible output, the magnetic circuit of the motor should have as small a reluctance as possible, and the magnetic leakage should likewise be reduced as much as possible. The former is attained by the use of toothed or perforated armatures with very small air gaps; and the latter by proper selection of the type. The armature should be made most effective by providing it with a great number of turns; the sparking which would thus result under ordinary conditions being checked by the use of carbon brushes which are set radially in order to enable reversibility in the direction of rotation of the motor. The weight efficiency of various railway motors is given in Table CII., p. 435.

(3) *Speed, and Reduction Gearing.*

The speed of the motor naturally depends upon the car velocity desired, upon the size of the car wheels, and upon the method used for the mechanical transmission of the motion from the armature-shaft to the car axle. The maximum speed of the car, according to local conditions (size of town, amount of traffic in streets, etc.) varies from 8 to 15 miles per hour, the greatest speed of the car axle, therefore, provided that 30-inch wheels are used with the slow, and 33-inch wheels with the fast running cars, ranges between 90 and 150 revolutions, respectively.

The methods of transmission most commonly employed in electric railway cars are the double and single spur gearing, and the direct coupling; worm gearing, bevel gearing, link-chains, and crank-rods being used only in single cases. The employment of *double spur gearing* was necessary with the earlier railway motors which were run at from 1000 to 1200 revolutions per minute, and which, therefore, had to have their speed reduced in the ratio of from 10:1 to 15:1. High-speed railway motors, however, on account of the noise and wear connected with the presence of four gear wheels for each motor, that is eight gears per car, proved too inconvenient

and too expensive to maintain, and low-speed motors of from 400 to 500 revolutions per minute, necessitating but a *single* spur gearing with a reduction ratio of from 4 : 1 to 5 : 1, were next resorted to. If the spur gears for such single reduction motors are provided with broad and carefully cut teeth, and are run in oil, both noise and wear are very small, and the efficiency is comparatively high. *Worm gearing* can be employed for any speed ratio within the limits of railway motor reduction, and by proper design very high efficiencies may be attained. If the worm is carefully cut from a solid piece of tool steel, and the rim of the worm wheel made of hard phosphor bronze, and if the dimensions are so chosen that an initial speed of 20 to 40 feet per second is obtained, the efficiency when run in oil may reach 90 per cent. and over.[1] If no speed reduction at all is desired, that is to say, if the motor is to be directly coupled with the car axle, its normal speed must be between 100 and 150 revolutions per minute. From tests made by Professor S. H. Short,[2] the saving of power consumed in operating a directly coupled, gearless street car motor is found to be from 10 to 30 per cent. as compared with double spur gearing, and from 5 to 10 per cent. as compared with single spur gearing, according to the load.

In order to show what has been done in the way of compact design and weight-efficiency of railway motors of various speed reductions, the following Table CII. has been prepared, giving the specific weight, the speed, kind and ratio of reduction, the type and dimensions of the frame, the space-efficiency, and the size of the armature, of the most common railway motors in practical use. The figures given for the dimensions of the field frame do not include any supporting or suspension brackets, lugs, or other extensions that may be attached to, or cast in one with the frame, but relate only to the magnetic portion of the field casting. This is done to bring all the space efficiencies to a common basis, thus enabling a fair comparison of the various types:

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[1] See "Schneckengetriebe in Verbindung mit Elektromotoren," by Emil Kolben, *Elektrotechn. Zeitschr.*, vol. xvi. p. 514 (August 15, 1895).

[2] "Gearless Motors," by Sidney H. Short, *Electrical Engineer*, vol. xiii. p. 386 (April 13, 1892); *Electrical World*, vol. xix. p. 263 (April 16, 1892).

TABLE CII.—GENERAL DATA OF MOST COMMON RAILWAY MOTORS.

NAME OF MOTOR.	Capacity. HP.	Weight Complete, Including Gears. Lbs.	Weight Efficiency. Lbs. p. HP.	Speed at Normal Load. Revs. p. Min.	Ratio of Speed Reduction.	Kind of Gearing.	TYPE.	Number of Poles.	Dimensions of Field Frame. Inches. Length, ∥ Axle.	Width, ⊥ Axle.	Height.	Space Occupied by Field Frame. Cu. Ins.	Space-Efficiency. Cu. ins. per HP.	Size of Armature. Inches. External Diameter.	Length of Core.	Kind of Armature. Sm.=Smooth T.=Toothed
Gen'l Elec., No. 6 Edison	15	1,765	117.7	1,130	9 : 1	D. Spur	Horseshoe	2	10¼	24½	15	3,770	251	9¾	10¼	Sm. Drum
" " No. 14 Edison	20	1,600	80.0	450	5 : 1	S. Spur	Iron Clad	4	11½	26	18½	5,530	277	15⅞	11⅜	T. Ring
" " No. 16 Edison	30	2,270	75.7	410	4.78 : 1	" "	" "	4	13¼	30½	23½	9,500	317	18¼	13⅛	" "
" " G. E. 800......	25	1,685	68.4	590	4.78 : 1	" "	" "	2	20¼	18	24	8,740	350	16	8	T. Drum
" " G. E. 1,000*...	40	2,185	54.6	500	3.94 : 1	" "	" "	4	16⅝	23¼	24	9,270	232	14½	8½	" "
" " G. E. 1,200....	45	2,975	66.2	425	3.53 : 1	" "	" "	4	24⅜	23⅝	24	13,800	307	16	13⅛	" "
" " G. E. 2,000....	100	4,650	46.5	790	3.18 : 1	" "	" "	4	25½	22½	33	18,900	189	18	13¼	" "
Oerlikon Works, 12 P. S.†	12	1,975	164.5	1,150	14 : 1	Worm	Horseshoe	2	22	24	10¾	5,670	473	10½	7½	Sm. Drum
" " 17 P. S.†	17	1,950	114.7	450	4.9 : 1	S. Spur	Radial M. P.	4	24	22½	22½	12,150	715	15	11	T. Drum
Short, Single Reduction...	20	2,220	111.0	550	5 : 1	" "	Axial M. P.	4	32½	24¾	23¼	18,700	935	21¾	5⅜	T. Ring
Walker, No. 3........	25	2,100	84.0	500	4.78 : 1	" "	Iron Clad	4	19⅞	21	22⅝	9,430	377	12¾	7½	T. Drum
" No. 4.........	30	2,250	75.0	500	4.78 : 1	" "	" "	4	22	21	22¼	10,270	342	12¾	8	" "
" No. 5.	35	2,400	68.6	500	4.78 : 1	" "	" "	4	22½	21	22¼	10,500	300	12¾	10	" "
" No. 10........	50	3,040	60.8	550	3.90 : 1	" "	" "	4	26¼	23¾	25¾	16 050	321	15	10	" "
" No. 15........	75	4,100	54.7	600	3.29 : 1	" "	" "	4	29½	24¾	31	22,600	301	18	8⅝	" "
" No. 20	100	4,700	47.0	600	3.375 : 1	" "	" "	4	24½	26	33¾	21,500	215	18	13	" "
Westinghouse, No. 3‡. ...	30	2,750	91.7	300	3.45 : 1	" "	Radial M. P.	4	21	22⅞	22⅞	11,000	367	11⅝	15	" "
" No. 12A...	30	2,200	73.3	685	4.86 : 1	" "	" "	4	21½	23½	23½	11,350	378	11⅝	8½	" "
" No. 38....	50	2,200	44.0	520	3.56 : 1	" "	" "	4	22¾	24	24	13,100	262	13⅞	11	" "

* Curves showing the performance of this motor are given in *Electrical World*, vol. xxviii. p. 176 (August 8, 1896).

† The dimensions given for the Oerlikon motors are rough approximations only, the correct figures not having been at hand.

‡ For description and test of this motor see "Dynamo Electric Machinery," by Silvanus P. Thompson, 5th edition, p. 542.

(4) *Speed Regulation.*

In order to effect the variation of the speed of railway motors within wide limits it is desirable that their field magnets should be series wound. The strength of the magnetic field can then be regulated either by inserting resistance into the main circuit, in connection with partial short-circuiting of the field coils, or by altering the combination of the magnet spools, or by series-parallel grouping of the armatures and field coils of the two motors.

In the *Resistance Method* the insertion of rheostat-resistance into the main circuit, by reducing the effective E. M. F., causes a decrease in the speed of the motor; in this case the cross-section of the magnet wire must be so dimensioned as to carry the maximum current, but the number of turns must be chosen far greater than is required for the production of the requisite number of ampere-turns at maximum current and maximum speed. For, almost the full field strength must be obtained with a comparatively small current-intensity, and it it therefore necessary to short-circuit a portion of the magnet coils at maximum load. That is to say, in order to raise the torque of the motor for increased loads, only one of the two factors determining the same is increased, namely the current strength in the armature, while the field current remains the same. In order to do this without excessive sparking, caused by the fact that the brushes, not being adjustable, are never at the neutral points of the resultant field, carbon brushes must be used, whose large contact resistance considerably reduces the current in the coils short-circuited by the brushes.

The *Combination Method* of speed regulation consists in suitably changing the grouping of the magnet-spools. For this purpose it is necessary to wind the magnet coil in sections, equal portions of which are placed on each magnet, and to connect the terminals of these sections, usually three in number, to a switch, or controller, of proper design. At the maximum load of the motor the three sections are connected in parallel, and for this combination, therefore, the cross-section of the winding is to be calculated. For starting the car all sections are connected in series, and, if no precaution were taken, the magnet winding would, in consequence, have to

carry the full starting current, which may be 4 to 6 times the maximum normal current. In order to avoid overheating and damage due to this starting current, a starting rheostat must be placed in circuit, the resistance of this rheostat being so dimensioned that the starting current is brought down in strength to that of the maximum working current.

While with the two former methods of speed regulation the two motors of the car are permanently connected in parallel, in the *Series-Parallel Method* of control, finally, both the armatures and magnet-coils of the two motors can be grouped in any desired combination. The same number of combinations is therefore possible with less elements, and only two sections per magnet-coil are necessitated. Since by placing both armatures and all four field-sections in series the starting current is considerably reduced, less resistance is needed in the starting rheostat, and a saving of energy is effected by this method. For calculating the carrying capacity of the magnet-wire the last two positions of the series-parallel controller are essential: for maximum speed the two motors, each having one coil cut out, are placed in parallel; and in the position for the next lower speed both motors with their two coils in series are grouped in parallel.

(5) *Selection of Type.*

The most important consideration in the selection of the type for a railway motor is the condition that there should be no external magnetic leakage, as otherwise the neighboring iron parts of the truck may seriously influence the magnetic distribution, and, furthermore, small iron objects, such as nails, screws, etc., may be attracted into the gap-space and may injure the armature. In order to protect the motor from dampness and mechanical injuries, such types are to be preferred in which the yoke surrounds the armature, and which therefore can easily be so arranged that the frame completely encases all parts of the machine. The types possessing the latter feature are the iron-clad types, Figs. 203 to 207, §72, and Figs. 217 to 220, §73, the radial outerpole type, Fig. 208, and the axial multipolar type, Fig. 212; and as can be seen from the preceding Table CII., these are in fact the forms of machines that are used in modern railway motor design.

b. CALCULATIONS CONNECTED WITH RAILWAY MOTOR DESIGN.[1]

(1) *Counter E. M. F., Current, and Energy Output of Motor.*

Inserting into the formula for the counter E. M. F.,

$$E' = \frac{N_c}{n'_p} \times \frac{N}{60} \times \Phi \times 10^8,$$

the value of the useful flux from §§ 86 and 87,

$$\Phi = \frac{\mathfrak{F}}{\mathcal{R}} = \frac{\frac{4\pi}{10} \times AT}{\mathcal{R}} = \frac{N_{se} \times I}{\frac{10}{4\pi} \times \mathcal{R}} = \frac{N_{se} \times I}{\mathcal{R}'},$$

where $\mathfrak{F}$ = magnetomotive force, in gilberts;
$AT = N_{se} \times I$ = magnetizing force, in ampere-turns;
$\mathcal{R}$ = reluctance of magnetic circuit, in oersteds;
$\mathcal{R}' = \frac{10}{4\pi} \times \mathcal{R} = \frac{10}{4\pi} \times \frac{1}{\mu} \times \frac{l''_m}{S_m}$,
μ = permeability of magnet-frame, at normal load;
l''_m = length of magnetic circuit, in inches;
S_m = area of magnet-frame, in square inches;

we obtain:

$$E' = \frac{N_c \times N_{se}}{\mathcal{R}'} \times \frac{I}{n'_p} \times \frac{N}{60} \times 10^8. \quad ..(395)$$

If the internal resistance of the motor, *i. e.*, armature resistance plus series field resistance, is designated by r, and the line potential by E, the current flowing in the armature, therefore is:

$$I = \frac{E - E'}{r} = \frac{E - \frac{N_c \times N_{se}}{\mathcal{R}'} \times \frac{I}{n'_p} \times \frac{N}{60} \times 10^8}{r},$$

[1] See "Some Practical Formulæ for Street-Car Motors," by Thorburn Reid, *Electrical Engineer*, vol. xii. p. 688 (December 23, 1891); "Capacity of Railway Motors," by E. A. Merrill, *Electrical Engineer*, vol. xvii. p. 231 (March 14, 1894).

and solving for I, we have:

$$I = \frac{E}{r + \frac{N_c \times N_{se}}{\mathcal{R}'} \times \frac{1}{n'_p} \times \frac{N}{60} \times 10^8}. \quad (396)$$

Hence the work done by the motor:

$$P'' = E' \times I = \frac{N_c \times N_{se}}{\mathcal{R}'} \times \frac{I^2}{n'_p} \times \frac{N}{60} \times 10^8. \quad (397)$$

N_c, N_{se}, and n'_p are constants of the motor, and $\mathcal{R}'$ varies somewhat with the saturation of the field, but may be considered practically constant; if, therefore, we unite all constants by substituting:

$$K = \frac{N_c \times N_{se}}{\mathcal{R}'} \times \frac{1}{n'_p} \times \frac{10^8}{60},$$

the above formulæ (395), (396), and (397) become:

$$E' = K \times I \times N, \quad \ldots\ldots\ldots\ldots (398)$$

$$I = \frac{E}{r \times K \times N}, \quad \ldots\ldots\ldots\ldots (399)$$

and

$$P'' = K \times I^2 \times N. \quad \ldots\ldots\ldots\ldots (400)$$

The value of the constant K can be readily calculated from the windings of the machine and from the dimensions and flux densities of its magnetic circuit. If, however, the values of E, I, and N for any load are given, and it is required to find the counter E. M. Fs., the currents, and the mechanical outputs for other loads, then K can, far simpler and more accurately, be determined by substituting the given values in:

$$K = \frac{E - I \times r}{I \times N}; \quad \ldots\ldots\ldots\ldots (401)$$

which is obtained from (399) by transformation.

(2) *Speed of Motor for Given Car Velocity.*

The speed of the motor required to move the car at a given velocity, with a given reduction gear, is:

$$N = \frac{\text{feet per min.}}{\frac{d_w}{12} \times \pi} \times z = \frac{5280 \times 12 \times v_m \times z}{60 \times \pi \times d_w}$$

$$= \frac{304 \times v_m \times z}{d_w}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(402)}$$

in which v_m = speed of car, in miles per hour;
z = ratio of speed reduction, *i. e.*, ratio of armature revolutions to those of the car axle;
d_w = diameter of car wheel, in inches.

(3) *Horizontal Effort, and Capacity of Motor Equipment for Given Conditions.*

The power required to propel a car depends upon five things: friction, grade, condition of track, curvature of track, and speed. No accurate formula can be given for the resistance due to friction, condition of track, and curvature, for this resistance will vary largely at different times with the same car, depending upon the care with which the bearings and gears are oiled, and whether the track is wet or dry, clean or dusty, or muddy. A good average practical value of the specific traction resistance, verified by numerous tests, is 30 pounds per ton of weight on the level, and $(30 \pm 20 \times g)$ pounds per ton on grades, g being the percentage of the grade, that is, the number of feet rise or fall, respectively, in a length of 100 feet. The horizontal force necessary to overcome the traction-resistance caused by a total weight of W_t tons, therefore, is:

$$f_h = W_t \times (30 \pm 20 \times g) \text{ pounds}, \quad \ldots\ldots \mathbf{(403)}$$

and the power, in watts, required to exert this horizontal effort, at a speed of v_m miles per hour, will be:

$$P'' = \frac{f_h \times \text{ft. per min.} \times 746}{33{,}000}$$

$$= \frac{f_h \times \frac{5280}{60} v_m \times 746}{33{,}000} = 2\, f_h \times v_m. \quad \ldots\ldots \mathbf{(404)}$$

In order to facilitate the calculation of the propelling power, or of the motor capacity required for given conditions of traction, the following Table CIII. has been calculated, which gives the power required to propel one ton at different grades and speeds, and which, therefore, furnishes P'' by simply multiplying the respective table-value by the total weight, W_t tons, to be propelled, *i. e.*, the weight of car plus passengers (average weight of passenger = 125 lbs.):

TABLE CIII.—SPECIFIC PROPELLING POWER REQUIRED FOR DIFFERENT GRADES AND SPEEDS.

PERCENTAGE OF GRADE,	HORSE-POWER REQUIRED TO PROPEL 1 TON, IF RATED SPEED OF CAR, v_m, IN MILES PER HOUR, IS:							
g	8	10	12	15	18	20	25	30
0	.64	.80	.96	1.21	1.45	1.61	2.01	2.41
1	1.07	1.34	1.61	2.01	2.41	2.68	3.35	4.02
2	1.50	1.88	2.25	2.82	3.38	3.76	4.69	5.63
3	1.93	2.41	2.90	3.62	4.34	4.83	6.03	7.24
4	2.36	2.95	3.54	4.42	5.31	5.90	7.37	8.85
5	2.78	3.48	4.17	5.22	6.26	6.97	8.71	10.44
6	3.22	4.02	4.83	6.03	7.23	8.05	10.04	12.06
7	3.65	4.56	5.47	6.84	8.20	9.12	11.40	13.67
8	4.07	5.09	6.11	7.63	9.15	10.18	12.73	15.28
9	4.50	5.62	6.75	8.43	10.10	11.25	14.07	16.89
10	4.93	6.16	7.39	9.24	11.07	12.32	15.40	18.50
12	5.78	7.23	8.68	10.84	13.01	14.47	18.10	21.70
15	7.07	8.84	10.60	13.25	15.90	17.70	22.10	26.55

From (404) the horizontal pull required to exert a given power at given speed is found thus:

$$f_h = \frac{33,000 \times 60}{5280} \times \frac{P''}{746} \times \frac{1}{v_m} = 375 \times \frac{hp}{v_m}. \quad (405)$$

Giving to hp values from 15 to 60 horse-power, and to v_m from 8 to 30 miles per hour, the following Table CIV. is obtained, which at a glance gives the horizontal effort, or draw-bar pull, exerted by any motor-capacity at a given speed, whereupon, from (403), the load W_t, in tons, can be computed, which the equipment under consideration is able to propel at any given grade:

TABLE CIV.—HORIZONTAL EFFORT OF MOTORS OF VARIOUS CAPACITIES AT DIFFERENT SPEEDS.

RATED CAPACITY OF MOTOR EQUIPMENT.	PULL AT PERIPHERY OF WHEEL, f_h, IN POUNDS, AT RATED SPEED OF CAR, v_m, IN MILES PER HOUR, OF:							
hp	8	10	12	15	18	20	25	30
15	703	563	469	375	313	281	225	188
20	938	750	625	500	417	375	300	250
25	1,172	938	781	625	521	469	375	313
30	1,406	1,125	938	750	625	563	450	375
40	1,875	1,500	1,250	1,000	833	750	600	500
50	2,344	1,875	1,562	1,250	1,043	938	750	625
60	2,812	2,250	1,875	1,500	1,250	1,125	900	750

A simple graphical method of determining the car velocity and the current consumption under various conditions of traffic is shown in § 133, Chapter XXVIII.

(4) *Line Potential for Given Speed of Car and Grade of Track.*

The E. M. F. required at the motor terminals to drive a car up a particular grade at a certain rate of speed may be found as follows. From (399) we have:

$$E = I \times (r + K \times N), \quad \ldots\ldots\ldots (406)$$

in which everything is known except E and I. But I can be obtained from formula (400), provided we know the work P'' that is to be done by the motor under the prevailing conditions. The value of P'' being given by (404), the current I can be expressed by transposition of formula (400), and by substituting the expression so found into (406) the required E. M. F. is obtained:

$$E = (r + K \times N) \times \sqrt{\frac{2\, f_h \times v_m}{K \times N}}. \quad \ldots (407)$$

Inserting into (407) the value of N found from (402), we have:

$$E = .81 \times \left(r + \frac{304 \times K \times v_m \times z}{d_w} \right) \times \sqrt{\frac{f_h \times d_w}{K \times z}}. \quad (408)$$

Knowing E, we are enabled to determine the size of wire required in the feeders to maintain a certain speed at any point on the line.

CHAPTER XXVI.

CALCULATION OF UNIPOLAR DYNAMOS.

118. Formulæ for Dimensions Relative to Armature Diameter.

Assuming the armature diameter of a unipolar dynamo as given, the ratio of the working density of the lines in the material chosen for the frame to the flux-density permissible in the air gaps will determine the dimensions of the frame. The armature consisting in a solid iron or steel core without winding, the only air gap necessary is the clearance required for untrue running, and, on account of the short air gaps so obtained, a comparatively high field density, namely, $\mathcal{H}'' =$ 40,000 lines per square inch (or $\mathcal{H} =$ 6200 lines per square centimetre) can be admitted. The practical working densities, as given in Table LXXVI., § 81, are:

$\mathcal{B}'' =$ 90,000 lines per square inch ($\mathcal{B} =$ 14,000 lines per square centimetre), for wrought iron,
$\mathcal{B}'' =$ 85,000 lines per square inch ($\mathcal{B} =$ 13,200 lines per square centimetre), for cast steel, and
$\mathcal{B}'' =$ 45,000 to 40,000 lines per square inch ($\mathcal{B} =$ 7000 to 6200 lines per square centimetre), for cast iron.

By comparison, then, it follows that the area of the gap spaces should be about twice the cross-section of the frame, if wrought iron or cast steel is used, and about equal to the frame section if cast iron is employed.

The cylinder type, on account of its smaller diameter and more compact form, being more practical than the disc type of unipolar machines, the former only will here be considered, inasmuch as it will not be difficult to derive similar formulæ for the latter. Moreover, since for the same size of armature

a cast-iron frame requires about twice the weight of a cast-steel one, the use of the former material is limited to special cases, and formulæ are given only for machines having cast-steel magnets.

Adopting the general design indicated by Fig. 31, § 11, good practical dimensions of the frame are obtained by making the

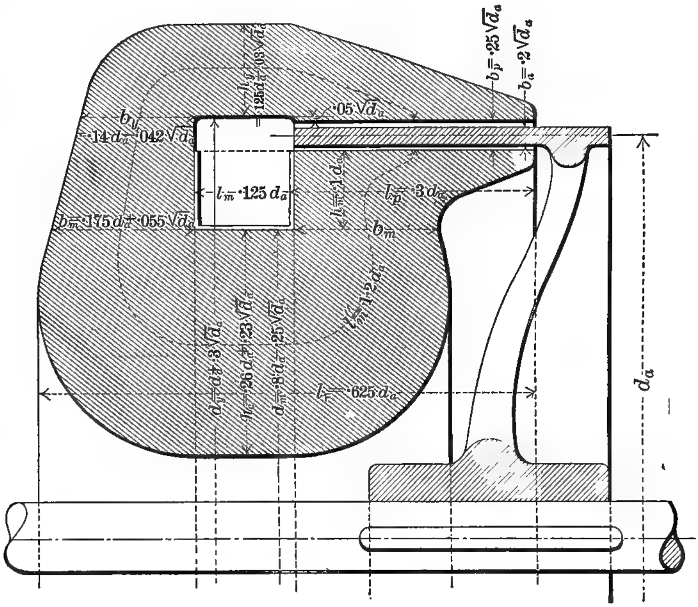


Fig. 316.—Dimensions of Cast Steel Unipolar Cylinder Dynamo.

active length of the armature conductor, that is, the length of the poles, see Fig 316:

$$l_p = .3d_a, \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(409)}$$

d_a being the mean diameter of the armature-cylinder; and by providing for the winding an annular space of length:

$$l_m = .125d_a, \quad \ldots\ldots\ldots\ldots\ldots \textbf{(410)}$$

and height:

$$h_m = .1d_a, \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(411)}$$

The gap area, then, will be:

$$S_g = d_a \pi \times .3d_a = .94d_a^2,$$

and the cross-section of the magnet frame, in order to have a magnetic density of 85,000 lines per square inch, must be:

$$S_m = .94 \times \frac{40,000}{85,000} \times d_a^2 = .44\, d_a^2.$$

The radial thickness of the armature, being that of the rim of a pulley of diameter d_a, is taken:

$$b_a = .2\sqrt{d_a}, \quad \ldots\ldots\ldots\ldots\ (412)$$

which, by adding

$$.05\sqrt{d_a}$$

for clearance, makes the total distance between the two pole faces:

$$b_p = .25\sqrt{d_a}. \quad \ldots\ldots\ldots\ldots (413)$$

Allowing $.05\sqrt{d_a}$ for the recess at the outer pole face, the internal diameter of the yoke is found:

$$d_y = d_a + .3\sqrt{d_a};$$

and the diameter at the bottom of the annular winding groove, or the diameter of the magnet core is:

$$d_m = d_a - .25\sqrt{d_a} - 2 \times .1 d_a = .8 d_a - .25\sqrt{d_a}.$$

The thicknesses of the frame section at these diameters must be:

$$b_y = \frac{S_m}{d_y \times \pi} = \frac{.44\, d_a^2}{(d_a + .3\sqrt{d_a})\,\pi} = .14\, d_a - .042\sqrt{d_a}, \quad (414)$$

and

$$b_m = \frac{S_m}{d_m \times \pi} = \frac{.44 d_a^2}{(.8 d_a - .25\sqrt{d_a})\,\pi}$$

$$= .175 d_a + .055\sqrt{d_a}, \quad \ldots\ldots\ldots\ldots\ \ldots\ldots\ldots (415)$$

respectively.

For the radial thicknesses of the outer and inner tube portions of the field frame we have the equations:

$$h_y = \frac{S_m}{(d_y + h_y)\,\pi} = \frac{.44 d_a^2}{(d_a + .3\sqrt{d_a} + h_y)\,\pi} = \frac{.14 d_a^2}{d_a + .3\sqrt{d_a} + h_y}$$

and

$$h_c = \frac{S_m}{(d_m - h_c)\pi} = \frac{.44 d_a^2}{(.8\,d_a - .25\sqrt{d_a} - h_c)\pi}$$

$$= \frac{.14\,d_a^2}{.8d_a - .25\sqrt{d_a} - h_c},$$

respectively, from which we obtain, for the radial thickness of the yoke:

$$h_y = -\frac{d_a + .3\sqrt{d_a}}{2} + \sqrt{\frac{(d_a + .3\sqrt{d_a})^2}{4} + .14d_a^2}$$

$$= .125d_a - .03\sqrt{d_a}, \quad \text{.......................... (416)}$$

and for the radial thickness of the core portion of the frame:

$$h_c = -\frac{.8d_a - .25\sqrt{d_a}}{2} - \sqrt{\frac{(.8d_a - .25\sqrt{d_a})^2}{4} - .14d_a^2}$$

$$= .26d_a + .23\sqrt{d_a}. \quad \text{........................ (417)}$$

The total axial length of the frame is:

$$l_F = .3d_a + .125d_a + 2d_a = .625d_a; \quad \text{..... (418)}$$

and for the mean length of the magnetic circuit in the frame we find by scaling the path:

$$l''_m = 1.2d_a. \quad \text{.............. (419)}$$

119. Calculation of Armature Diameter and Output of Unipolar Cylinder Dynamo.

All the dimensions of the machine being given, by § 118, as multiples of the armature diameter, d_a, the dimensioning of the frame is reduced to the calculation of d_a.

In order to obtain a formula for the armature diameter, we express the polar area in two ways: electrically, as the quotient of flux, Φ, and field density, $\mathcal{H}''$, and geometrically, as a cylinder surface of diameter d_a and length $.3d_a$.

The number of parallel circuits as well as the number of conductors in the unipolar armature is unity, and the lines are cut but once in each revolution, the useful flux necessary to

generate E volts at the speed of N revolutions, therefore, from formula (137), § 56, is:

$$\Phi = \frac{.6 \times E \times 10^9}{N},$$

hence the gap area can be expressed, electrically, by:

$$S_g = \frac{\Phi}{\mathcal{H}''} = \frac{6 \times E \times 10^9}{N \times \mathcal{H}''}, \quad \text{.........(420)}$$

while geometrically we have from Fig. 316:

$$S_g = d_a \times \pi \times .3 d_a = .94 d_a^2. \quad \text{.....(421)}$$

Equating (420) and (421), we obtain:

$$.94 d_a^2 = \frac{6 \times E \times 10^9}{N \times \mathcal{H}''},$$

from which follows:

$$d_a = 80{,}000 \times \sqrt{\frac{E}{N \times \mathcal{H}''}}. \quad \text{......(422)}$$

Inserting in this the value of the field density given in § 118, namely, $\mathcal{H}'' = 40{,}000$ lines per square inch, we find:

$$d_a = 400 \times \sqrt{\frac{E}{N}}, \quad \text{..........(423)}$$

in which d_a = mean diameter of armature, in inches;
E = E. M. F. required, in volts;
N = speed, in revolutions per minute.

From (423) the armature diameter of a unipolar cylinder dynamo can be computed which generates the required E. M. F. at a *given* speed. If, however, the minimum value of d_a, at the maximum safe speed permissible, is desired, N must be eliminated from the above equation (423), and replaced by the peripheral velocity. For this purpose the value of d_a from (423) is inserted into the equation

$$d_a = 230 \times \frac{v_c}{N} \text{ [see (30), § 21];}$$

by this process we obtain:

$$400 \times \sqrt{E \times N} = 230\, v_c,$$

or:

$$N = .33 \times \frac{v_c^2}{E}, \quad \text{.............(424)}$$

and this in (423) gives:

$$d_a = 400 \times \sqrt{\frac{E^2}{.33\, v_c^{\,2}}} = 690 \times \frac{E}{v_c}. \quad ..(425)$$

The values of v_c, *i. e.*, the limiting safe velocities, for the materials used in unipolar armatures are:

$v_c = 400$ feet per second, for forged steel;
$v_c = 300$ feet per second, for wrought iron and cast steel;
$v_c = 200$ feet per second, for cast iron.

Inserting these values into (425), the following formula giving the minimum armature diameter for unipolar cylinder dynamos of E volts E. M. F. are arrived at:

$$\left.\begin{array}{ll} \text{For } \textit{forged steel} \text{ armature:} & d_a = \frac{690}{400} \times E = 1.73\, E. \\ \text{For } \textit{wrought iron} \text{ or } \textit{cast} & \\ \quad \textit{steel} \text{ armature:} & d_a = \frac{690}{300} \times E = 2.3\, E. \\ \text{For } \textit{cast iron} \text{ armature:} & d_a = \frac{690}{200} \times E = 3.45\, E. \end{array}\right\} \quad (426)$$

The corresponding minimum speeds are found by formula (424), as follows:

$$\left.\begin{array}{ll} \text{For } \textit{forged steel} \text{ armature:} & N = .33 \times \frac{400^2}{E} = \frac{53{,}000}{E}. \\ \text{For } \textit{wrought iron} \text{ or } \textit{cast} & \\ \quad \textit{steel} \text{ armature:} & N = .33 \times \frac{300^2}{E} = \frac{30{,}000}{E}. \\ \text{For } \textit{cast iron} \text{ armature:} & N = .33 \times \frac{200^2}{E} = \frac{13{,}200}{E}. \end{array}\right\} \quad (427)$$

The *output* of the machine is limited only by the carrying capacity of the armature; the current carrying cross-section of the latter is:

$$d_a\, \pi \times .2 \sqrt{d_a} = .2\, \pi\, d_a^{\frac{3}{2}},$$

and since iron or steel will carry at least 200 amperes per square inch, the current capacity, in amperes, is:

$$I = 200 \times .2\, \pi \times d_a^{\frac{3}{2}} = 125 d_a^{\frac{3}{2}}, \quad ....(428)$$

which, at E volts E. M. F., gives the output of the dynamo, in watts:

$$P = E \times I = 125 \times E \times d_a^{\frac{3}{2}}. \quad \text{.....}\mathbf{(429)}$$

120. Formulæ for Unipolar Double Dynamo.

In duplicating the design shown in Fig. 316, a unipolar dynamo with an armature of twice the effective length of the former is obtained, Fig. 317.

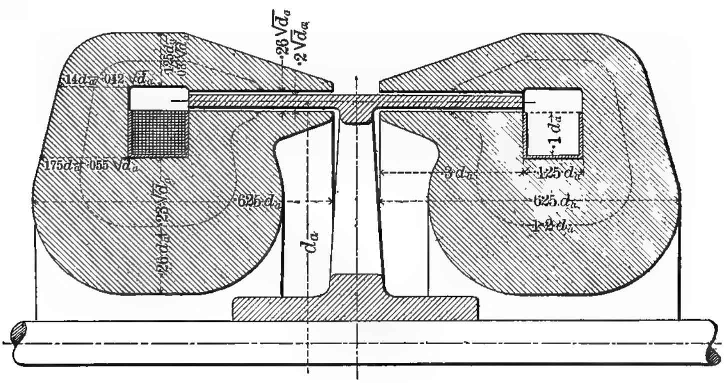


Fig. 317.—Dimensions of Cast-Steel Unipolar Double Dynamo.

The pole area for this type is:

$$S_g = 2 \times d_a \pi \times .3 d_a = 1.88 d_a^2, \quad \text{....}\mathbf{(430)}$$

hence, by equating (430) and (420), the diameter is obtained:

$$d_a = 56{,}400 \times \sqrt{\frac{E}{N \times \mathcal{H}''}}, \quad \text{........}\mathbf{(431)}$$

which, for $\mathcal{H}'' = 40{,}000$ lines per square inch, becomes:

$$d_a = 282 \times \sqrt{\frac{E}{N}}. \quad \text{................}\mathbf{(432)}$$

The minimum diameter which produces E volts is:

$$d_a = 282 \times \sqrt{\frac{E^2}{.33 v_c}} = 490 \times \frac{E}{v_c}, \quad \text{..}\mathbf{(433)}$$

from which,

$$\left.\begin{array}{ll}\text{for } \textit{forged steel} & \\ \quad\text{armature:} & d_a = \dfrac{490}{400} \times E = 1.22\,E; \\ \text{for } \textit{wrought iron} \text{ or } \textit{cast steel} & \\ \quad\text{armature:} & d_a = \dfrac{490}{300} \times E = 1.63\,E; \\ \text{and for } \textit{cast iron} & \\ \quad\text{armature:} & d_a = \dfrac{490}{200} \times E = 2.45\,E. \end{array}\right\} \quad \ldots .(434)$$

For the double machine, the current carrying capacity and the output are found from the same formulæ (428) and (429) respectively, as for the single-frame machine, and since the diameter of the frame is smaller, also its current intensity, and, in consequence, its total output will be smaller than that of a single-cylinder machine of the same E. M. F.

121. Calculation of Magnet Winding for Unipolar Cylinder Dynamos.

The dimensions of both the single and double cylinder types being generally expressed as multiples of the armature diameter, see Figs. 316 and 317, the magnetizing forces required for the various portions of their magnetic circuit can be computed from the following formulæ.

The magnetizing force required for the air gaps, their density being $\mathcal{H}'' = 40{,}000$, is:

$$at_g = .3133 \times 40{,}000 \times .05\,\sqrt{d_a} \times 1.2 = 750\,\sqrt{d_a}, \quad ..(435)$$

where 1.2 is taken to be the probable factor of field deflection, see Table LXVI., § 64.

Magnetizing force required for armature:

$$\left.\begin{array}{l} at_a = m''_a \times .2\,\sqrt{d_a}. \\ \text{Since } \mathcal{B}''_a = 40{,}000 \text{ lines, we have:} \\ \quad \text{for } \textit{wrought iron:}\ at_a = 1.5\,\sqrt{d_a} \\ \quad \text{for } \textit{cast steel:}\ \ \ at_a = 1.8\,\sqrt{d_a} \\ \quad \text{for } \textit{cast iron:}\ \ \ \ at_a = 17.6\,\sqrt{d_a} \end{array}\right\} \quad \ldots .(436)$$

Magnetizing force required for magnet frame (*cast steel* of density $\mathcal{B}''_m = 85{,}000$):

$$at_m = m''_m \times 1.2 d_a = 44 \times 1.2 d_a = 53 d_a. \quad ..(437)$$

There being no armature-reaction, the total number of ampere-turns, AT, required for excitation at full output, see (227), § 89, is the sum of the magnetizing forces obtained by formulæ (435), (436), and (437).

The voltage of a unipolar dynamo being comparatively small, but of constant value if the speed is kept constant, the excitation should be effected by a shunt-winding.

The dimensions of the magnet-coil being fixed by the design, the mean length of one turn, the radiating surface, and the weight, respectively, can be expressed thus:

$$l_T = (d_m + h_m)\,\pi = (.8d_a - .25\sqrt{d_a} + .1d_a)\,\pi$$
$$= 2.83d_a - .785\sqrt{d_a}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(438)}$$
$$S_M = (d_m + 2h_m)\,\pi \times l_m = (d_a - .25\sqrt{d_a})\,\pi \times .125d_a$$
$$= .39d_a^2 - .1\sqrt{d_a^3}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(439)}$$

and

$$wt_m = l_T \times l_m \times h_m \times .21$$
$$= (2.83d_a - .785\sqrt{d_a}) \times .125d_a \times .1d_a \times .21$$
$$= .0074d_a^3 - .002\sqrt{d_a^5}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(440)}$$

The gauge of the wire, then, is determined by means of formula (319), and the temperature increase that is obtained by filling the entire space provided for this purpose, is found from (329). See example, § 149.

If the temperature rise corresponding to the given dimensions should be higher than desired in a particular case, the cross-section of the winding space must be suitably increased, preferably by extending its length, l_m, and a greater weight of wire must be employed.

CHAPTER XXVII.

CALCULATION OF DYNAMOTORS, GENERATORS FOR SPECIAL PURPOSES, ETC.

122. Calculation of Dynamotors.

Dynamo-electric generators which are energized, not by mechanical power, but by the electric current derived from another source of electricity, are, in general, called *Secondary Generators*, and serve the purpose of *transforming* a current of one kind or voltage into a current of another kind or voltage.

According to the nature of the currents to be transformed and to that of the secondary currents generated, secondary generators can be divided into two classes:

(1) Secondary generators for transforming continuous currents of any voltage into a continuous current of any other voltage;

(2) Secondary generators for transforming continuous currents of any voltage into single-phase or polyphase alternating currents of any voltage, or *vice versa.*

Secondary generators of the first class may be of two kinds:

(1) *Motor-dynamos*, or those in which a separate motor operates a dynamo, both machines either being mounted on a common base and having a common shaft, or being entirely separate and having their shafts coupled together;

(2) *Dynamotors*, or those in which the motor and dynamo are placed upon the same armature, or upon two separate armatures revolving in the same magnetic field, the former arrangement being the usual case.

Motor-dynamos and dynamotors are used for the following purposes:

(1) For transforming high-voltage currents transmitted from a central station to distributing centers located at convenient places into currents suitable for lighting, etc.

(2) For transforming ordinary lighting currents into com-

paratively large currents at very low voltage, as used for electro-metallurgical purposes, telegraph or telephone operating, meter testing, etc.

(3) For compensating the drop in voltage on long mains by inserting into the mains at the distant point a series motor driving a generator armature placed as a shunt across the mains. An arrangement of this kind is called a *Booster*.

(4) For charging accumulators at a higher voltage than that of the line, so that lamps may be operated either directly from the circuit or from the cells.

(5) For 3-wire and 5-wire systems of distribution, a number of armatures or windings on the same shaft being connected

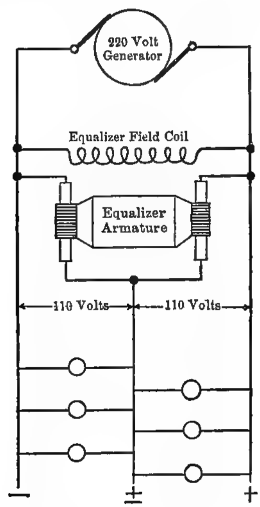


Fig. 317*a*.—Connections of Equalizing Dynamo.

across the various pairs of mains, so that, if the potential drops at any one pair of mains, its armature will feed this pair, driven by the other armatures as motor. Such a device is called an *Equalizing Dynamo*. In the *Eddy Company's* equalizing system, a dynamotor having two 110-volt windings is connected across the terminals of a 220-volt generator for the purpose of enabling its use for supplying a 110-volt 3-wire system. The two dynamotor armature windings are con-

nected in series, and the neutral wire is run from the common connection of the two windings, as shown in Fig. 317*a*. When the system is unbalanced, that armature winding of the dynamotor which is connected with the side of the smaller load (having the lesser drop) acts as a motor, runs the armature, and thereby causes the other armature to run as a generator, thus raising the pressure of the heavier loaded side of the system.

(6) For starting and controlling large motors, especially those used for driving large printing presses supplied from lighting circuits. Printing presses have many sets of gears

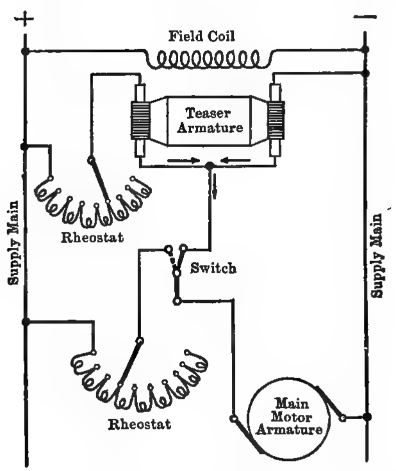


Fig. 317*b*.—Connections for Teaser System of Motor Control.

and possess very large moments of inertia, so that an unusually large torque is required to start them. Sometimes the starting torque is as much as 5 or 6 times the normal torque of the motor when running at full load. The torque exerted by a motor depends upon the strength of the current flowing through its armature. Since the current which is required to produce the normal running torque of these motors is already of considerable magnitude, it is desirable that a continuous-current transformation by means of a dynamotor be employed to avoid drawing the excessive starting current from the line.

The dynamotor used for this purpose is designed so that its motor side may be supplied by the line voltage while its generator side is usually wound for a voltage of about one-fifth that of the line. By this arrangement, which has been introduced by the *Bullock Electrical Manufacturing Co.*, and is by them called the *Teaser System* of Motor Speed Control, the excessive current for starting a motor is derived without necessitating an increase of the supply current above the normal amount. The two teaser armature windings are connected in series with a rheostat across the supply mains; the dynamotor field winding is excited directly from the line; the negative brush of the motor side is connected with the positive brush of the dynamo side; see Fig. 317*b*. At starting, the main motor armature is supplied from the generator end of the dynamotor with a voltage somewhat less than one-fifth of the line voltage, depending upon the magnitude of the regulating resistance; thus, the current which passes through the main motor is about five times as great as that taken from the supply mains, so that the required amount of starting torque is produced with normal supply current. Since the speed of the motor is dependent upon the E. M. F. impressed upon it, the starting speed is only about one-fifth of the normal running speed. The E. M. F. of the motor, and with it its speed, is raised by manipulating the dynamo-regulating resistance, and when the proper speed is attained the main motor connections are switched to the supply mains through a second series-regulating resistance, thus establishing the required conditions for running under working load. Regulation of the resistances and changes of the connections are accomplished by the aid of a controller, and thus the motor may be operated by the manipulation of a single hand-wheel.

Secondary generators of the second class consist in a combination of a continuous-current motor with an alternating-current generator, or of an alternating-current motor with a continuous-current generator. They are usually called *Rotary Converters*, and are employed in all cases where an alternating current is to be derived from continuous-current supply circuits, or continuous currents from alternating circuits. Since one of the windings involves an alternating current design, the calculation of rotary converters will be taken up in the

second volume of this book under the head of *Alternating-Current Machinery.*

The calculation of a *motor-dynamo* consists in the design of a motor which is supplied by the given current, and in the design of a generator which produces the desired current and voltage when run at the speed attained by the driving motor. The former design is accomplished by means of the formulæ contained in Chapter XXV., the latter by the methods given in Parts II. to VI.; hence the calculation of motor-dynamos need not be specially considered.

The calculation of *dynamotors*, however, involves some points of special interest, and is separately treated for this reason. The armature of a dynamotor, as previously stated, is provided with two separate windings, each connected to its own commutator, usually placed at opposite sides of the armature core. These two armature windings may be placed one above the other upon the core, or they may be interspersed by leaving suitable spaces upon the core surface for the second winding, when putting on the first. The electrical activity of the generator winding is equal to that of the motor winding, therefore the space occupied by each winding will be approximately the same, and half the winding space of the armature should be apportioned to each. The armature and frame of a dynamotor will, consequently, be of a size and weight corresponding to a machine of double the capacity to be transformed by it.

Since the magnetomotive force of the motor armature winding, or *primary* winding, is opposite in direction to that of the generator winding, or *secondary* winding, and since these two magnetomotive forces are nearly equal to one another and are produced in the same core, they will practically neutralize each other, the result being that in a dynamotor there is no appreciable armature-reaction, and the brushes never require to be shifted during variations of load.

The size of the machine depends upon the speed, the latter being chosen with respect to the heating of armature and bearings only, for, the transformation itself is not influenced by it, because, in calculating the motor portion of the armature, any change in the selection of the speed, for the same winding, calls for an alteration of the field density in exactly

the inverse proportion, so that the product of conductor-velocity and field density remains constant, and the E. M. F. produced in the generator winding, therefore, is always the same.

The field magnets of a dynamotor must, at least in part, always be excited from the primary circuit, that is, from the motor side, since otherwise the motor would not start. In case of transformation from low to high tension the fields are usually shunt wound, but in transforming from very low to high pressure it is more economical to start the motor action by a few turns of series winding connected to the motor circuit and to supply the remainder of the field excitation by a shunt winding from the secondary or generator side, which commences to be actuated as soon as the machine has started to run.

The counter E. M. F. of the motor winding is found by deducting from the primary voltage the drop due to internal motor resistance, and the E. M. F. active in the generator winding is the sum of the secondary voltage required and of the potential absorbed by the secondary winding. The quotient of these two E. M. Fs. gives the ratio of the number of armature turns of the primary to that needed in the secondary winding. The active length and the cross-section of either the primary or the secondary armature conductor is then calculated in the ordinary manner, and the winding so obtained is arranged upon an armature of twice the winding space necessary to accommodate it. The number of conductors and the area of the other winding, then, is simply obtained in multiplying or dividing, respectively, by the ratio of the E. M. Fs. to be induced in the two windings. For practical example see §150.

If the primary E. M. F. be E_1 volts, the primary current I_1 amperes, the resistance of the primary winding r_1 ohms, and the number of primary armature turns N_{a_1}, while the corresponding quantities in the secondary circuit are E_2, I_2, r_2, and N_{a_2}, respectively, the counter E. M. F. of the primary winding will be:

$$E'_1 = E_1 - I_1 r_1,$$

and the E. M. F. induced in the generator armature:

$$E'_2 = E_2 + I_2 r_2 = k\,E_1 + I_2 r_2,$$

where

$$k = \frac{E_2}{E_1}$$

is the given *ratio of transformation.* The ratio of the E. M. Fs. induced in the two windings, therefore, is:

$$\frac{E'_2}{E'_1} = \frac{k\,E_1 + I_2\,r_2}{E_1 - I_1\,r_1}.$$

But since the weight of copper in the two windings is approximately equal, the drop in the primary winding will practically be:

$$I_1\,r_1 = \frac{I_2\,r_2}{k},$$

so that the ratio of the number of turns of the secondary to that of the primary winding becomes:

$$\frac{N_{a_2}}{N_{a_1}} = \frac{E'_2}{E'_1} = \frac{k\,E_1 + I_2\,r_2}{E_1 - \frac{I_2\,r_2}{k}}$$

$$= \frac{k\,(k\,E_1 + I_2\,r_2)}{k\,E_1 - I_2\,r_2} = k \times \frac{E_2 + I_2\,r_2}{E_2 - I_2\,r_2}. \qquad \text{.........(441)}$$

The terminal E. M. F. of the generator side can then be expressed thus:

$$E_2 = E'_2 - I_2\,r_2 = E'_1 \times \frac{N_{a_2}}{N_{a_1}} - I_2\,r_2$$

$$= (E_1 - I_1\,r_1)\,\frac{N_{a_2}}{N_{a_1}} - I_2\,r_2$$

$$= \left(E_1 - \frac{I_2\,r_2}{\frac{N_{a_2}}{N_{a_1}}}\right)\frac{N_{a_2}}{N_{a_1}} - I_2\,r_2 = \frac{N_{a_2}}{N_{a_1}} - 2\,I_2\,r_2. \qquad \text{..(442)}$$

The machine, therefore, as far as its efficiency is concerned, acts as though it were a motor of terminal E. M. F.

$$E_2 = E_1 \times \frac{N_{a_2}}{N_{a_1}} \text{ volts,}$$

with an internal resistance of $2\,r_2$ ohms, that is, twice the resistance of the secondary winding.

123. Designing of Generators for Special Purposes.

a. Arc Light Machines (Constant Current Generators).

Ordinary arc lamps for commercial use are so adjusted that the pressure required to force the current through the arc is from 45 to 50 volts. A 2000 candle-power lamp will then require a current intensity of about 10 amperes, a 1200 candle-power lamp a current of about 6.5 amperes, and a 600 candle-power lamp a current of about 4 amperes. The energy consumed in the arc will therefore be about 450 watts for each 2000 candle-power lamp, about 300 watts for each 1200 candle-power lamp, and about 200 watts for each 600 candle power lamp. An arc light dynamo for n lamps must therefore have a capacity of $450n$, or $300n$, or $200n$ watts, respectively, and, since arc lamps are usually arranged in series, must be able to give an E. M. F. of from 50 to $n \times 50$ volts, and a constant current of 10, or 6.5, or 4 amperes, respectively. For search-lights and lighthouse reflector lamps higher currents are used, up to 200 amperes or more; but only a few of these are ever fed from the same dynamo, which, consequently, is of a comparatively low voltage.

For all arc lamps, however, the constancy of the current is essential, and in arc light dynamos, therefore, the current must be kept practically constant for all variations of load. The problems to be considered in the design of constant current machines are so radically different from those of a constant potential dynamo, that, in general, a well-designed machine of the one class will not answer for the other.

The ordinary shunt dynamo has the tendency to regulate for constant current,[1] because the induced E. M. F., if the magnetic circuit is suitably dimensioned, is proportional to the ampere-turns in the field, and if the resistance and the reaction of the armature are negligible, the machine will at any voltage just give the ampere-turns required to produce this voltage; that is to say, it will produce any voltage required by the conditions of the external circuit. This theoretical condition is

[1] See "Test of a Closed Coil Arc Dynamo," by Professor R. B. Owens and C. A. Skinner; discussion by C. P. Steinmetz; *Transactions Am. Inst. E. E.*, vol. xi. p. 441 (May 16, 1894); *Electrical World*, vol. xxiv. p. 150 (August 18, 1894); *Electrical Engineer*, vol. xviii. p. 144 (August 12, 1894).

fulfilled in practice if the variable shunt excitation, which for low saturations is proportional to the terminal voltage, is augmented by the constant exciting force necessary to compensate for the drop of E. M. F. due to armature resistance and for the cross ampere-turns due to armature reaction. Thus, a shunt dynamo with a constant separate excitation will fulfill the condition of giving a terminal E. M. F. proportional to the external resistance, and consequently a constant current, for all voltages below the bend of the magnetic saturation curve; that is, for all voltages for which the magnetic density is below 25,000 lines per square inch in cast iron, and below 70,000 lines per square inch in wrought iron or cast steel. (See Fig. 256, § 88).

Such a shunt machine will be a constant current dynamo, and will do very well for feeding incandescent lamps in series, but will be very unsatisfactory as an arc light generator, because it does not regulate quickly enough. If the load is changed suddenly, as often occurs in arc light working, it would take too long a time before the magnetism changes to the altered conditions of load and excitation, and thus either a sudden rush or a sudden decrease of current would take place. In an arc light machine the current intensity must not go above or below its normal value when the load is suddenly varied; the armature, therefore, must regulate instantly; that is to say, a small change of the armature current must essentially influence the effective field—if necessary, destroy it, for even when short-circuiting the machine the field may not disappear entirely, but may only be so distorted as to be ineffective with regard to the terminal voltage. Consequently a machine of a large and unvariable field flux and of very large armature reaction is required, so that the armature magnetomotive force is of nearly the same magnitude as the field M. M. F., and very large compared with the resultant effective M. M. F. necessary to produce the magnetism.

All successful arc light generators are based upon this principle of regulating for constant current by their armature reaction, and in their design, therefore, the following conditions, which lead to a great armature reaction, have to be fulfilled [see formulæ (244) and (248), § 93]: (1) The number of turns on the armature must be great; (2) the distortion of the field

must be large; (3) the number of bifurcations of the armature current must be small; (4) the length of path of the field lines of force in the polepieces must be great and its area small; (5) the length of path of the armature lines of force in the polepieces must be small and its area large; and (6) the polepieces must require a high specific magnetizing force. Conditions (1), (3), and (4) will be fulfilled if a ring armature of small axial length, and therefore of large diameter, is chosen, and if the polepieces are shaped so as to have large circumferential projections; condition (2) points to an armature with smooth core, and condition (3) makes a bipolar type preferable, while (6) calls for high densities in the polepieces, and is most nearly attained by the use of highly saturated cast iron for that part of the magnetic circuit. In order to have constant flux, a constant current dynamo must be series wound and worked to very high densities in the magnetic circuit, the latter being the more insensitive to sudden changes in the exciting power the higher it is saturated. If wrought iron or cast steel is used in the magnet frame its cross-section should be so dimensioned that the resulting magnetic density has a value between 110,000 and 120,000 lines of force per square inch (= 17,000 to 18,500 lines per square centimetre), and in case of cast iron, between 60,000 and 75,000 lines per square inch (= 9300 to 11,500 lines per square centimetre). The radial thickness of the armature core should be chosen so as to obtain in the minimum armature cross-section a density of from 110,000 to 130,000 lines per square inch (= 17,000 to 20,000 lines per square centimetre) in case of bipolar machines, and from 100,000 to 120,000 lines per square inch (= 15,500 to 18,500 lines per square centimetre) for multipolar machines. This high saturation of the armature is required for still another purpose, viz., to guard against sudden rise of the E. M. F. when the armature current is broken. For, since the magnetomotive force effective in producing the field magnetism, if current is flowing in the armature, is the difference between the total field M. M. F. and the armature M. M. F., the effective M. M. F., when the current is broken, will rise by the amount of the armature reaction and become equal to the total field M. M. F. But, the total M. M. F. being very large compared with the effective M. M. F. necessary to send the normal flux through the armature, an

enormous E. M. F. would be produced in the moment of opening the circuit if the saturation in the armature core were capable of a corresponding increase. In using the above densities, however, ever so great an increase of M. M. F. cannot raise the saturation, and thereby the voltage, seriously.

For the reasons set forth in § 43, open coil windings are frequently used in arc light dynamo armatures, although good results have also been attained with closed coil windings.

In the manner explained in the foregoing, a machine can be designed which automatically keeps the current intensity constant under all loads without any artificial means; it will, however, require an enormous magnetizing force on both field and armature in order to obtain very close regulation. But if artificial regulation is employed, very much less magnetizing force is needed, since then only just enough ampere-turns are sufficient so as not to get too large a fluctuation of the armature current by a very sudden change of load before the regulator can act; hence, the arc light regulator is merely for the purpose of making the inherent automatic regulation of the machine still closer.

There are two distinct systems of arc dynamo regulation: (1) By generating the maximum voltage at all times, but taking off by the brushes only such a portion of it as is required by the load. This is effected by shifting the brushes from the neutral line; in a closed coil armature this has the effect that the E. M. F. induced in some of the coils is in the opposite direction to that induced in the other coils in the same half of the armature, and their algebraical sum, consequently, can be made any part of the maximum E. M. F.; in an open coil armature the brushes in the neutral position collect the current from the group of coils having maximum E. M. F., by moving them either way; therefore, groups will be connected to the brushes which have a smaller E. M. F. than the maximum potential of the machine. This method of regulation is employed in the Edison, Thomson-Houston, Fort Wayne, Sperry, Western Electric, Standard Electric, and Bain arc machines. (2) By changing the whole E. M. F. generated by the dynamos as the load varies. The E. M. F. depends upon the number of conductors, the cutting speed, and the field density. It is impracticable to vary the former two while the machine is run-

ning, but the field density can easily be adjusted. The field strength depends upon the number of turns on the magnets and upon the current passing through them, and can therefore be varied by changing either of them. The variation of the number of field turns is performed by automatically cutting out, or short-circuiting, a portion of them, and the regulation of the field current, by placing a variable shunt across the field winding. The Excelsior arc light machine is regulated in the former manner, while the Brush and the Schuyler dynamos have a variable shunt.

The employment of external regulation introduces another problem. Whether the brushes are shifted in a constant field, or whether they remain stationary in a changing field, the position of the neutral line relative to the brush contact diameter varies with every change of the load, and means must be provided to collect the current without sparking in any position. The best solution of this problem is, of course, to so design the dynamo that the field is perfectly uniform all around the armature, for then the brushes will actually commutate in any position of the field. To attain this, a low density is required in the gap, from 10,000 to 20,000 lines per square inch (= 1550 to 3100 lines per square centimetre); hence the pole area must be made as great as possible by large extending polepieces. If this solution is not feasible in practice, but if the resultant density at any position of the brush varies with the amount of shifting necessary to bring the brush to that position, sparkless commutation can be obtained by varying the frequency of commutation; that is, the circumferential width of the brush, in employing two brushes connected in parallel, and shifting the one against the other.

b. Dynamos for Electro-Metallurgy.

For electroplating, electrotyping (galvano-plastics), electrolytic precipitation of metals (refining of crude metals and extracting of metals from ores), electro-smelting (reduction of metals), and for other electrolytical purposes, low electromotive forces and very large current intensities are requisite, as the quantity of metal extracted from the electrolyte depends upon the intensity of the current only, and not upon its potential. The latter, however, affects the quality of the deposit,

for, if too great an E. M. F. is permitted, the precipitate will not be homogeneous. The E. M. F. required for any electrolytical process is the sum of the counter E. M. F. of the electrolytic cell, or the E. M. F. of chemical reaction, and the drop of potential caused by the resistance of the electrolyte.

In dynamos for very low voltage, in order not to reduce the speed too much, as this would unduly increase the weight and cost, both the number of convolutions on the armature and the field density must be brought down to their minimum values. Machines with weak fields give trouble in sparking on account of the armature reaction; dynamos with few massive conductors and few divisions in the commutator are subject to sparking, and are liable to heat from local eddy currents. Electro-metallurgical machines, therefore, should be designed with short magnetic circuit, especially the length of the flux-path in the polepieces should be as small as possible. The polepieces should further have a large cross-section in the direction of the field flux, but a small transverse area and a great length for the lines of force set up by the armature current; that is to say, the armature itself should be of small diameter and of comparatively great length (hence, preferably a smooth-drum armature), and the polepieces should embrace only a small portion of its periphery, and, if possible, be provided with longitudinal slots parallel to the direction of the field flux. In order to avoid eddy currents as much as possible, a stranded conductor, or a multiplex winding (see § 44), or both combined, should be used, and the poles should be either elliptically bored, or given slanting pole corners, or, if of wrought iron, should be provided with cast-iron tips (see § 76). If it is desired to use the machine for different voltages, the polepieces may be designed in accordance with Fig. 173, § 76.

Dynamos for electrolytical purposes must be shunt wound, as otherwise they are liable to have their polarity reversed by the action of the counter E. M. F.

In case of *bipolar* types the field density of metallurgical dynamos, according to their size, should range between 7000 and 20,000 lines per square inch (= 1100 to 3100 lines per square centimetre), if the polepieces are of *cast iron*, and between 10,000 and 30,000 lines per square inch (= 1550 to 4650 lines per square centimetre), if they are of *wrought iron*.

or *cast steel.* For *multipolar* types the corresponding values are 9000 to 30,000 lines per square inch (= 1400 to 4650 lines per square centimetre), and 15,000 to 40,000 lines per square inch (= 2300 to 6200 lines per square centimetre), respectively. The densities employed in the field frame are slightly less than those given in Table LXXVI., § 81, namely, about 80,000 lines per square inch (= 12,500 lines per square centimetre) for *wrought iron* and *cast steel,* and about 35,000 lines per square inch (= 5500 lines per square centimetre) for *cast iron.* The armature core densities are given in Table XXII., § 26.

c. Generators for Charging Accumulators.

Owing to the well-known fact that the counter E. M. F. of a storage battery gradually rises about 25 per cent. during charging, generators to serve the purpose of charging accumulators, in order to keep the charging current constant, should be so designed that their voltage increases automatically with increasing load. Such machines, therefore, must be excited by a shunt winding, and must have a very massive field frame of consequent low magnetic saturation. The former is necessary to cause an automatic increase of the magnetizing force with increasing external load, and the latter to effect a corresponding rise of the flux-density, and thereby of the E. M. F. generated in the armature.

Thus for generating the minimum voltage, at start of the charging period, the magnetic density in the frame should be from 30,000 to 35,000 lines per square inch (= 4600 to 5500 lines per square centimetre) in case of *cast-iron* magnets, and from 70,000 to 80,000 lines per square inch (= 11,000 to 12,500 lines per square centimetre) in case of *wrought-iron* or *steel* magnets. The armature should have a smooth core of large cross-section, so that the reluctance of the gap remains constant, and therefore the total reluctance of the circuit approximately constant for the entire range of the magnetizing force.

In central station working the usual practice is to employ the charging dynamos also for directly supplying the lighting circuits, either separately or by connecting them in parallel to

the accumulators at the time of maximum load. In this case the dynamos must be capable (1) of supplying a constant minimum potential, namely the lamp pressure, which is not to vary with change of load, and (2) of giving a voltage from 25 to 30 per cent. higher, *i. e.*, the charging E. M. F. which must automatically regulate for variation of load. These two contradictory conditions can be fulfilled by designing a shunt dynamo of low magnetic density in armature core and magnet frame, and by providing the armature core with high teeth of such peripheral thickness that the flux required for the generation of the lamp-potential is sufficient to almost completely saturate the same, the density in the teeth at lamp-pressure to be 130,000 lines per square inch (= 20,000 lines per square centimetre) or more. The reluctance of the gap for light loads, up to the lamp-pressure, will then increase with the load, and as the magnetizing force in a weakly magnetized shunt dynamo also varies directly with the load, the flux, and thereby the E. M. F., will remain constant. But as soon as the saturation of the teeth is reached, that is to say, as soon as the machine is used for voltages above that of the lighting circuit, the gap reluctance, then being equivalent to that of air, will become constant, hence the E. M. F. of the machine will vary in direct proportion with the load, as long as all parts of the magnetic circuit are well below the point of saturation.

d. Machines for Very High Potentials.

For transmission of power to long distances, for testing purposes, and for laboratory work, dynamos of 10,000 volts and over are sometimes needed. Professor Crocker,[1] in an address before the Electrical Congress, Chicago, August 24, 1893, has given the chief points to be observed in the successful construction of such machines, as follows: (1) The insulation must be excellent, and for no two parts that have the full potential between them should measure less than 1000 megohms; (2) the side-mica of the commutator should be at least $\frac{1}{16}$ of an inch, and the end insulations at least $\frac{1}{4}$ of

[1] "On Direct Current Dynamos for Very High Potential," by F. B. Crocker, *Electrical World*, vol. xxii. p. 201 (September 9, 1893).

an inch thick, and, if possible, the surface distance at the ends should be increased by having the insulation project, the number of commutator divisions can then be so chosen that the potential between adjacent bars is 100 volts per pair of poles; (3) hard, smooth, and fine-grained carbon brushes should be used, as the employment of metallic brushes, owing to the film of the brush-material that is rubbed into the surface of the mica insulation, and which at a voltage of 10,000 or above, is a sufficiently good conductor to carry many watts of electrical energy, would lead to the destruction of the commutator; (4) the brush-pressure should not be any greater than necessary to insure good contact, because otherwise a layer of carbon dust might be produced on the commutator, when a similar effect as with metallic brushes, but not to the same degree, would be caused; (5) the armature should have a slotted core (toothed or perforated), and should be wound with double silk-covered wire, the former with the object of reducing the reluctance of the magnetic circuit and enabling the employment of very high field-densities, from $1\frac{1}{4}$ to $1\frac{1}{2}$ those given in Tables VI. and VII., § 18; (6) the magnet frame should be well saturated, densities of about 100,000 lines per square inch (= 15,500 lines per square centimetre) for wrought iron or cast steel, and of about 50,000 lines per square inch (= 7750 lines per square centimetre) for cast iron being best suited for the purpose; (7) the potential of the frame must be kept at one-half the terminal E. M. F., a condition which, however, is fulfilled if the machine is highly insulated; and (8) for reasons of economy, the field excitation of a high potential machine has to be supplied by a series winding, as otherwise the space occupied by the covering of the wire, and thereby the winding depth, would become excessive and a waste of copper, besides increased labor and difficulty in handling the extremely fine wire, would result.

e. Multi-Circuit Arc Dynamos.

The one serious objection to the use of the ordinary arc dynamo is the restriction of its capacity by the limitation of the line voltage. Since 7500 volts is considered the maximum for commercial work, ordinary arc machines are able to supply

only 150 open or 100 inclosed arc lamps, corresponding to a maximum capacity of about 75 KW, or about 50 KW, respectively.

In the *multi-circuit dynamo* a number of separate and independently regulable circuits are supplied, and thus it is possible to build arc-lighting generators of much larger capacity without exceeding the limit of line voltage.

Each pair of poles, the portion of armature thereunder, and the corresponding pair of brushes, constitute a separate series machine, the current leading from one brush to one of the line circuits, through two of the field coils, through the other brush of the pair, and dividing through the armature winding to the first-named brush. Each pair of brushes is carried upon a separate rocker segment, and the regulation is effected by shifting the separate rockers.

In the *Brush* and in the *Rushmore* multi-circuit arc machines there is a small oil-pressure cylinder for each rocker, and oil is admitted under pressure to one end or the other by a small magnet-controlled valve, and thus each pair of brushes occupies at all times a position on the commutator corresponding to the line voltage required. All regulating mechanism is placed in a small tank in the base of the machine, which also contains a small rotary oil pump, belted to the armature shaft to supply oil at the needed pressure. By this arrangement each circuit is able to be abruptly open-circuited or short-circuited under any load without the least effect upon the other circuit.

f. Double-Current Generators.

A *double-current generator* is a dynamo producing at the same time two different currents, either two continuous currents of different voltages, or one continuous current and an alternating current, or two alternating currents of different phase-relations or of different frequencies. A double-current generator is therefore nothing more or less than a dynamotor which is driven by mechanical power, and the same principles of construction apply as set forth in § 122.

Though numerous writers have during the last few years

recommended the use of double-current generators, they have not been introduced into practice to any extent, chiefly for the reason that there is usually no advantage gained by combining two generators for different purposes into one machine, while in most cases such a combination would be likely to complicate matters very much by introducing unnecessary difficulties.

The only possible benefit obtained by the use of double-current generators is the ability to supply two kinds of currents and still have but one type of station machinery. On the other hand, however, there are two decided disadvantages: (1) interference with the regulation of one system by certain fluctuations in the load of the other system; (2) in case of alternating current, the necessity of expensive station transformers capable of carrying the total alternating current load, and of auxiliary regulating apparatus. The latter is required on account of the poor regulating properties of the alternating current side of a double-current generator, and even the use of the auxiliary regulators will not always insure satisfactory results. Furthermore, the continuous current output of a continuous-alternating generator limits the frequency to about 35 cycles (§ 74), making compulsory the use of a low-frequency alternating current. This low frequency may or may not be of disadvantage, according to the nature of the plant and to conditions of operation.

For plants of such size as to permit the employment of very large units, facilitating the use of many magnet poles, double-current generators might be tolerable, provided the characters of the two loads are such as to justify the expense, but even in this case it is questionable whether any actual operating advantage would be conferred by double-current machines as compared with ordinary alternators and direct-current dynamos.

124. Prevention of Armature-Reaction.

Not only the heating, but also, even to a higher degree, the amount of sparking at the brushes limits the output of an armature. The increased sparking with rise of load is due to the interference of the magnetic field set up by the current flowing in the armature, the tendency of the latter being to

produce a cross magnetization through the armature core, at right angles to the useful lines of force, resulting in the distortion of the field of the dynamo, that is, in increased field density under the trailing pole corners, and in decreased density under the leading pole-corners; see Fig. 140, § 64, and Fig. 270, § 93. This distortion, depending in a given machine directly upon the magnetizing force of the armature, naturally increases with the current furnished by the dynamo, and the result is that the amount of shifting of the neutral line, or diameter of commutation, and therefore the sparking at the brushes, increases with the load on the machine. Consequently, it becomes necessary to change the position of the brushes to meet every variation in load, and unless the pole-tips or the armature-teeth are saturated (see § 22), a point of loading is soon reached for which no diameter of sparkless commutation can be found, and the output of the machiné has reached a maximum at this point, notwithstanding the fact that the load may be below that allowed by a safe heating limit. In order, therefore, to increase the output of a dynamo, the armature reaction itself, or its distorting effects, must be checked.

Besides the means for this purpose already alluded to in §§ 22, 76, and 122, consisting in specially shaping the pole-pieces, the air gaps, and the armature teeth so as to increase the reluctance of the cross-magnetization path, either permanently or proportionably to the load, three distinct methods for preventing armature reaction have recently been devised: (*a*) Balancing of armature cross-magnetization by means of special field coils (Professor H. J. Ryan); (*b*) compensation by additional armature winding (Wm. B. Sayers); and (*c*) checking of armature reaction by the employment of auxiliary magnet poles (Professor Elihu Thomson).

a. Ryan's Balancing Field Coil Method.[1]

This method, which in principle was first suggested by Fischer-Hinnen,[2] and independently also by Professor G.

[1] "A Method for Preventing Armature Reaction," by Harris J. Ryan and Milton E. Thompson, *Transactions Am. Inst. E. E.*, vol. xii. p. 84 (March 20, 1895); *Electrical World*, vol. xx. p. 329 (November 19, 1892); *Electrical Engineer*, vol. xix. p. 293 (March 27, 1895).

[2] "Berechnung Elektrischer Gleichstrom Maschinen," by J. Fischer-Hinnen, Zürich, 1892.

Forbes, Professor S. P. Thompson, and W. H. Mordey, consists, in general, in surrounding the armature with a stationary winding exactly equal in its magnetizing effects to the armature winding, but directly opposed to the latter, and thus completely balancing all armature-reaction. It is practically carried out by placing a number of balancing coils, one per pole, having a total number of turns equal to that of the armature, into longitudinal slots cut into the polepieces parallel to the shaft, and by connecting these coils in series to the armature, thus making their magnetizing force of equal number of ampere-turns as, but of opposite direction to, that of the armature. The two M. M. Fs. thus counterbalance and neutralize each other, leaving the field-flux practically unchanged at all loads of the machine. By this means all sparking due to distortion of the field is prevented, and only the sparking due to the self-induction in the short-circuited coil, and to the current reversal in the same, is left. In order to check the latter, each pole-space is provided with a commutation magnet, or lug, which is made the centre of the respective balancing coil, and which is energized by an additional winding consisting in a few extra turns of the balancing coil. If no current is flowing in the armature, and therefore also the balancing coils are without current, the commutation magnet is not energized and the field opposite the latter is neutral, but as soon as load is put on the armature the commutation lug is magnetized by the additional turns of the balancing coil, and a reversing field for the short-circuited armature-coil is created; the strength of this reversing field, being energized by the armature current, increases with the load, thus fulfilling the conditions for sparkless commutation.

Fig. 318 shows two half polepieces slotted to receive a balancing coil of eight turns, the half-turns being numbered consecutively to indicate the manner in which the coil is wound. In Fig. 319 the field of a bipolar dynamo with commutation lugs and balancing coils is represented; the two polepieces in this case are in one piece, the commutation lugs being arranged in the centre line. The same effect, however, can be produced by connecting each two polepieces by a *pole-bridge*, Fig. 320, or by employing a special *pole-ring*, Fig. 321, carrying the commutation lugs as well as the balancing coils. In any case

the slots *A* and *B*, adjoining the commutation lugs, *C*, are larger than the remaining slots, for the purpose of receiving the extra turns for magnetizing the commutation lugs.

The disadvantages of this method are (1) increased reluctance of the magnetic circuit on account of reducing, by virtue of the slots for the balancing coils, the cross-section of the pole-pieces; this requires additional field-excitation; (2) increased magnetic leakage owing to the close proximity of the pole-tips,

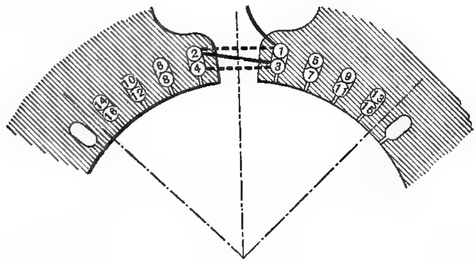


Fig. 318.—Polepiece Provided with Ryan Balancing Coils.

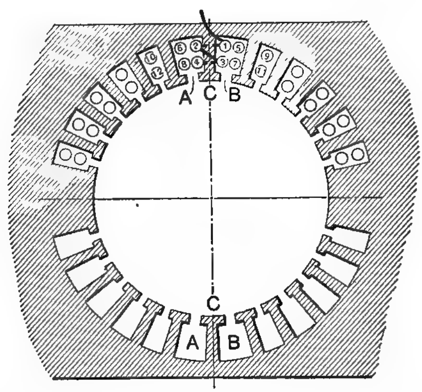


Fig. 319.—Bipolar Dynamo Field with Commutation Lug and Ryan Balancing Coils.

or to the bridging of the pole spaces, necessitated to form the commutation lugs; this leakage must also be made up by extra field-winding; (3) reduction of the ventilating space around the armature, and consequent increased heating of the latter; and (4) increased weight and cost of machine. The increase

in exciting power due to (1) and (2) alone may be sufficient to overcome an additional length of air gap large enough to

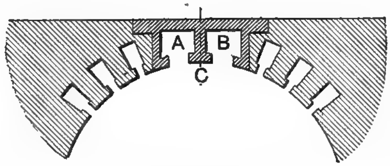


Fig. 320.—Dynamo Field with Pole Bridge, Carrying Commutation Lug for Ryan Balancing Coil.

nearly or quite check the armature reaction without the use of balancing coils.

b. Sayers' Compensating Armature Coil Method.[1]

While in the former method the compensating coils are placed on the fields, in the present one additional series windings are put on the armature; a series dynamo on this principle, therefore, requires no field winding at all, and a compound machine is to be provided with shunt coils only. This end is attained by connecting the main loops of the armature to the commutator-bars by means of connecting coils which form open circuits except when in contact with the brushes; then

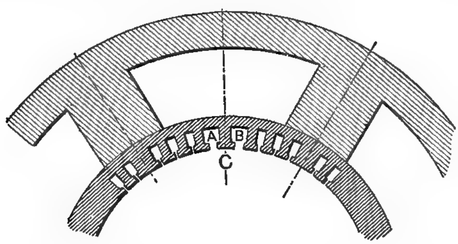


Fig. 321.—Dynamo Field Frame with Pole-Ring for Ryan Balancing Coils.

they carry the whole armature current, and thus exercise their function of creating a sufficient E. M. F. to balance the self-

[1] "Reversible Regenerative Armatures and Short Air Space Dynamos," by W. B. Sayers; *Trans. Inst. El. Eng.*, vol. xxii. p. 377 (July, 1893), and vol. xxiv. p. 122 (February 14, 1895); *Electrical Engineer* (London), vol. xv. (new series) p. 191 (February 15, 1895); *Electrician* (London), vol. xxxvi. p. 341 (January 10, 1896).

induction of the short-circuited armature coils. These "commutator-coils" form loops under the field-poles and thereby produce a forward field, which excites the magnets. By this means it is possible to control sparking, to reduce the magnetic reluctance of the frame and, in consequence, the exciting power, and to raise the weight-efficiency.

The sparking being under perfect control, the brushes in a generator can be placed backward, instead of giving them a forward lead, and the armature-current consequently exercises a helpful magnetizing action instead of having a destroying effect as in the ordinary case.

Fig. 322 shows the principle of this winding, *A*, *A*, being the main armature coils, and *B*, *B*, the compensating, or commutator coils.

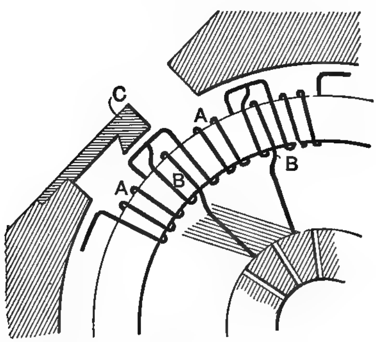


Fig. 322.—Diagram of Sayers' Compensating Armature Winding.

An auxiliary magnet, or pole extension, *C*, having a similar function as the commutator lug in the previous method, is employed to supply the proper strength of the reversing field for the short-circuited armature coil.

Sayers uses toothed and perforated armature cores, placing the main winding at the bottom and the commutator coils at the top in each slot. In order to keep down self-induction, the opening at the top of the slot, that is, the distance between the tooth-projections, should be made as wide as can be done without exceeding the limit where appreciable loss would occur through eddies in the polar surfaces of the field magnets. For the latter reason the width of this opening should not exceed $1\frac{1}{2}$ times the length of the air space; Sayers

usually makes it about $1\frac{1}{4}$ times that length. The number of conductors in each slot must be as small as is consistent with considerations of cost of manufacture, and since the number of commutator segments should be as small as possible, it is advantageous to connect the armature winding so that the conductors in two or more pairs of slots form but one coil. By placing the conductors of opposite potential, or connected at the time of commutation to opposite brushes, into separate slots, the self-induction of the armature winding can be reduced to about one-half.

For reversible motors the rocking arm carrying the brush-holders is mounted on the shaft so as to move freely between two stops, the friction of the brushes, upon reversal of direction, changing the position of the brushes automatically and without sparking from the stop at one side to that of the other, the stops being so adjusted as to keep the brushes in proper position for sparkless commutation.

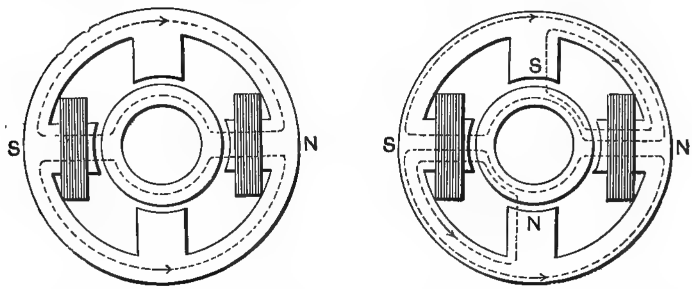
While in the case of a generator it may be inadvisable to reduce the air space below a given value on account of the crowding up of lines due to the large armature reaction, causing a diminution in the total flux, in the case of a motor this action can be taken advantage of, and the air space reduced to a safe mechanical clearance; the reduction of the total flux due to crowding up will then tend to compensate for drop of pressure due to dead resistance, so that in the case of a motor we obtain the happy concurrence of lightest weight and minimum cost with best regulating qualities.

c. Thomson's Auxiliary Pole Method.[1]

By the employment of auxiliary, or blank poles, one between each two active or excited poles, the current in the armature is made to react under load to magnetize a portion of the field frame which at no load is neutral or nearly so. The armature reaction may thus be made to give rise to a magnetic flux sufficient, or even more than sufficient, to compensate for its diminishing effect upon the useful field flux. This result is

[1] "Compounding Dynamos for Armature Reaction," by Elihu Thomson, *Trans. A. I. E. E.*, vol. xii. p. 288 (June 26, 1895); *Electrical Engineer*, vol. xx. p. 35 (July 10, 1895).

accomplished by dividing each field pole into a portion which is left unwound and a portion which is wound and excited in shunt, or separate. At no load, only the wound polar portions act to generate the open circuit E. M. F.; as the load is put on, the unwound auxiliary poles become active in consequence of a magnetic flux developed in them by the armature current itself, that is in consequence of the armature M. M. F. The disposition of the poles is shown in Figs. 323 and 324, the un-



Figs. 323 and 324.—Magnetic Circuits of Dynamos with Thomson Auxiliary Poles, at no Load and with Current in Armature.

wound poles being presented to the armature at right angles to the useful field flux. Fig. 323 gives the magnetic circuits at no load when the unwound poles are neutral, magnetically, while in Fig. 324 the grouping of the magnetic circuits is represented, if current is flowing in the armature, the cross-flux then being taken up by the auxiliary poles and led off into the backs of the wound poles, thereby strengthening the useful field instead of weakening it.

By properly choosing the position and spread of the auxiliary poles in relation to that of the main poles, and by adjusting the magnetizing force of the field relatively to that of the armature, the effect of compounding, or any degree of over-compounding, may easily be obtained, or the blank poles may be made adjustable in position so as to vary the effect of the armature M. M. F. upon them.

125. Size of Air Gaps for Sparkless Collection.

Although from the magnetic standpoint as small an air gap as possible is desired, the distance between armature core and polepieces should not be cut down too much, for the following

reasons: (1) With a very small air space the excitation is too low to maintain a stiff field at full load; (2) eddy currents become troublesome; (3) a great difficulty arises in maintaining the armature exactly centred, which is much more essential in a multipolar than in a bipolar machine; and (4) dynamos constructed with a very small air gap require a larger angle of lead, and do not generate as high a voltage as others of the same type having a larger air gap; this is due to the greater armature reaction, which causes a greater distortion of the lines, and owing to this increased obliquity of the lines, a short air gap may have a greater reluctance than a longer one; in fact, there is a certain value for each dynamo, beyond which there is no advantage in diminishing the air gap, as the obliquity of the lines becomes too great.

For sparkless collection of the current the gaps should be so proportioned that the magnetizing force required to give the correct flow of lines for the normal voltage and speed is the sum of the magnetizing force necessary to balance the armature cross turns, and of the magnetizing force required to give a reversing field of sufficient strength to effect sparkless collection. If it is less than this amount, there will be sparking, while if it is greater, the excess constitutes a useless waste of energy.

The magnetizing force necessary to produce the proper strength of the reversing field has been found by Claude W. Hill [1] to be 11.25 times the ampere-turns per armature coil in machines with *ring* armatures and *wrought-iron* magnets, and from 26.5 to 29.6 times the ampere-turns per coil in *drum* armatures of various sizes. Taking 12 and 30 times the magnetizing force of one armature coil, for ring and drum armatures, respectively, the length of the air gaps for sparkless collection can be derived as follows:

By (228), § 90, the ampere-turns needed for the air gaps are:

$$at_g = .3133 \times \mathcal{H}'' \times l''_g = .3133 \times \mathcal{H}'' \times k_{12} \times (d_p - d_a);$$

the magnetizing force necessary to compensate armature-reactions, by (250), is:

[1] "Armatures and Magnet-Coils," by Claude W. Hill, *Electrical Review* (London), vol. xxxvii. p. 227 (August 23, 1895).

$$at_r = k_{14} \times \frac{N_a \times I'}{2\, n'_p} \times \frac{k_{13} \times \alpha}{180°};$$

and the ampere-turns required to produce the proper strength of the reversing field by means of the above figures based upon Hill's results, can be expressed by:

$$at_s = 12 \times n_a \times \frac{I'}{2\, n'_p}, \text{ for } \textit{ring} \text{ armatures,}$$

$$at_s = 30 \times n_a \times \frac{I'}{2\, n'_p}, \text{ for } \textit{drum} \text{ armatures.}$$

For sparkless collection then we must make:

$$at_g = at_r + at_s,$$

or, for *ring* armatures:

$$\cdot 3133 \times \mathcal{H}'' \times k_{12} \times (d_p - d_a)$$
$$= k_{14} \times \frac{N_a\, I'}{2\, n'_p} \times \frac{k_{13} \times \alpha}{180} + 12\, n_a \times \frac{I'}{2\, n'_p},$$

whence:

$$(d_p - d_a) = \frac{n_a\, I'}{2\, n'_p} \times \frac{k_{14} \times n_c \times \frac{k_{13} \times \alpha}{180} + 12}{\cdot 3133 \times k_{12} \times \mathcal{H}''}; \quad \textbf{(443)}$$

and similarly for *drum* armatures:

$$(d_p - d_a) = \frac{n_a\, I'}{2\, n'_p} \times \frac{k_{14} \times n_c \times \frac{k_{13} \times \alpha}{180} + 30}{\cdot 3133 \times k_{12} \times \mathcal{H}''}. \quad \textbf{(444)}$$

126. Iron Wire for Armature and Magnet Winding.

Small dynamos, up to 5 KW. capacity, are very uneconomical, for the reason that the armature-winding with its binding wires occupies a comparatively large depth, which with the clearance between the finished armature and the polepieces makes the air gaps unduly large. The leakage factor, being the quotient of the total permeance (which in small machines is particularly large on account of the comparatively large surfaces and small distances in the frame) and of the useful permeance (which is extra small owing to the long air gaps), is therefore very high, and a comparatively large exciting power is required in consequence.

For the purpose of removing the main cause of low efficiency of small dynamos, viz., excessive ratio of gap-space to armature diameter, it has been repeatedly suggested [1] to employ *iron wire* for winding the armature. It is certain that the winding of an armature with iron wire will materially reduce the reluctance of the gap-spaces, and thereby will economize in exciting power, (1) directly, by lessening the total reluctance of the magnetic circuit of which the air gap is the predominant portion, and (2) indirectly by reducing the magnetic leakage of the machine. Magnetically, therefore, the use of iron wire for the armature coils offers a great advantage over copper. Another advantage of employing iron wire for winding the armatures of small machines is the increase of the total effective length of the armature conductor thereby made possible. In order to limit the leakage across the tips of the polepieces, the distance between the pole-corners must be larger than the length of the two gaps; in small copper-wound armatures this distance therefore is excessive compared with large dynamos, even if it is reduced so that quite a good deal of leakage does take place across the pole-tips; and, if iron armature coils are employed, may be considerably decreased, thereby rendering a larger portion of the armature circumference useful, and increasing the effective length of the armature conductor, while the ratio of the decreased pole-distance to the gap-length, which then only consists in the height taken up by binding and in the mechanical clearance, will even be greater, and thus effect a decrease in the percentage of leakage from pole to pole.

On the other hand, the electrical resistivity of iron being about six times that of commercial copper, for the same current output an iron wire of about six times the cross-section of a copper wire will be required, and this will occupy about six times the space on the ends of a drum, or in the interior of a ring armature, eventually necessitating an increase in the diameter of the latter. Since the winding is very deep, and consists of magnetic conducting material, the outer layers will form a shorter path for the magnetic lines than the inner ones, so that only a portion of the useful flux will cut the inner

[1] See editorial, *Electrical Engineer*, vol. xviii. p. 150 (August 22, 1894).

layers, and the latter therefore will not generate their full share of E. M. F. The presence of the iron wire in the interior of the ring armature, moreover, would allow magnetic lines to cross the internal ring-space, and these, in cutting the winding, would produce an E. M. F. opposite in direction to the E. M. F. of the machine, thus reducing the latter by its amount. Finally, the total revolving mass of iron in the armature being greater in the case of iron coils, both the hysteresis and eddy current losses will be in excess of those in a copper-wound armature.

As to cost, the fine copper wire commonly used in small armatures is difficult to insulate with thin cotton covering, and, therefore, expensive silk insulation is usually applied, while an iron wire of six times its area, that is, about 2½ times its diameter, may conveniently be insulated with the cheaper cotton. But since the weight of the iron wire, on account of its sixfold area and of the higher winding space and consequent larger armature-heads, is at least seven to eight times that of a corresponding copper winding, it is doubtful whether there is a direct saving in cost by the employment of iron wire. Furthermore, an increase in the length of armature shaft and machine-base being necessitated by the much larger heads, while the reduction of the gap reluctance and of the magnetic leakage effects a saving in magnet-wire and a decrease in field frame area and length of magnetic circuit, the cost of the machine frame is influenced positively as well as negatively, and it will depend upon the circumstances in every single case whether copper or iron armature coils are preferable.

It has also been recommended to use iron wire for winding the magnet coils. In this case the winding itself may be considered a part of the magnetic circuit, hence the cores may be diminished in area and the length of the wire thereby reduced, but on account of the insulation on the winding, its reluctivity is much greater than that of the solid core, and the winding therefore can only take the place of a portion of the core much smaller than itself, leaving the outside diameter of the magnet cores still larger than if wound with copper wire. Owing to this increase in diameter, the core surfaces are increased and their distance apart is diminished, hence the per-

meance of the path between them is increased and rise for greater leakage is given, unless the frame area, by the simultaneous use of iron for the armature coils, is reduced sufficiently to make up for this increase in diameter. In case of a small shunt-wound machine, the magnet wire is extremely fine, and the reduction of both the number of turns and their mean length would necessitate the selection of a still smaller sized wire in order to have a sufficiently high resistance in the field coils, and then the use of iron wire would be particularly desirable. From these considerations it follows that the advisability of using iron wire for the magnet coils likewise depends upon the circumstances connected with the machine in question.

The fact, however, that various makers have practically tried iron wire armature and magnet windings without adopting their use for small dynamos, seems to indicate that there is nothing to be gained by the change.

CHAPTER XXVIII.

DYNAMO-GRAPHICS.

127. Construction of Characteristic Curves.

The majority of the practical problems connected with the construction of dynamo-electric machines can readily be solved *graphically*, by the use of certain curves, technically called *characteristics*, which express the dependence upon one another of the various quantities involved. For distinction the curves relating to quantities of the external circuit are termed *external characteristics*, while those referring to quantities within the machine itself are known as *internal characteristics.*

In most problems the *magnetic characteristics*, showing the variation of the E. M. F. with increasing magnetizing power, is employed, and the construction of this curve, from the data of the machine calculated, forms, therefore, the fundamental problem of dynamo-graphics.

This problem is solved by means of the formula for the total magnetizing force of the machine. Inserting into (227) the values given in (228), (230), (238), and (250), Chapter XVIII., we obtain for the total number of ampere-turns per magnetic circuit, in English measure:

$$AT = .3133 \times l''_g \times \mathcal{H}'' + l''_a \times m''_a + l''_m \times m''_m + k_{14} \times \frac{N_a \times I'}{2n'_p} \times \frac{k_{13} \times \alpha}{180^\circ}. \quad \ldots\ldots\ldots (\mathbf{445})$$

But m''_a and m''_m depend upon the values $\mathcal{B}''_a$ and $\mathcal{B}''_m$ of the magnetic densities of the armature core and the magnet-frame, respectively, and, since

$$\mathcal{B}''_a = \mathcal{H}'' \times \frac{S_g}{S_a} \quad \text{and} \quad \mathcal{B}''_m = \lambda \times \mathcal{H}'' \times \frac{S_g}{S_m},$$

S_g, S_a, S_m being the areas of the magnetic circuit in air gaps, armature core, and field frame, and λ the leakage factor, it is

evident that m''_a and m''_m depend upon the value of the field-density $\mathcal{H}''$, and can be mathematically expressed as "functions" of $\mathcal{H}''$, thus:

$$m''_a = f_1(\mathcal{H}'') \text{ and } \quad m''_m = f_2(\mathcal{H}''),$$

$f_1(\mathcal{H}'')$ and $f_2(\mathcal{H}'')$ reading "function one of $(\mathcal{H}'')$" and "function two of $(\mathcal{H}'')$", respectively.

Furthermore, since the angle of brush lead, α, is the arc whose trigonometric tangent is the quotient of armature ampere-turns by total field ampere-turns (see equation (245), page 349), and is, consequently, depending upon the field-density, the compensating ampere-turns at_r can likewise be expressed as a function of $\mathcal{H}''$, as follows:

$$at_r = k_{14} \times \frac{N_a \times I'}{2n'_p} \times \frac{k_{13} \times \text{arc tan} \dfrac{n'_p \times n_z \times AT}{N_a \times I'}}{180^\circ}$$

$$= \frac{N_a \times I'}{2n'_p} \times f_3(\mathcal{H}'').$$

The total number of ampere-turns per magnetic circuit can therefore be expressed thus:

$$AT = .3133 \times l''_g \times \mathcal{H}'' + l''_a \times f_1(\mathcal{H}'') + l''_m \times f_2(\mathcal{H}'') + \frac{N_a \times I'}{2n'_p} \times f_3(\mathcal{H}''). \qquad \ldots\ldots(\mathbf{446})$$

Every term being a function of the field density $\mathcal{H}''$, a curve for AT can be obtained by plotting the curves for the components at_g, at_a, at_m, and at_r, for different ordinate values of $\mathcal{H}''$, and subsequently adding the abscissæ. By transforming the ordinates from field density into the corresponding proportional values of E. M. F., which can be done by simply changing the scale, the magnetic characteristic of the dynamo is then obtained; the characteristic gives the E. M. F. at the terminals as a function of the magnetizing power.

In Fig. 325 the curves OA, OB, OC, and OD represent the ampere-turns required at various densities for the air gaps, for the armature core, for the field frame, and for compensating the armature reaction, respectively. OA is a straight line owing to the fact that in air the ampere-turns required are propor-

tional to the density desired. OB is the saturation curve for laminated wrought iron, and OC that for the material employed in the field frame, for values of the density ranging from zero to the maximum employed at the largest overload the machine

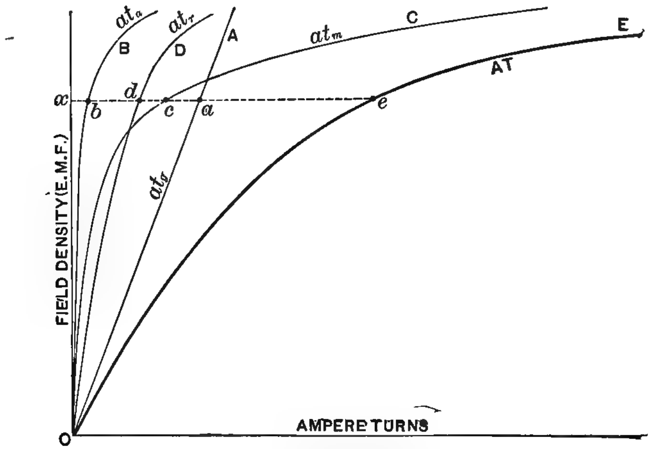


Fig. 325.—Construction of Magnetic Characteristic of Dynamo, from its Components.

is intended for. Any point, e, on the characteristic curve OE is then obtained by adding all the abscissæ of OA, OB, OC, and OD that have the same ordinate Ox, thus:

$$xe = xa + xb + xc + xd.$$

If the magnet frame consists of two or three different materials, either two or three distinct curves, as the case may be, have to be plotted instead of the curve OC, or one single curve may be laid out in which the addition of the component abscissæ has been made by the formula:

$$m''_{\mathrm{m}} \times l''_{\mathrm{m}}$$
$$= m''_{\mathrm{w.i.}} \times l''_{\mathrm{w.i.}} + m''_{\mathrm{c.i.}} \times l''_{\mathrm{c.i.}} + m''_{\mathrm{c.s.}} \times l''_{\mathrm{c.s.}}, \quad \ldots . (447)$$

$m''_{\mathrm{w.i.}}$ corresponding to a density of $\lambda \times \mathcal{H}'' \times \dfrac{S_g}{S_{\mathrm{w.i.}}}$, $m''_{\mathrm{c.i.}}$ corresponding to $\lambda \times \mathcal{H}'' \times \dfrac{S_g}{S_{\mathrm{c.i.}}}$, and $m''_{\mathrm{c.s.}}$ to $\lambda \times \mathcal{H}'' \times \dfrac{S_g}{S_{\mathrm{c.s}}}$.

In cases where the armature reaction is small and where the magnetic density in the armature core is low, that is, in all machines except those designed for certain special purposes (see § 123), the curves *OB* and *OD* are very nearly straight lines, and can be united with curve *OA* by means of the approximate formula:

$$\begin{aligned} at_{\text{gar}} &= at_{\text{g}} + at_{\text{a}} + at_{\text{r}} \\ &= .3133 \times \mathcal{H}'' \times l''_{\text{g}} + k_{20} \times \lambda \times \mathcal{H}'' \times \frac{S_{\text{g}}}{S_{\text{a}}} \times l''_{\text{a}} \\ &\qquad + .00001 \times N_{\text{a}} \times \frac{I'}{2\,n'_{\text{p}}} \times \mathcal{H}'' \\ &= \mathcal{H}'' \times \left(.3133 \times l''_{\text{g}} + k_{20} \times \lambda \times \frac{S_{\text{g}}}{S_{\text{a}}} \times l''_{\text{a}} \right. \\ &\qquad \left. + .00001 \times N_{\text{a}} \times \frac{I'}{2\,n'_{\text{p}}} \right), \quad \ldots\ldots\ldots\ldots (448) \end{aligned}$$

thus simplifying the construction of the magnetic characteristic into the addition of the abscissæ of but a single curve and a

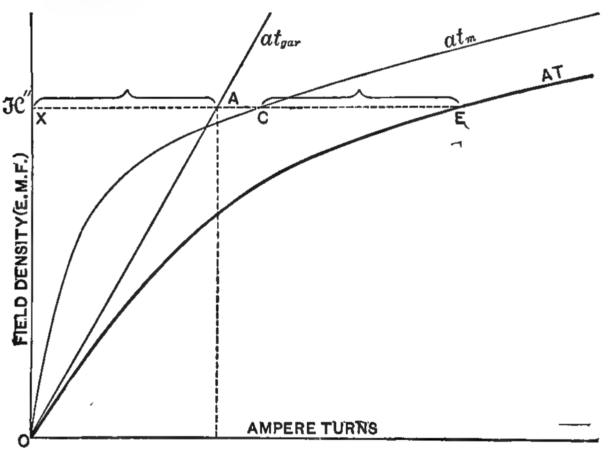


Fig. 326.—Simplified Method of Constructing Magnetic Characteristic.

single straight line. Formula (448) gives practically accurate results if the mean density in the armature core, § 91, at maximum load of dynamo, is within 80,000 lines per square inch, or 12,500 lines per square centimetre, and if the values of the

constant k_{20} for different mean maximum load densities are taken from the following Table CV.:

TABLE CV.—FACTOR OF ARMATURE AMPERE-TURNS FOR VARIOUS MEAN FULL-LOAD DENSITIES.

English Units.			Metric Units.		
Mean Density in Armature Core at Maximum Output. Lines p. sq. in. $\mathfrak{B}''_a$	Ampere-Turns per inch of Magnetic Circuit in Armature Core. m''_a	Constant in Approximate Formula for Armature Ampere-Turns. $k_{20} = \frac{m''_a}{\mathfrak{B}''_a}$	Mean Density in Armature Core at Maximum Output. Lines per cm² $\mathfrak{B}_a$	Ampere-Turns per cm. of Magnetic Circuit in Armature Core. m_a	Constant in Approximate Formula for Armature Ampere-Turns. $k'_{20} = \frac{m_a}{\mathfrak{B}_a}$
25,000	4.5	.00018	4,000	1.8	.00045
30,000	5.5	.00018	5,000	2.35	.00047
35,000	6.5	.00019	6,000	2.85	.000475
40,000	7.5	.00019	7,000	3.35	.00048
45,000	8.5	.00019	8,000	3.95	.00049
50,000	9.6	.00019	9,000	4.8	.00053
55,000	11.1	.00020	10,000	6.1	.00061
60,000	13	.00022	10,500	7	.00067
65,000	15.7	.00024	11,000	8	.00073
70,000	19.6	.00028	11,500	9.4	.00082
75,000	24.7	.00033	12,000	10.8	.00090
80,000	31.2	.00039	12,500	12	.00096

For calculations in metric units the coefficient of gap ampere-turns, .3133, must be replaced by .8 (see § 90), and the value .0000645 is to be taken for the factor of compensating ampere-turns, instead of .00001, which has been averaged from a great number of bipolar and multipolar dynamos, having drum as well as ring, and smooth as well as toothed and perforated armatures. In the majority of cases the value of this factor, in English units, ranges between .0000075 and .0000125, while the actual minimum and maximum limits found were .0000040 and .0000160, respectively. The metric value is derived from the average in English measure by multiplying with the number of square centimetres in one square inch.

The simplified process of constructing the characteristic, then, is as follows: The value of the combined magnetizing force, at_{gar}, calculated from (448) for any one, preferably high, value of the field density, $\mathfrak{H}''$, is plotted as abscissa XA, Fig. 326, with that value, XO, of $\mathfrak{H}''$ as ordinate, and the point A

thus found is connected with the co-ordinate centre O, by a straight line. Next the saturation curve OC of the field frame is plotted by computing

$$\lambda \times \mathcal{H}'' \times \frac{S_g}{S_m}$$

for a series of values of $\mathcal{H}''$, multiplying the corresponding magnetizing forces, m''_m, taken from Table LXXXVIII., p. 336, or LXXXIX, p. 337, or from Fig. 259, p. 338, for the respective material, with the length l''_m, of the magnetic circuit in the field frame, and connecting the points so obtained by a continuous curved line OC. In case of a composition frame this process is to be performed according to formula (447), that is to say, by adding all the component magnetizing forces for each value of the density $\mathcal{H}''$. The required characteristic OE is then obtained by drawing horizontal lines, such as XE in Fig. 326, and making CE, measured from curve OC, equal to the distance XA of line OA from the axis OX.

Example: To construct the characteristic of a bipolar generator of 125 volts and 160 amperes at 1200 revolutions per minute, having a ring armature and a cast-iron field frame, the following data being given: Length of magnetic circuit in cast iron, $l''_m = 80$ inches; in armature core, $l''_a = 15$ inches; in gap spaces, $l''_g = 1\frac{1}{32}$ inch. Mean area in cast iron, $S_m = 79$ square inches; in armature, $S_a = 50$ square inches; in gaps, $S_g = 158$ square inches. Number of armature conductors, $N_c = 216$. Coefficient of magnetic leakage, $\lambda = 1.25$.

If the field frame, as in the present case, consists of but one material, the magnetization curve for that material—of which a supply may be prepared for this purpose—can be directly utilized. It is only necessary to multiply the scale of the abscissæ by l''_m, and to divide that of the ordinates by $\lambda \times \frac{S_g}{S_m}$; in the present case the magnetizing force per inch length of circuit is to be multiplied by 80 to obtain the total number of ampere-turns, and the density per square inch of field frame is to be divided by

$$1.25 \times \frac{158}{79} = 2.5$$

in order to reduce the ordinates to the corresponding values of the field density. In this manner the second scales in Fig. 327, marked "Total Number of Ampere-turns" and "Field

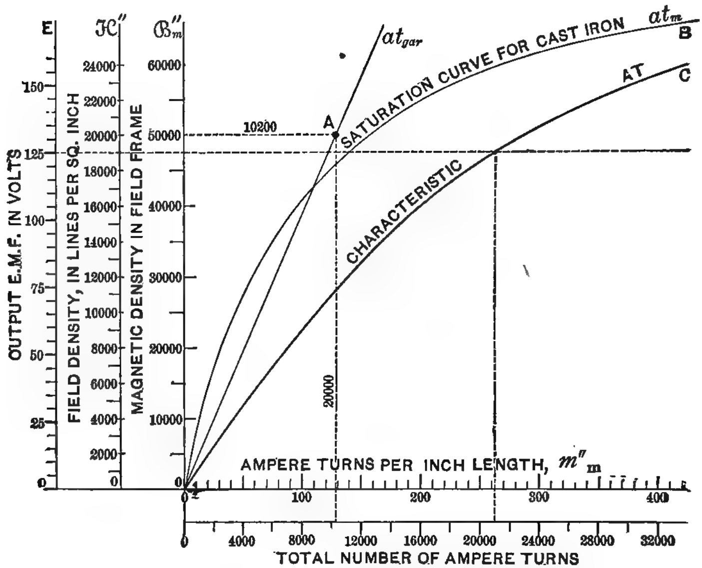


Fig. 327.—Practical Example of Construction of Characteristic.

Density," respectively, are obtained, and now the line at_{gar} can be plotted. For this purpose the mean density in the armature core at maximum output, and from this the value of the constant k_{20} must first be determined. From formula (138) we have, for the useful flux, at normal load:

$$\Phi = \frac{6 \times (125 + 5) \times 10^9}{216 \times 1200} = 3{,}000{,}000 \text{ maxwells},$$

hence,

$$\mathfrak{B}''_a = \frac{\Phi}{S_a} = \frac{3{,}000{,}000}{50} = 60{,}000 \text{ lines per square inch},$$

for which Table CV. gives:

$$k_{20} = .00022.$$

Calculating now the value of at_{gar} for $\mathcal{H}'' = 20,000$, we find by formula (448):

$$at_{gar} = 20,000\left(.3133 \times 1\tfrac{1}{32} + .00022 \times 1.25 \times \frac{158}{50} \times 15 \right.$$
$$\left. + .00001 \times 216 \times \frac{160}{2}\right)$$
$$= 20,000\,(.324 + .013 + .173)$$
$$= 20,000 \times .510 = 10,200 \text{ ampere-turns.}$$

Plotting this value as abscissa for an ordinate of $\mathcal{H}'' = 20,000$, the point A is obtained, which, when connected with the co-ordinate centre O, gives the line OA, representing the sum of the gap, armature, and compensating ampere-turns for any field density. The addition of the abscissa of this line to those of the curve OB, which gives the magnetizing force, at_m, required for the field frame, furnishes the required characteristic. In order to read the ordinates in volts, a third scale of ordinates is yet to be added; since the field density at full load is

$$\mathcal{H}'' = \frac{\Phi}{S_g} = \frac{3,000,000}{158} = 19,000,$$

this third scale is obtained by placing "125 volts" opposite that density, and by subdividing accordingly, the resulting scale giving the output E. M. F. for varying magnetizing force.

128. Modification in the Characteristic Due to Change of Air Gap.[1]

In practice it often becomes necessary to change the length of the air gap in order to secure sparkless collection of the current (compare § 125), and it is then important to investigate the influence of different air gaps upon exciting power and E. M. F.

The characteristic OBC, Fig. 328, for the original air gap constructed according to § 127, is replaced by the curve ABC, consisting of the straight-line portion, AB, and of the curved

[1] Brunswick, *L'Eclairage Elec.*, August 31, 1895; *Electrical World*, vol. xxvi. p. 349 (September 28, 1895).

portion, BC. Since for low densities the magnetizing force required for the iron portion of the magnetic circuit is very small, the straight line portion, AB, can be considered as the magnetizing force due to the air gap alone, and therefore the curved portion, BC, as the sum of the elongation, BD, of this straight line plus the magnetizing force due to the iron. Any

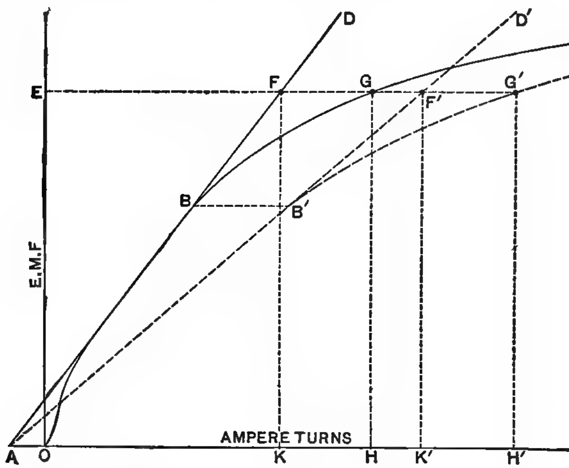


Fig. 328.—Conversion of Characteristics for Different Air Gaps.

change in the length of the air gaps will, consequently, for any given ordinate, OE, only alter the abscissa, EF, of the straight line AD, but will leave unaffected the abscissa-difference, FG, between the curve BC and the straight line BD. Hence the new characteristic OC' for an increased air gap is obtained by increasing the abscissa EF to EF', in the ratio of the old to the new air gap, and by adding to the abscissa thus found the original difference between BC and BD, making $F'G' = FG$. Then OH' is the magnetizing force required to produce the E. M. F. OE, corresponding to the point G' on the new characteristic; the portion OK' of the magnetizing force is the exciting power used for the new air gap, and $K'H'$ that for the remaining parts of the magnetic circuit, and is therefore independent of the air gap.

129. Determination of the E. M. F. of a Shunt Dynamo for a Given Load.[1]

If E, Fig. 329, is the E. M. F. developed by the machine at no load, viz.:

$$E = I_{sh} \times r_{sh},$$

and if the E. M. F., E_1, at a certain load corresponding to an armature current of I amperes is to be found, draw OA, by connecting the co-ordinate centre, O, with the point A on the

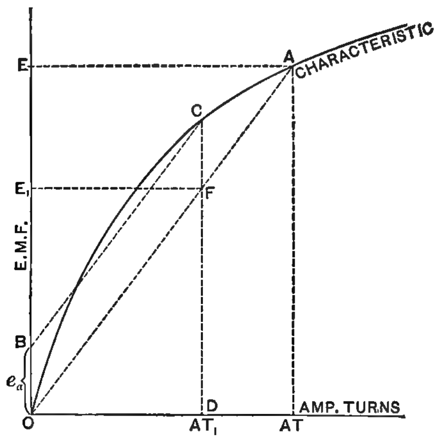


Fig. 329.—Determination of E. M. F. of Shunt Dynamo for Given Load.

characteristic corresponding to the E. M. F. E, then make OB equal to the total drop of E. M. F. caused by the armature current I, or

$$OB = e_a = I \times r_a + e_r,$$

where $I \times r_a$ is the drop caused by the armature resistance r_a, and e_r that due to armature reaction. The latter may approximately be taken as half the former, *i. e.*.

$$e_r = \tfrac{1}{2} I \times r_a,$$

thus making the total drop

$$e_a = 1.5 \times I \times r_a.$$

[1] Picou, "Traité des Machines dynamo-électriques."

The point B thus being located, draw $BC \parallel OA$, and from the intersection, C, of this parallel line with the characteristic curve drop the perpendicular CD upon the axis of abscissæ. The portion FD of CD, from its intersection, F, with OA to the axis of abscissæ, is the required E. M. F., $DF = E_1$, while $OD = AT_1$ is the corresponding exciting force of the field magnets.

The characteristic shows that the drop, CF, is the greater the lower the saturation of the machine.

130. Determination of the Number of Series Ampere-Turns for a Compound Dynamo.

Let the E. M. F., which is to be kept constant, be represented by E, Fig. 330. Draw EA parallel to the axis of

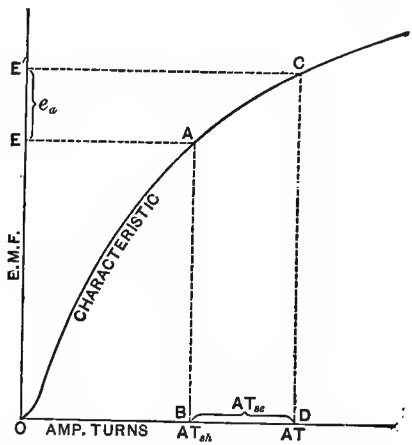


Fig. 330.—Determination of Compound Winding.

abscissæ, and from A on the characteristic drop the perpendicular AB. The length OB then gives the ampere-turns required on open circuit, that is, the shunt excitation AT_{sh}. If e_a again denotes the total drop of E. M. F. caused by the armature current at the given load (see § 129), then in order to keep the external E. M. F., E, constant, an internal E. M. F., $E' = E + e_a$, must be generated. Drawing $E'C \parallel OB$, and $CD \perp OB$, we find that the latter requires a total magnetizing force of $OD = AT$ ampere-turns.

Hence the number of series ampere-turns necessary for compounding:

$$BD = AT_{se} = AT - AT_{sh},$$

the series excitation being the difference between the total number of ampere-turns required for the generation of E' volts, and the shunt excitation needed for E volts.

131. Determination of Shunt Regulators.[1]

Shunt regulators are employed: (*a*) to keep the output E. M. F. constant at variable load and constant speed; (*b*) to keep the E. M. F. constant for variable speed; (*c*) to keep the E. M. F. constant if both the load and the speed are variable; and (*d*) to effect any variation in the E. M. F.

a. Regulators for Shunt Machines of Varying Load.

In Fig. 331, E is the constant potential of the dynamo, r_m, the magnet resistance, r_r, the resistance of the shunt regula-

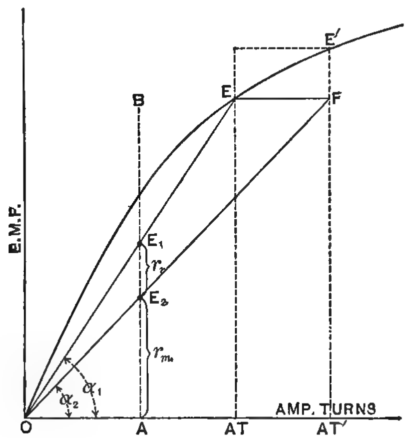


Fig. 331.—Shunt Regulating Resistance for Constant Potential at Varying Load.

tor, and N_m the number of convolutions per magnetic circuit. The dynamo is driven by a motor of constant speed, and so

[1] "Lösung einiger praktischer Fragen über Gleichstrom-Maschinen auf graphischem Wege," by J. Fischer-Hinnen, *Elektrotechn. Zeitschr.*, vol. xv. p. 397 (July 19, 1894).

arranged that at full load all resistance of the regulator is cut out. The resistance is to be found which has to be put in series with the field magnets in order to keep the potential on open circuit the same as at full load.

In the manner shown in § 129 the exciting current intensities, I_m and I'_m, at no and full load, respectively, are first determined by finding the magnetizing forces AT and AT', for the E. M. Fs. E, and $E' = E + e_a$, respectively, and dividing the same by the given number of shunt-turns, thus:

$$I_m = \frac{AT}{N_m}, \quad \text{and } I'_m = \frac{AT'}{N_m}.$$

Then, according to Ohm's Law:

$$\left.\begin{aligned} r_m + r_r &= \frac{E}{I_m} = \frac{N_m \times E}{AT} = \tan \alpha_1, \\ \text{and} \qquad r_m &= \frac{E}{I'_m} = \frac{N_m \times E}{AT'} = \tan \alpha_2. \end{aligned}\right\} \quad \ldots \mathbf{(449)}$$

The values of r_m and $(r_m + r_r)$ can be directly found as follows: In the distance $OA = N_m$ (Fig. 331) draw AB parallel to the axis of the ordinates; find point F by drawing $EF \parallel OA$ and $E'F \parallel AB$; and draw the lines OE and OF. These will intersect AB in points E_1 and E_2, respectively, for which hold the following relations:

$$\tan \alpha_1 = \frac{AE_1}{OA} = \frac{N_m \times E_1}{N_m} = E_1 = r_m + r_r,$$

and:

$$\tan \alpha_2 = \frac{AE_2}{OA} = \frac{N_m \times E_2}{N_m} = E_2 = r_m.$$

The required regulating resistance, therefore, is directly:

$$r_r = E_1 - E_2.$$

Example: A shunt dynamo for 100 volts and 40 amperes having an armature resistance of $r_a = .12$ ohm, a magnet winding of $N_m = 4200$ turns per magnetic circuit, and the magnetic characteristic shown in Fig. 332, is to be provided with a regulator for constant pressure at variable load.

The drop at 40 amperes is:

$$e_a = 1.5 \times I \times r_a = 1.5 \times 40 \times .12 = 7.2 \text{ volts},$$

and the characteristic gives, for $E = 100$ and $E' = 100 + 7.2 = 107.2$ volts, respectively:

Magnetizing force at no load, $AT =$ 8100 ampere-turns;
Magnetizing force at full load, $AT' =$ 10,500 ampere-turns.

Hence, by (449):

$$r_m + r_r = \frac{N_m \times E}{AT} = \frac{4200 \times 100}{8100} = 51.8 \text{ ohms},$$

$$r_m = \frac{N_m \times E'}{AT'} = \frac{4200 \times 100}{10{,}500} = 40.0 \text{ ohms},$$

and

$$r_r = 51.8 - 40.0 = 11.8 \text{ ohms}.$$

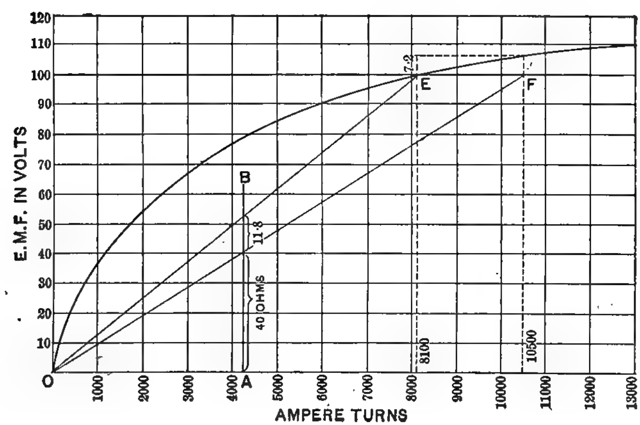


Fig. 332.—Practical Example of Graphical Determination of Shunt Regulator for Constant Potential at Varying Load.

These values can also be directly derived from the characteristic by erecting, at $OA = 4200$, the perpendicular AB, and by drawing the lines OE and OF; the resistances can then be read off on AB from the scale of ordinates.

b. Regulators for Shunt Machines of Varying Speed.

If N is the normal, and N_1 the maximal, or minimal abnormal speed, as the case may be, then the speed ratio,

$$\frac{N_1}{N} = n,$$

is greater or smaller than 1, according to whether the speed-variation is in the form of an increase or of a decrease. In

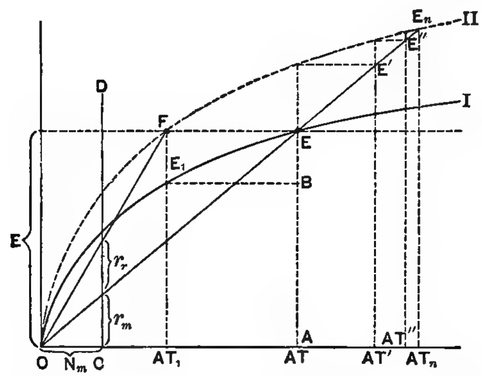


Fig. 333.—Shunt Regulating Resistance for Constant Potential at Increasing Speed.

order to obtain the characteristic of the machine for the abnormal speed, all ordinates of the original characteristic, I, must be multiplied by the speed-ratio, n. The result of this multiplication is shown by curve II, Fig. 333, for increasing, and by curve II, Fig. 334, for decreasing speed.

If the point E on curve I, corresponding to the E. M. F. at normal speed, N, is connected with O, then the intersection, E_n, of the line OE with curve I, is the E. M. F. which the machine would yield at the speed N_1. For, in the first moment, the E. M. F. E, Fig. 333, on account of the increased speed, will rise to the amount E'; at the same time, however, the exciting current rises, and with it the magnetizing force increases from AT to AT', causing an increase of the E. M. F. to E'', on account of which the magnetizing power is further increased to AT'', and so on, until at E_n the equilibrium is reached. But the potential of the machine is to be kept con-

stant; for this purpose, that magnetizing force, AT_1, is to be found which produces the E. M. F. E at the speed N_1. This, however, can be done without the use of curves II, which therefore need not be constructed at all. For, since the num-

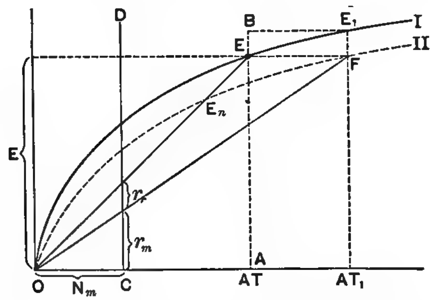


Fig. 334.—Shunt Regulating Resistance for Constant Potential at Decreasing Speed.

ber of ampere-turns required to produce E volts at N_1 revolutions is identical with the magnetizing force needed to generate

$$E_1 = \frac{E \times N}{N_1} = \frac{E}{n}$$

volts at normal speed, N, it follows that it is only necessary to draw $EA \perp OA$, to make

$$AB = E_1 = \frac{E}{n},$$

and to draw $BE_1 \parallel OA$. The abscissa of the intersection, E_1, of this parallel with the characteristic I is the required number of ampere-turns, AT_1. The latter will be smaller than AT if $n > 1$, and greater if $n < 1$; in the former case, therefore, the excitation must be reduced by adding resistance, while in the latter case it must be increased by cutting out resistance. AT_1 being known, the regulating resistance can be computed as follows:

For $N_1 > N$:

$$r_m + r_r = \frac{E}{\frac{AT_1}{N_m}} = \frac{E \times N_m}{AT_1}, \quad r_m = \frac{E \times N_m}{AT};$$

whence:

$$r_r = E \times N_m \times \left(\frac{1}{AT_1} - \frac{1}{AT} \right). \quad \ldots (450)$$

For $N_1 < N$:

$$r_m + r_r = \frac{E \times N_m}{AT}, \quad r_m = \frac{E \times N_m}{AT_1};$$

or:

$$r_r = E \times N_m \times \left(\frac{1}{AT} - \frac{1}{AT_1} \right). \quad \ldots . (451)$$

If at distance $OC = N_m$ a parallel, CD, to the axis of ordinates is drawn, then resistances can be directly derived graphically, as shown in Figs. 333 and 334.

Example: A dynamo of 125 amperes current output, having the characteristic OA, Fig. 335, is to be regulated to give a constant potential of 120 volts for a speed variation of 9 per cent. below and 10 per cent. above the normal speed; to determine the magnet and regulator resistance, if at normal speed a current consumption of 3.2 per cent. is prescribed.

Under the given conditions the speed ratio and corresponding E. M. F. for increasing speed is:

$$n = \frac{N_1}{N} = \frac{N + 0.10\, N}{N} = 1.1; \quad E_1 = \frac{E}{n} = \frac{120}{1.1} = 109 \text{ volts};$$

and for decreasing speed:

$$n' = \frac{N'_1}{N} = \frac{N - 0.09\, N}{N} = .91\,; \quad E'_1 = \frac{E}{n'} = \frac{120}{.91} = 132 \text{ volts}.$$

For these E. M. Fs. the characteristic furnishes the following magnetizing forces:

Ampere-turns at normal speed, $AT = 20{,}000$;
Ampere-turns at maximum speed, $AT_1 = 15{,}400$;
Ampere-turns at minimum speed, $AT'_1 = 27{,}600$.

Hence:

$$N_m = \frac{20{,}000}{.032 \times 125} = 5000 \text{ convolutions};$$

and consequently:

$$r_{\mathrm{m}} + r_{\mathrm{r}} = \frac{5000 \times 120}{15,400} = 39.0 \text{ ohms.}$$

$$r_{\mathrm{m}} = \frac{5000 \times 120}{27,600} = 21.8 \text{ ohms.}$$

$$r_{\mathrm{r}} = 39.0 - 21.8 = 17.2 \text{ ohms.}$$

This value is directly given by the ordinate scale in the diagram, Fig. 335, being the distance between the lines OF and

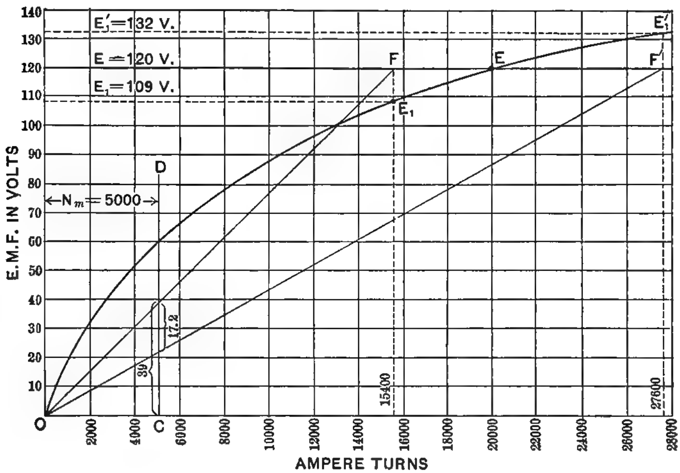


Fig. 335.—Practical Example of Graphical Determination of Shunt Regulator for Constant Potential at Varying Speed.

OF_1, measured on the ordinate CD, in distance $OC = N_{\mathrm{m}} =$ 5000 from the co-ordinate centre.

c. Regulators for Shunt Machines of Varying Load and Varying Speed.

In this case the required resistance must be capable of keeping the potential the same at no load and maximum speed as at full load and minimum speed. The former of these two extreme cases—no load and maximum speed, N_1, —has already been treated under subdivision *b*; to consider the latter case—full load and minimum speed—reference is

had to the open circuit curves I and II, Fig. 336, for normal speed, N, and for minimum speed, N_2, respectively.

If AT ampere-turns are requisite to produce, at normal speed and on open circuit, the potential, E, to be regulated,

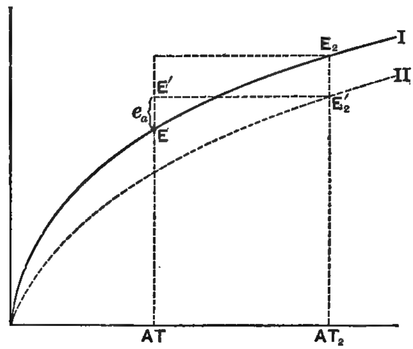


Fig. 336.—Shunt Regulating Resistance for Constant Potential at Variable Load and Variable Speed.

the magnetizing force for minimum speed is found by determining the abscissa AT_2 for

$$E' = E + e_a$$

on curve II, which at the same time also is the abscissa for the potential

$$E_2 = \frac{E'}{n_2}$$

on curve I, n_2 being the ratio of minimum to normal speed. The value of AT_2 can therefore be derived without plotting curve II, by adding to E the drop e_a, dividing the sum by

$$n_2 = \frac{N_2}{N},$$

and finding the abscissa for the potential so obtained. If the magnetizing force for open circuit and maximum speed is AT_1, the desired regulating resistance for variable load and variable speed is:

$$r_r = N_m \times E \times \left(\frac{1}{AT_1} - \frac{1}{AT_2}\right), \quad \ldots . (452)$$

where N_m is the number of turns per magnetic circuit.

Example: A shunt dynamo having a potential of 60 volts, a drop in the armature of 3 volts, a current-intensity of 50 amperes, 6 per cent. of which is to be used for excitation at full load, and having the characteristic given in Fig. 337, is to

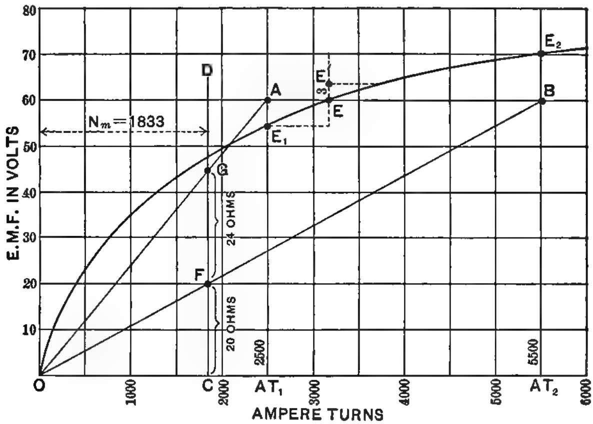


Fig. 337.—Practical Example of Graphical Determination of Shunt Regulator for Constant Potential at Varying Load and Varying Speed.

be regulated for a speed variation of 10 per cent. above and below normal speed, and for loads varying from zero to full capacity.

For no load and maximum speed we have in this case:

$$n_1 = \frac{N_1}{N} = \frac{N + .1N}{N} = 1.1,$$

$$E_1 = \frac{E}{n_1} = \frac{60}{1.1} = 54.5 \text{ volts},$$

$AT_1 = 2500$ ampere-turns (from Fig. 337); and for full load and minimum speed:

$$E' = E + e_a = 60 + 3 = 63 \text{ volts},$$

$$n_2 = \frac{N_2}{N} = \frac{N - .1N}{N} = .9,$$

$$E_2 = \frac{E'}{n_2} = \frac{63}{.9} = 70 \text{ volts},$$

$AT_2 = 5500$ ampere-turns (from Fig. 337),

$$N_m = \frac{AT_2}{I_{sh}} = \frac{5500}{.06 \times 50} = 1833 \text{ convolutions.}$$

Connecting the points A and B, in which the 2500 and 5500 ampere-turn lines, respectively, intersect the 60-volt line, with the co-ordinate centre O, and erecting, at $OC = 1833$, the perpendicular CD, the intersections F and G are obtained, and the lengths CF and FG give the required resistances of the magnet-winding and of the shunt-regulator, respectively. The result thus found can be checked by the following computation:

$$r_m + r_r = \frac{N_m \times E}{AT_1} = \frac{1833 \times 60}{2500} = 44 \text{ ohms,}$$

$$r_m = \frac{E}{I_{sh}} = \frac{60}{.06 \times 50} = 20 \text{ ohms,}$$

$$r_r = 44 - 20 = 24 \text{ ohms;}$$

or, directly, by formula (452):

$$r_r = 1833 \times 60 \times \left(\frac{1}{2500} - \frac{1}{5500}\right) = 24 \text{ ohms.}$$

d. Regulators for Varying the Potential of Shunt Dynamos.

The potential of the machine is to be adjustable between a minimum limit E_1 and a maximum limit E_2, and the adjusted potential is to be kept constant for varying load. These conditions are fulfilled by so proportioning the magnet-winding and the regulator-resistance that at full load the maximum potential E_2 is generated with the regulator cut out entirely, and that at no load the minimum potential E_1 is produced with all the regulator-resistance in circuit.

From the characteristic, Fig. 338, the magnetizing forces AT_1, corresponding to the potential E_1 at no load, and AT_2, corresponding to the potential E_2 at full load, or to the internal E. M. F., $E'_2 = E_2 + e_a$, are obtained; and if again N_m denotes the number of field-convolutions per magnetic circuit, we have:

$$r_m + r_r = \frac{N_m \times E_1}{AT_1}$$

and

$$r_m = \frac{N_m \times E_2}{AT_2},$$

from which follows:

$$r_r = N_m \times \left(\frac{E_1}{AT_1} - \frac{E_2}{AT_2}\right). \quad ..(453)$$

In order to derive the values of the resistances r_m and r_r graphically, the points E_1, on the characteristic, and E''_2, on the ampere-turn line AT_2, are connected with O, and a per-

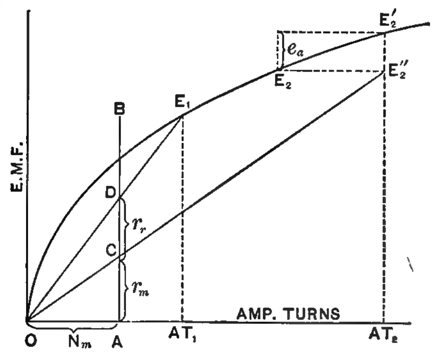


Fig. 338.—Shunt Regulating Resistance for Adjusting Potential Between Given Limits at Varying Load.

pendicular, AB, is erected upon the axis of abscissæ at the distance $OA = N_m$ from the ordinate axis. Then the portions AC and CD of AB, cut off by the lines OE''_2 and OE_1, represent the required resistances r_m and r_r, respectively.

132. Transmission of Power at Constant Speed by Means of Two Series Dynamos.[1]

Since two exactly identical series machines do not solve the problem of transmission at constant speed with varying load, it is now to be investigated graphically, how generator and motor must be designed, electrically, for that purpose.

[1] J. Fischer-Hinnen, *Elektrotechn. Zeitschr.*, vol. xv. p. 400 (July 19, 1894).

Let I, Fig. 339, represent the external characteristic, giving the E. M. F. as function of the current intensity of the generator, and also of the motor when run as a generator, thereby indicating that both machines are identical in design.

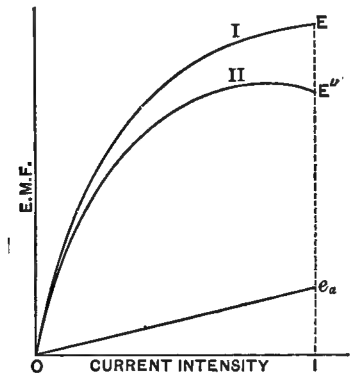


Fig. 339.—External Characteristics of Generator and Motor of Identical Design.

If R is the total resistance of both machines plus the resistance of the line, the total drop of E. M. F. at any current-intensity, I, is

$$e_a = I \times R,$$

and the E. M. F. at the motor terminals, therefore:

$$E'' = E - I \times R \text{ volts.}$$

By plotting the straight line Oe_a and subtracting the ordinate values from those of curve I, we obtain curve II, which represents the external characteristic of the motor. The speed of the motor for any load is then found by taking its E. M. F. E'' at the current-intensity, I, corresponding to that load, from the characteristic II, and inserting it into the formula:

$$N'' = \frac{E'' \times 60 \times 10^8}{N''_a \times n''_p \times \Phi''} = \frac{K'' \times E''}{\Phi''}, \quad \text{..(454)}$$

where N'' = speed of motor, at certain load;

E'' = E. M. F. required on motor-terminals for that load;

N''_a = number of turns on the motor armature;

n''_p = number of bifurcations of current in armature;
Φ'' = number of useful lines of force;
$K'' = \dfrac{60 \times 10^8}{N''_a \times n'_p}$ = constant for motor under consideration.

But since the E. M. F. of the generator is, with similar denotation:

$$E = \frac{N \times N_a \times n'_p \times \Phi}{60 \times 10^8} = \frac{N \times \Phi}{K},$$

it follows:

$$K = \frac{N \times \Phi}{E},$$

or

$$K'' = \frac{K'' \times K}{K} = \frac{K''}{K} \times \frac{N \times \Phi}{E},$$

and formula (454) becomes:

$$N'' = N \times \frac{E''}{E} \times \frac{K''}{K} \times \frac{\Phi}{\Phi''}$$

$$= N \times \frac{K''}{K} \times \frac{\Phi}{\Phi''} \times \frac{E - I \times R}{E}. \quad \ldots\ldots(455)$$

From this follows that constancy of speed cannot be obtained by means of two identical machines, for, in that case we would have $K'' = K$, and $\Phi'' = \Phi$, or

$$\frac{K'' \Phi}{K \Phi''} = 1,$$

for which formula (455) would show that N'' is an inverse function of Φ, that is, of the current-intensity, I, of which the flux is a direct function.

But by making $K'' \Phi$ greater than $K \Phi''$ in the same proportion as E exceeds $E - I \times R$, constant speed at varying load can be attained. K and K'' are constants for the respective machines, and therefore cannot be varied proportional to I; the flux Φ, however, is a direct function of the exciting power, and is inversely proportional to the reluctance of the magnetic circuit; approximate constancy of N'', consequently, can be produced (1) by making the motor of a higher reluctance than

the generator, either by increasing the length of the air gap or by reducing the section of the iron in the former, or (2) by making the magnetizing force of the generator greater than that of the motor by winding a greater number of field turns on its magnets. The proper way, however, is to select for the motor a somewhat smaller type, corresponding to the smaller capacity required for it, and to so design its magnet frame, air gap, and windings as to create a characteristic whose ordinates for any current intensity are proportional to the corresponding ordinates of curve II, Fig. 339.

133. Determination of Speed and Current Consumption of Railway Motors at Varying Load.[1]

The speed of the car and the current required for the motor equipment are to be found for different grades of track, *i. e.*, for varying propelling power.

To solve this problem, the speed characteristic of the motor

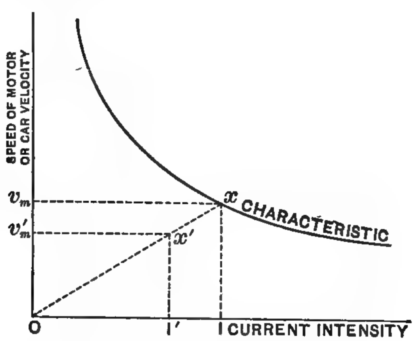


Fig. 340.—Speed Characteristic of Railway Motor.

—giving the motor speed, or still better, the car velocity, as a function of the current-intensity—is plotted.

Let W_t = total weight to be propelled, in tons;
v_m = velocity of car, in miles per hour;
g = grade of track, in per cent., *i. e.*, number of feet of rise in a horizontal distance of 100 feet;

[1] J. Fischer-Hinnen, *Elektrotechn. Zeitschr.*, vol. xv. p. 401 (July 19, 1894).

I = current required to propel W_t tons, at $g\%$ grade, with a velocity of v_m miles per hour;
E' = potential of line;
η_e = mean electrical efficiency of railway motor;

then we have, by formula (384), § 117:

$$I = \frac{P''}{E' \times \eta_e} = \frac{2 \times W_t \times v_m \times (30 + 20 \times g)}{E' \times \eta_e}. \quad \ldots (456)$$

In this v_m is not known, but since the car velocity increases in direct proportion with the current strength, I, it is only

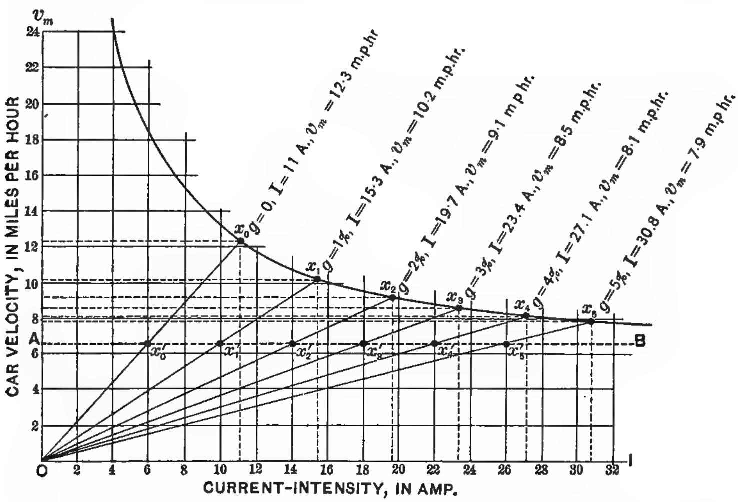


Fig. 341.—Practical Example of Graphical Determination of Car-Velocity and Current-Consumption at Different Grades.

necessary to calculate, from (456), one value, I', for any value, v'_m, of the velocity. By plotting the result, the point x', Fig. 340, is obtained; and if we now connect x' with O, and prolong the line Ox' until it intersects the speed-characteristic at x, the co-ordinates of this point, x, are the required values I and v_m for current consumption and car velocity, respectively, for the particular grade in question.

Example: An electric railway car of a seating capacity of 34 passengers weighs $2\frac{1}{2}$ tons, its electrical equipment $1\frac{1}{4}$ tons; the average efficiency of the motors from one-fifth to

full load is 82 per cent., and the line potential is 500 volts. Its speed characteristic is given in Fig. 341. The car velocity attained at, and the current required for, different grades up to 5 per cent. is to be determined for maximum load.

Including the conductor and motorman, the full carrying capacity of the car is 36 persons, which, at an average of 125 pounds per head, make a total load of $2\frac{1}{4}$ tons; the maximum weight to be propelled, therefore, is:

$$W_t = 2\frac{1}{2} + 1\frac{1}{4} + 2\frac{1}{4} = 6 \text{ tons.}$$

Inserting the given data into (356), we obtain:

$$I = \frac{2 \times 6 \times v_m \times (30 + 20g)}{500 \times .82} = .02927 \times v_m \times (30 + 20g).$$

For $v_m = 6.83$ miles per hour, the equation for the current takes the following convenient form:

$$I' = .02927 \times 6.83 \times (30 + 20g) = 2 \times (3 + 2g),$$

from which, for: $g = 0\,\%,\ 1\,\%,\ 2\,\%,\ 3\,\%,\ 4\,\%,\ 5\,\%$
we find: $I' = 6,\ 10,\ 14,\ 18,\ 22,\ 26$ amperes.
In order to derive therefrom the actual speeds, and current intensities corresponding to the same, a line AB, Fig. 341, is drawn parallel to the axis of abscissæ and at a distance $OA = 6.83$ from it. Upon this line AB the points x'_0, x'_1, ...x'_5, corresponding to the above values of I' are found and connected with O. Then the co-ordinates of the intersections x_0, x_1, ...x_5 of the lines Ox'_0, Ox'_1, ...Ox'_5 with the characteristic are the required amounts of I and v_m for the various grades.

PART VIII.

PRACTICAL EXAMPLES OF DYNAMO CALCULATION.

CHAPTER XXIX.

EXAMPLES OF CALCULATIONS FOR ELECTRIC GENERATORS.

134. Calculation of a Bipolar, Single Magnetic Circuit, Smooth Ring, High-Speed Series Dynamo: 10 Kilowatts. Single Magnet Type. Cast Steel Frame. 250 Volts. 40 Amps. 1200 Revs. p. Min.

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—According to § 15, p. 49, the percentage of polar arc, in this case, should be between .75 and .85; the machine being a small one, we take $\beta_1 = .78$, for which, in Table IV., p. 50, we find, by interpolation, a unit armature induction of $e = 64 \times 10^{-8}$ volt per foot per bifurcation. The number of bifurcations in bipolar machines is $n'_p = 1$.

Next we consult Tables V. and V*a*, § 17. From Table V*a* it is seen that the given speed is the average high speed for 10 KW output. Hence a value of v_c near the average conductor velocity given in Table V. for a 10 KW high-speed ring armature must be taken. Though the average, 85 feet per sec., is a very good value for the case under consideration, we will here take $v_c = 80$ ft. p. sec., which will result in a somewhat better shape for the armature. Table VI., p. 54, gives an average field density of $\mathcal{H}'' = 19{,}000$ lines per square inch, the value for cast iron polepieces being preferable, although the present machine has cast steel poles, because otherwise the magnetizing force required would be rather great to be produced by one single magnet. The total E. M. F., finally, that must be generated in order to yield the required 250 volts at the brushes is found by Table VIII., p. 56, to be about $E' = 250 + .12 \times 250 = 280$ volts. Hence, by means of formula (26), p. 65, the length of the active armature conductor required is:

$$L_a = \frac{280 \times 10^8}{64 \times 80 \times 19{,}000} = 288 \text{ feet.}$$

2. *Sectional Area of Armature Conductor, and Selection of Wire.*—Taking a current density of 500 circular mils per ampere, we find the cross-section of the armature conductor, according to formula (27), § 20:

$$\delta_a^2 = \frac{500 \times 40}{2} = \mathbf{10{,}000} \text{ circular mils.}$$

Referring to a wire gauge table we find that a single wire of this area would be rather too thick, and therefore difficult to wind on a small armature; we consequently select a gauge of half the above section, taking 2 No. 15 B. W. G. wires having a total sectional area of $2 \times 5184 = 10{,}368$ circular mils. The diameter of No. 15 B. W. G. wire is $\delta_a = .072''$ bare, and $\delta'_a = .088''$ when insulated for 250 volts with a $.016''$ double cotton covering.

3. *Diameter of Armature Core.*—Applying formula (30), § 21, the mean winding diameter of the armature, corresponding to the given speed of $N = 1200$ revolutions per minute, is found:

$$d'_a = 230 \times \frac{80}{1200} = 15.3'';$$

and from this the core diameter can be deduced by means of Table IX., § 21, thus:

$$d_a = .98 \times 15.3 = \mathbf{15} \text{ inches.}$$

Approximately, d_a could also have been derived from Table XI., by multiplying the respective table-diameter by the ratio of the table-speed to the speed prescribed:

$$d_a = 14 \times \frac{1250}{1200} = 14.6''.$$

4. *Length of Armature Core.*—The number of wires per layer, if the entire circumference of the armature were to be occupied by winding, is by formula (35), § 23:

$$n'_w = \frac{15 \times \pi}{.088} = 535.$$

Allowing 16 per cent. of the circumference for spaces between the coils, we have:

$$n_w = .84 \times 535 = 448,$$

the exact result, 449, being replaced by the nearest even and readily divisible number. Table XVIII., § 23, gives the height of the winding space, $h = .325''$, and Table XIX., § 24, the thickness of core insulation, $a = .040''$, allowing $.040''$ more for binding (see p. 75), by formula (39) the number of layers is obtained:

$$n_1 = \frac{.325 - .080}{.088} = 3.$$

Remembering that the armature conductor consists of 2 wires in parallel, we insert the values found into formula (40) and find the length of the armature core:

$$l_a = \frac{12 \times 2 \times 288}{448 \times 3} = 5\tfrac{1}{8} \text{ inches.}$$

5. *Arrangement of Armature Winding.*—The voltage of the machine being below 300, the potential between adjacent commutator bars will be within the limit of sparklessness, if the number of armature coils is chosen between 40 and 60. There are three numbers which fulfill this condition, viz.:

$$n_c = \frac{448 \times 3}{2} \div 12 = 56,$$

$$n_c = \frac{448 \times 3}{2} \div 14 = 48,$$

and

$$n_c = \frac{448 \times 3}{2} \div 16 = 42.$$

In practice that number would have to be taken for which the tools, and possibly even the entire commutator, of an existing machine could be used; here, however, although for the smallest number the cost of the commutator as well as that of winding and connecting would be the lowest, we will take $n_c = 56$, because this number is preferable to the others on account of the more symmetrical arrangement of the winding it produces. For, in dividing the total number of wires on the armature, $448 \times 3 = 1344$, by the different values of n_c, we obtain for the number of wires per armature coil the figures 24, 28, and 32, respectively, and as $24 = 8 + 8 + 8$, $28 = 9 + 9 + 10$, and $32 = 10 + 11 + 11$, it follows

that in the first case alone the number of wires per layer is uniform, while for each of the two latter windings the number of wires in one of the three layers would differ by 1 from the other two. Substituting, therefore, $n_c = 56$ into (46) the number of convolutions per coil is obtained:

$$n_a = \frac{448 \times 3}{56 \times 2} = 12,$$

that is to say, the armature winding is composed of **56** coils, each having **12** turns of **2** No. **15** B. W. G. wires.

The arrangement of the winding is shown by the diagram, Fig. 342, which represents the cross-section of one armature

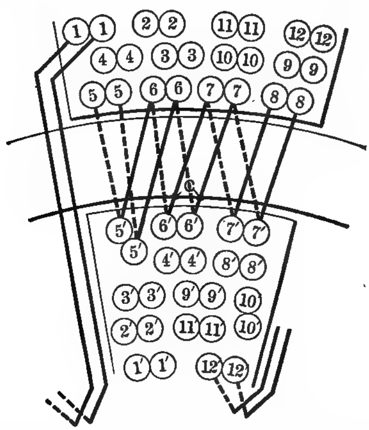


Fig 342.—Arrangement of Armature Winding, 10-KW. Single-Magnet Type Generator.

coil. In order to have both ends of the coil terminate at the outside layer, at the inner circumference of the armature, and at the commutator end, as is most desirable for convenience in connecting and for avoidance of crossings, the centre, *C*, of each coil must be placed at the inner armature circumference on the commutator end, and, starting from *C*, one-half, *C* 7, 7′, 8, 8′. . . . 12 must be wound right-handedly, and the other half, *C* 6′, 6, 5′, 5 . . . . 1, left-handedly, as indicated. The winding in the interior of the armature is shown arranged in five layers, this being necessary on account of the smaller interior circumference.

6. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—The useful magnetic flux, according to formula (138), § 56, being

$$\Phi = \frac{6 \times 280 \times 10^9}{56 \times 12 \times 1200} = 2,083,000 \text{ maxwells},$$

the radial depth of the armature core, by (48), § 26, is obtained:

$$b_a = \frac{2,083,000}{2 \times 80,000 \times 5\frac{1}{8} \times .90} = 2\tfrac{7}{8} \text{ inches.}$$

In this the density in the minimum core section is taken at the upper of the limits prescribed by Table XXII., while the ratio k_2 is selected from Table XXIII., under the assumption that .010″ iron discs with oxide coating are employed.

Subtracting twice the radial depth from the core diameter, we find the internal diameter of the armature core:

$$15 - 2 \times 2\tfrac{7}{8} = 9\tfrac{1}{4} \text{ inches,}$$

and the arithmetical mean of the external and internal diameters is the mean diameter of the core:

$$d'''_a = \frac{1}{2}(15 + 9\tfrac{1}{4}) = 12\tfrac{1}{8} \text{ inches.}$$

Inserting the value of b_a into formula (234), § 91, the maximum depth of the armature core is obtained:

$$b'_a = 2\tfrac{7}{8} \times \sqrt{\frac{15}{2\frac{7}{8}} - 1} = 5.92 \text{ inches;}$$

hence, by (232) and (233), the minimum and maximum cross-sections:

$$S''_{a_1} = 2 \times 5\tfrac{1}{8} \times 2\tfrac{7}{8} \times .90 = \mathbf{26.5} \text{ square inches,}$$

and

$$S''_{a_2} = 2 \times 5\tfrac{1}{8} \times 5.92 \times .90 = \mathbf{54.7} \text{ square inches,}$$

respectively. Dividing with these areas into the useful flux, we find the maximum and minimum densities:

$$\mathcal{B}''_{a_1} = \frac{2,083,000}{26.5} = 78,700 \text{ lines per square inch,}$$

and

$$\mathcal{B}''_{a_2} = \frac{2,083,000}{54.7} = 38,100 \text{ lines per square inch,}$$

for which Table LXXXVIII., p. 336, gives the specific magnetizing forces

$$m''_{a_1} = 29.5, \text{ and } m''_{a_2} = 7.1 \text{ ampere-turns per inch.}$$

According to formula (231), therefore, the mean specific magnetizing force is:

$$m''_a = \frac{1}{2}(29.5 + 7.1) = 18.3 \text{ ampere-turns per inch,}$$

and to this, according to Table LXXXVIII., corresponds an average density of:

$$\mathfrak{B}''_a = \mathbf{68{,}500} \text{ lines per square inch.}$$

7. *Weight and Resistance of Armature Winding; Insulation Resistance of Armature.*—The poles being situated exterior to the armature, as in Fig. 59, § 27, formula (53) gives the total length of the armature conductor:

$$L_t = \frac{2 \times (5\frac{1}{8} + 2\frac{7}{8}) + .325 \times \pi}{5\frac{1}{8}} \times 288 \doteq 955 \text{ feet.}$$

Hence, by (58), p. 101, the bare weight of the armature winding:

$$wt_a = .00000303 \times 10{,}368 \times 955 = \mathbf{30} \text{ pounds.}$$

The same result can also be obtained by means of the specific weight given in the wire gauge table; No. 15 B. W. G. wire weighing .0157 pound per foot, and two wires being connected in parallel, we have:

$$wt_a = 2 \times 955 \times .0157 = 30 \text{ pounds.}$$

From this the covered weight of the winding is deduced by means of formula (59) and Table XXVI., thus:

$$wt'_a = 1.078 \times 30 = \mathbf{32\frac{1}{2}} \text{ pounds.}$$

The resistance of the armature winding, at 15.5° C., is obtained from (61), § 29:

$$r_a = \frac{1}{4 \times 2} \times 955 \times \left(\frac{10.5}{5184}\right) = \mathbf{.24} \text{ ohm.}$$

By Fig. 343 the surface of the armature core is:

$$2 \times (5\tfrac{1}{8} + 2\tfrac{7}{8}) \times 12\tfrac{1}{8} \times \pi = 610 \text{ square inches;}$$

if oiled muslin whose average resistivity, by Table XX., § 24, is 650 megohms per square inch-mil at 30° C., and 650 ÷ 25 = 26 megohms per square inch-mil at 100° C., is employed to make up the 40 mils of core insulation given by Table XIX., the insulation resistance of the armature is found:

$$\frac{650 \times 40}{610} = \mathbf{42.6} \text{ megohms at } 30° \text{ C.,}$$

and

$$\frac{26 \times 40}{610} = \mathbf{1.7} \text{ megohm at } 100° \text{ C.}$$

8. *Energy Losses in Armature, and Temperature Increase.*—The energy dissipated by the armature winding, by formula (68), § 31, is found:

$$P_a = 1.2 \times 40^2 \times .24 = 460 \text{ watts.}$$

The frequency is:

$$N_1 = \frac{1200}{60} = 20 \text{ cycles per second;}$$

the mass of iron in the armature core, from (71), § 32:

$$M = \frac{12\tfrac{1}{8} \times \pi \times 2\tfrac{7}{8} \times 5\tfrac{1}{8} \times .90}{1728} = .292 \text{ cubic feet;}$$

for $\mathfrak{B}''_a = 68,500$, Table XXIX. gives the hysteresis factor:

$$\eta = 27.3,$$

and Table XXXIII., the eddy current factor:

$$\varepsilon = .034.$$

Hence, the energy losses due to the hysteresis and eddy currents, from (73), p. 112, and (76), p. 120, respectively:

$$P_h = 27.3 \times 20 \times .292 = 160 \text{ watts,}$$
$$P_e = .034 \times 20^2 \times .292 = 4 \text{ watts.}$$

By (65), p. 107, then, the total energy dissipation in the armature is:

$$P_A = 460 + 160 + 4 = \mathbf{624} \text{ watts.}$$

The heat generated by this energy, according to (79), § 34, is liberated from a radiating surface of

$$S_A = 2 \times 12\tfrac{1}{8} \times \pi \times (5\tfrac{1}{8} + 2\tfrac{7}{8} + 1\tfrac{3}{8}) = \mathbf{715} \text{ square inches,}$$

whence follows the rise in armature temperature, by (81), p. 127:

$$\theta_a = 42 \times \frac{624}{715} = \mathbf{36\tfrac{1}{2}}° \text{ C.,}$$

the specific temperature increase, $\theta'_a = 42°$ C., being taken from Table XXXVI. for a peripheral velocity of 80 feet per second, and for a ratio of pole area to radiating surface of

$$\frac{.78 \times 15\tfrac{7}{8} \times \pi \times 5\tfrac{7}{8}}{715} = .326.$$

Inserting the above value of θ_a into formula (63), p. 106, the armature resistance, hot, at 15.5 + 36.5 = 52 degrees, Centigrade, is obtained:

$$r'_a = .24 \times \left(1 + \frac{36.5}{250}\right) = \mathbf{.275} \text{ ohm.}$$

9. *Circumferential Current Density, Safe Capacity and Running Value of Armature; Relative Efficiency of Magnetic Field.*—From formula (84), § 37, the circumferential current density is obtained:

$$i_c = \frac{672 \times 20}{15 \times \pi} = \mathbf{285} \text{ amperes per inch,}$$

for which Table XXXVII. gives a temperature increase of 30° to 50° Cent., the result obtained being indeed within these limits.

For the maximum safe capacity we find, by formula (88), § 38, and by the use of Table XXXVIII.:

$$P' = 15^2 \times 5\tfrac{1}{8} \times .89 \times 1200 \times 19{,}000 \times 10^{-6}$$
$$= \mathbf{23{,}500} \text{ watts,}$$

and for the running value of the armature, by formula (90), § 39:

$$P'_a = \frac{280 \times 40}{30 \times 19{,}000} = \mathbf{.0197} \text{ watt per pound of copper at unit field density (1 line per square inch).}$$

The values of P' and P'_a show that the armature is a very good one, electrically, for, according to the former, an overload of over 100 per cent. can be stood without injury, and by comparing the latter with the respective limits of Table XXXIX. it is learned that the inductor efficiency is as high as in the best modern dynamos.

The relative efficiency of the magnetic field, by formula (155), § 59, is:

$$\Phi'_p = \frac{2{,}083{,}000}{280 \times 40} \times 80 = \mathbf{14{,}880} \text{ maxwells per watt at unit velocity,}$$

and, according to Table LXII., page 212, this is within the limits of good design.

10. *Torque, Peripheral Force, and Lateral Thrust of Armature.*—By means of formula (93), § 40, we obtain the torque:

$$\tau = \frac{11.74}{10^{10}} \times 40 \times 672 \times 2{,}083{,}000 = \mathbf{65.7} \text{ foot-pounds.}$$

and by (95), § 41, the force acting at each armature conductor:

$$f_a = .7375 \times \frac{280 \times 40}{80 \times 672 \times .89} = \mathbf{.173} \text{ pound.}$$

The force tending to move the armature toward the magnet core is found by formula (103), § 42; the reluctance of the path through the averted half of the armature being about 10 per cent. in excess of that through the armature half nearest to the magnet core, the field density in the former will be about 10 per cent. smaller than in the latter; that is to say, the stronger density, $\mathcal{H}''$, is about 5 per cent. above, and the weaker density, $\mathcal{H}''_2$, about 5 per cent. below the average density $\mathcal{H}''$, or

$$\mathcal{H}''_1 = 19{,}000 \times 1.05 = 20{,}000,$$
$$\text{and } \mathcal{H}''_2 = 19{,}000 \times .95 = 18{,}000;$$

hence the side thrust:

$$f_t = 11 \times 10^{-9} \times 15 \times 5\tfrac{1}{8} \times (20{,}000^2 - 18{,}000^2)$$
$$= \mathbf{64\tfrac{1}{2}} \text{ pounds.}$$

This pull is to be added to or subtracted from the belt pull, according to whether the dynamo is driven from the magnet or from the armature side.

11. *Commutator, Brushes, and Connecting Cables.*—The internal diameter of the wound armature being

$$9\tfrac{1}{4} - 2 \times (.040 + 5 \times .088) = 8\tfrac{1}{4} \text{ inches},$$

the brush-surface diameter of the commutator is chosen

$$d_k = 8\tfrac{1}{4} - 2 \times \tfrac{5}{8} = 7 \text{ inches},$$

by allowing $\frac{5}{8}''$ radially for the height of the connecting lugs, as shown in Fig. 343. If we make the thickness of the side

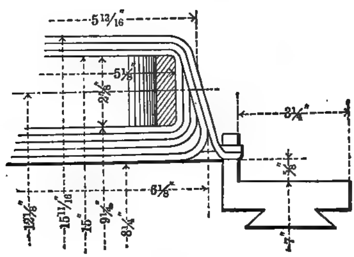


Fig. 343.—Dimensions of Armature and Commutator, 10-KW. Single-Magnet Type Generator.

mica $h_i = .030''$, Table XLVI., § 48, and if we fix the number of bars to be covered by the brush as $n_k = 1\frac{7}{8}$, the circumferential breadth of the brush contact, by (115), becomes:

$$b_k = 1\tfrac{7}{8} \times \left(\frac{7 \times \pi}{56} - .030\right) = .68''.$$

Adding to this the thickness of one side insulation, which is also covered by the brush, we obtain the breadth of the brush-bevel, .68 + .030 = 71″, which, for an angle of contact of 45°, gives the actual thickness of the brush as

$$\sqrt{\frac{.71^2}{2}} = \tfrac{1}{2} \text{ inch}.$$

Tangential carbon brushes being best suited for the machine under consideration, formula (118), page 176, gives the effective length of the brush contact surface:

$$l_k = \frac{40}{30 \times .68} = \mathbf{2} \text{ inches,}$$

which we subdivide into two brushes of 1 inch width, each. Allowing $\frac{1}{8}''$ between them for their separation in the holder, and adding $\frac{7}{16}''$ for wear, we obtain the length of the brush-surface from (114), page 169, thus:

$$l_c = (2 + \tfrac{1}{2}) \times (1 + \tfrac{1}{8}) + \tfrac{7}{16} = 2\tfrac{13}{16} + \tfrac{7}{16} = \mathbf{3\tfrac{1}{4}} \text{ inches.}$$

The best tension with which the brushes are to be pressed against the commutator is found by means of formulæ (119) to (121) and Table XLVII., as follows:

The peripheral velocity of the commutator is:

$$v_k = \frac{7 \times \pi \times 1200}{12} = 2200 \text{ feet per minute,}$$

hence the speed-correction factor for the specific friction pull, by (119), p. 179:

$$f'_k = \left(1 - \frac{2200 - 1000}{8000}\right) \times f_k = .85\, f_k .$$

Inserting the values into (120) and (121), the formulæ for the energies absorbed by contact resistance and by friction reduce to

$$P_k = .00268 \times \frac{\rho_k \times 40^2}{2 \times .68} = 3.15 \times \rho_k .$$

and

$$P_f = 6 \times 10^{-5} \times .85\, f_k \times 2 \times .68 \times 2200 = .1526\, f_k .$$

Taking from Table XLVII. the values of ρ_k and f_k for brush tensions of $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and 3 pounds per square inch, respectively, for tangential carbon brush and dry commutator, we find:

for $1\frac{1}{2}$ lbs. per sq. in., $P_k + P_f = 3.15 \times .15 + .1526 \times .95 = .618$ HP.;

" 2 " " $P_k + P_f = 3.15 \times .12 + .1526 \times 1.25 = .569$ HP.;

" $2\frac{1}{2}$ " " $P_k + P_f = 3.15 \times .10 + .1526 \times 1.6 = .559$ HP.;

" 3 " " $P_k + P_f = 3.15 \times .09 + .1526 \times 1.9 = .574$ HP.,

from which follows that the most economical pressure is about **2½** pounds per square inch of contact.

The proper cross-section of the connecting cables, by allowing 900 square mils per ampere, in accordance with Table XLVIII., § 50, is found to be:

$$40 \times 900 = 36,000 \text{ square mils,}$$

$$\text{or } 36,000 \times \frac{4}{\pi} = \mathbf{46,000} \text{ circular mils.}$$

Taking 7 strands of 3×7 wires each, or a 147-wire cable, each wire must have an area of

$$\frac{46,000}{147} = 315 \text{ circular mils,}$$

and the cable will have to be made up of No. 25 B. & S. wire, which is the nearest gauge-number.

12. *Armature Shaft and Bearings.*—By (123) and Table L., § 31, the diameter of the core portion of the shaft is:

$$d_c = 1.2 \times \sqrt[4]{\frac{280 \times 40}{1200}} = \mathbf{2\tfrac{1}{8}} \text{ inches;}$$

by (122), p. 185, and Table XLVII., the journal diameter:

$$d_b = .003 \times \sqrt{280 \times 40} \times \sqrt[4]{1200} = \mathbf{1\tfrac{7}{8}} \text{ inch;}$$

and, by (128), p. 190, and Table LIV., the length of the journal:

$$l_b = .1 \times 1\tfrac{7}{8} \times \sqrt{1200} = \mathbf{6\tfrac{1}{2}} \text{ inches.}$$

13. *Driving Spokes.*—Selecting 4-arm spiders, similar to those shown in Fig. 127, § 52, the leverage of the smallest spoke-section, determined by the radial depth of the armature, is $l_s = 3\frac{1}{2}''$, and the width of the spokes, fixed by the length of the armature core, is $b_s = 2''$; hence, by formula (126), p. 189, their thickness:

$$h_s = 4.25 \times \sqrt{\frac{11,200 \times 3\frac{1}{2}}{80 \times 8 \times 2 \times 7000}} = \tfrac{9}{32} \text{ inch.}$$

14. *Pulley and Belt.*—Taking a belt-speed of $v_B = 3500$ feet per minute, Table LVIII., § 54, the pulley diameter becomes, by (129), p. 191:

$$D_p = \frac{3.7 \times 3500}{1200} = \mathbf{10\tfrac{3}{4}} \text{ inches.}$$

the size of the belt, by Table LIX.:

$$h_B = \tfrac{3}{16} \text{ inch}, \; b_B = \mathbf{4} \text{ inches,}$$

and the width of the pulley:

$$b_p = 4 + \frac{1}{2} = \mathbf{4\tfrac{1}{2}} \text{ inches.}$$

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Magnetic Flux.*—From Table LXVIII., § 70, the average leakage factor for a 10-KW single-magnet type machine, with high-speed drum armature and cast-iron pole pieces, is $\lambda = 1.40$; the present machine having a ring armature and a cast-steel frame, the leakage is about $22 + 11 = 33$ per cent. less (see note to Table LXVIII., p. 263), and the leakage factor is reduced to $\lambda = 1.27$. The total flux, consequently, by (156):

$$\Phi' = 1.27 \times 2{,}083{,}000 = \mathbf{2{,}650{,}000} \text{ maxwells.}$$

2. *Sectional Area of Magnet Frame.*—According to formula (216) and Table LXXVIII., § 82, we obtain the cross-section of the magnet frame:

$$S''_m = \frac{2{,}650{,}000}{90{,}000} = \mathbf{29.4} \text{ square inches.}$$

The axial length of the frame, limited by the length of the armature core on the one hand and by the length over the armature winding on the other, being chosen $l_p = \mathbf{5\tfrac{7}{8}}''$, its thickness is:

$$\frac{29.4}{5\tfrac{7}{8}} = \mathbf{5} \text{ inches.}$$

3. *Polepieces and Magnet Core.*—The bore of the field is found by summing up as follows:

Diameter of armature core............	=	15.000 inches
Winding..................	$6 \times .088 =$	.528 "
Insulation and binding......	$2 \times .080 =$	.160 "
Clearance (Table LXI.)....	$2 \times \frac{5}{32} =$	.312 "
	$d_p =$	**16.000** inches

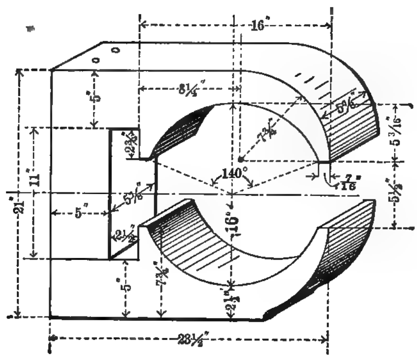


Fig. 344.—Dimensions of Field Magnet Frame, 10-KW Single-Magnet Type Generator.

Making the width at the centre of the polepiece one-half the full width, or $2\frac{1}{2}$ inches, the total height of the machine is obtained $16 + 5 = 21$ inches, leaving the length of the magnet core:

$$l_m = 21 - 2 \times 5 = \mathbf{11} \text{ inches.}$$

The distance between the pole-tips is obtained by formula (150) and Table LX., § 58, as:

$$l'_p = 5.5 \times (16 - 15) = \mathbf{5\frac{1}{2}} \text{ inches.}$$

The assumed percentage of polar arc corresponds to a pole angle of $\beta = 140°$, or a pole space angle of $\alpha = 180 - 140 = 40°$, and therefore furnishes:

$$l'_p = 16 \times \sin 20° = \mathbf{5\frac{1}{2}} \text{ inches.}$$

If the two values of l'_p so obtained differ from each other, the larger figure is preferable on account of smaller leakage.

The distance between the magnet core and the adjoining pole-tips is determined by Table LXXX., § 83. In this, the height of the magnet-winding for a 28 square inch rectangular core is given as $h_m = 2$ inches; allowing $\frac{1}{2}''$ clearance, we ob-

tain the desired distance, and making the width of the pole-shoes 16 inches, the total width of the frame becomes:

$$5 + 2\tfrac{1}{2} + 16 = \mathbf{23\tfrac{1}{2}} \text{ inches.}$$

Fig. 344 shows the field magnet frame thus dimensioned.

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap-Spaces.*—The actual field-density, by (142), p. 204, being

$$\mathcal{H}'' = \frac{2{,}083{,}000}{\frac{1}{2}(15 + 16) \times \frac{\pi}{2} \times .89 \times \frac{1}{2}(5\tfrac{1}{8} + 5\tfrac{7}{8})}$$

$$= 17{,}500 \text{ lines per square inch,}$$

the product of field density and conductor velocity is

$$\mathcal{H}'' \times v_c = 17{,}500 \times 80 = 1{,}400{,}000;$$

hence the permeance of the gaps, by Table LXVI. and formula (167), page 226:

$$\mathcal{P}_1 = \frac{15\tfrac{1}{2} \times \frac{\pi}{2} \times .89 \times 5\tfrac{1}{2}}{1.19 \times (16 - 15)} = \frac{119}{1.19} = \mathbf{100.}$$

2. *Permeance of Stray Paths.*—The area of the pole-shoe end surface, S_2, Fig. 164, is, according to Fig. 344:

$$S_2 = \frac{1}{2}\left(21 \times 8\tfrac{1}{4} + (7\tfrac{3}{4})^2 \times \frac{\pi}{2} + 7\tfrac{3}{4} \times 5\tfrac{1}{2} - 16^2 \times \frac{\pi}{4} - 5\tfrac{1}{2} \times \tfrac{7}{16}\right)$$

$$= 54 \text{ square inches.}$$

Substituting this value into (193), we obtain the total relative permeance of the waste field:

$$\mathcal{P}_3 = \frac{20\tfrac{7}{8} \times 5}{11 + 5 \times \frac{\pi}{2}} + \frac{2 \times 54 + 5\tfrac{7}{8} \times \left(8\tfrac{1}{4} + 7\tfrac{3}{4} \times \frac{\pi}{2}\right)}{5\tfrac{1}{2} + 7\tfrac{3}{4}}$$

$$+ \frac{2\tfrac{1}{2} \times 5\tfrac{7}{8}}{11} + \frac{5\tfrac{7}{8} \times 2\tfrac{3}{4}}{5\tfrac{1}{2} + 2\tfrac{3}{4}}$$

$$= 5.5 + 17.2 + 1.3 + 2.0 = \mathbf{26.}$$

3. *Leakage Factor.*—From (157), p. 218:

$$\lambda = \frac{100 + 26}{100} = \frac{126}{100} = \mathbf{1.26.}$$

This being smaller than, and only 1 per cent. different from, the leakage factor taken for the preliminary calculation of the total flux, we will use the value found from the latter in the subsequent calculations.

d. CALCULATION OF MAGNETIZING FORCES.

1. *Air Gaps.*—Length, by (166), p. 224:

$$l''_g = 1.19 \times (16 - 15) = 1.19 \text{ inch.}$$

Area, by (141), p. 204:

$$S''_g = 15\tfrac{1}{2} \times \frac{\pi}{2} \times .89 \times 5\tfrac{1}{2} = 119 \text{ square inches.}$$

Density, by (142), p. 204:

$$\mathcal{H}'' = \frac{2,083,000}{119} = 17,500 \text{ lines per square inch.}$$

Magnetizing force required, by (228), p. 339:

$$at_g = .3133 \times 17,500 \times 1.19 = \mathbf{6530} \text{ ampere-turns.}$$

2. *Armature Core.*—Length of path, by (236), p. 343:

$$l''_a = 12\tfrac{1}{8} \times \pi \times \frac{90^\circ + 20^\circ}{360^\circ} + 2\tfrac{7}{8} = 14\tfrac{1}{2} \text{ inches.}$$

Minimum area of circuit, by (232), p. 341:

$$S''_{a_1} = 2 \times 5\tfrac{1}{8} \times 2\tfrac{7}{8} \times .90 = 26.5 \text{ square inches.}$$

$$\therefore \mathcal{B}''_{a_1} = \frac{2,083,000}{26.5} = 78,700 \text{ lines p. sq. in.}; \quad m''_{a_1} = 29.5.$$

Maximum area of circuit, by (233), p. 341, and (234), p. 342:

$$S''_{a_2} = 2 \times 5\tfrac{1}{8} \times 2\tfrac{7}{8} \times \sqrt{\frac{15}{2\tfrac{7}{8}} - 1} \times .90 = 54.7 \text{ sq. in.}$$

$$\therefore \mathcal{B}''_{a_2} = \frac{2,083,000}{54.7} = 38,000 \text{ lines p. sq. in.}; \quad m''_{a_2} = 7.1.$$

Average specific magnetizing force, by (231), p. 341:

$$m''_a = \frac{29.5 + 7.1}{2} = 18.3 \text{ ampere-turns per inch.}$$

Magnetizing force required, by (230), p. 340:

$$at_a = 18.3 \times 14\tfrac{1}{2} = \mathbf{265} \text{ ampere-turns.}$$

3. *Magnet Frame* (all cast steel).—Length of portion with uniform cross-section (core and yokes):

$$l''_m = 11 + 2 \times (5 + 2\tfrac{1}{2}) = 26 \text{ inches.}$$

Area of magnet core and yokes:

$$S_m = 5 \times 5\tfrac{7}{8} = 29.4 \text{ square inches.}$$

Density:

$$\mathfrak{B}''_m = \frac{2,650,000}{29.4} = 90,000 \text{ lines per square inch.}$$

Specific magnetizing force (Table LXXXVIII., or Fig. 256):

$$m''_m = 57 \text{ ampere-turns per inch.}$$

Mean length of portion with varying cross-section (pole-pieces), from formula (243) and Fig. 344:

$$l''_p = 2\tfrac{3}{4} + 8\tfrac{1}{4} + 7\tfrac{3}{4} \times \frac{\pi}{2} - 5\tfrac{3}{16} = 18 \text{ inches.}$$

Minimum area:

$$S''_{p_1} = 5 \times 5\tfrac{7}{8} = 29.4 \text{ square inches.}$$

$$\therefore \mathfrak{B}''_{p_1} = \frac{2,650,000}{29.4} = 90,000 \text{ lines p. sq. in.}; \; m''_{p_1} = 57.$$

Maximum area:

$$S''_{p_2} = \left(15\tfrac{3}{8} \times \frac{\pi}{2} \times .78 + 2 \times \frac{1}{2}\right) \times 5\tfrac{7}{8} = 120 \text{ sq. in.}$$

$$\therefore \mathfrak{B}''_{p_2} = \frac{2,083,000}{120} = 17,400 \text{ lines p. sq. in.}; \; m''_{p_2} = 4.6.$$

Average specific magnetizing force, by formula (241):

$$m''_p = \frac{57 + 4.6}{2} = 30.8 \text{ ampere-turns per inch.}$$

Corresponding flux density by Table LXXXVIII., p. 336:

$$\mathfrak{B}''_p = 78,000 \text{ lines per square inch.}$$

Magnetizing force required for magnet frame, by (238), p. 344:

$$at_m = 57 \times 26 + 30.8 \times 18 = \mathbf{2035} \text{ ampere-turns.}$$

4. *Armature Reaction.*—According to Table XCI., § 93, the coefficient of armature-reaction for $\mathfrak{B}''_p = 78{,}000$, in cast steel, is $k_{14} = 1.25$, hence, by formula (250), the magnetizing force required to compensate the magnetizing effect of the armature winding:

$$at_r = 1.25 \times \frac{672 \times 40}{2} \times \frac{20°}{180°} = \mathbf{1870} \text{ ampere-turns.}$$

5. *Total Magnetizing Force Required.*—By (227), p. 339, we have:

$$AT = 6530 + 625 + 2035 + 1870 = \mathbf{10{,}700} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

Temperature increase at normal load not to exceed $\theta_m = 30°$ Centigrade. Voltage to be adjustable between 225 and 250, in steps of 5 volts each.

1. *Winding Proper* (*for* $E = 250$ *volts*).—Rounding the total magnetizing force to 11,000 ampere-turns, formula (287), § 99, gives the number of series turns:

$$N_{se} = \frac{11{,}000}{40} = \mathbf{275.}$$

The length of the mean turn, by (290), being

$$l_T = 2\,(5\tfrac{7}{8} + 5) + 2 \times \pi = 28 \text{ inches,}$$

the total length of the series field wire is obtained, by formula (288) p. 374:

$$L_{se} = \frac{275 \times 28}{12} = \mathbf{642} \text{ feet.}$$

Formulæ (278) and (282) give the radiating surface of the magnet:

$$S_{M_s} = 2 \times 11 \times (5\tfrac{7}{8} + 5 + 2 \times \pi) + 2 \times 2 \times (28 - 5\tfrac{7}{8})$$
$$= 466 \text{ square inches,}$$

hence by (294) the resistance required for the specified temperature increase:

$$r_{se} = \frac{30}{75} \times \frac{466}{40^2} \times \frac{1}{1 + .004 \times 30} = \mathbf{.104} \text{ ohm,}$$

and therefore by (294) the specific length of the magnet-wire:

$$\lambda_{se} = \frac{642}{.104} = \mathbf{6170} \text{ feet per ohm.}$$

The nearest gauge wire is No. 2 B. & S., which is too inconvenient to handle; we therefore take 2 No. 7 B. W. G. wires (.180″ + .012″ = .192″), which have a joint specific length of $2 \times 3138.6 = 6277$ feet per ohm. Allowing $\frac{3}{8}$ inch at each end of the magnet spool for insulation and discs, formula (297), p. 377, gives an effective winding depth of

$$h'_m = 275 \times \frac{2 \times .192^2}{11 - 2 \times \frac{3}{8}} = 1.9 \text{ inch.}$$

Actual resistance of magnet-winding (from wire gauge table):

$$r_{se} = \frac{642 \times .000319}{2} = \mathbf{.1025} \text{ ohm at } 15.5^\circ \text{ C.,}$$

or

$$r'_{se} = .1025 \times 1.12 = \mathbf{.115} \text{ ohm at } 45.5^\circ \text{ C.}$$

Weight of magnet winding, bare:

$$wt_m = 2 \times 642 \times .098 = \mathbf{126} \text{ pounds;}$$

weight, covered, from Table XXVI., § 28:

$$wt'_m = 1.0228 \times 126 = \mathbf{130} \text{ pounds.}$$

2. *Regulator* (*see diagrams*, § 100).—The difference of 5 volts between each of the five steps being

$$\frac{5 \times 100}{250} = 2 \text{ per cent. of the full load output,}$$

the shunt coil regulator has to be calculated for 90, 92, 94, 96, and 98 per cent. of the maximum E. M. F., the resistances of the five combinations, therefore, are:

Resistance, first	combination	$= \frac{90}{10} \times r'_{se} =$	$9 \times r'_{se}$,
" second	"	$= \frac{92}{8} \times r'_{se} =$	$11.5 \times r'_{se}$,
" third	"	$= \frac{94}{6} \times r'_{se} =$	$15.67 \times r'_{se}$,
" fourth	"	$= \frac{96}{4} \times r'_{se} =$	$24 \times r'_{se}$,
" fifth	"	$= \frac{98}{2} \times r'_{se} =$	$49 \times r'_{se}$,

By the proceeding shown in § 100 we then obtain the following formulæ for the resistances of the five coils:

$$r_{\mathrm{I}} = \frac{(11.5\, r'_{se} - r_1) \times (9\, r'_{se} - r_1)}{(11.5\, r'_{se} - r_1) - (9\, r'_{se} - r_1)}$$

$$= \frac{113.5\, r'^2_{se} - 20.5\, r'_{se}\, r_1 + r_1^2}{2.5\, r'_{se}}$$

$$= 45.5\, r'_{se} - 8.2\, r_1; \quad \text{.......................... (457)}$$

$$r_{\mathrm{II}} = \frac{(15.67\, r'_{se} - r_1) \times (11.5\, r'_{se} - r_1)}{(15.67\, r'_{se} - r_1) - (11.5\, r'_{se} - r_1)}$$

$$= \frac{160.2\, r'^2_{se} - 27.2\, r'_{se}\, r_1 + r_1^2}{4.167\, r'_{se}}$$

$$= 38.2\, r'_{se} - 6.5\, r_1; \quad \text{.......................... (458)}$$

r_{III} = resistance of third combination minus res. of leads

$$= 15.67\, r'_{se} - r_1; \quad \text{.......................... (459)}$$

r_{IV} = res. of fourth comb. minus res. third comb.

$$= (24 - 15.67)\, r'_{se} = 8.33\, r'_{se}; \quad \text{.................. (460)}$$

r_{V} = res. of fifth comb. minus res. fourth comb.

$$= (49 - 24)\, r'_{se} = 25\, r'_{se}. \quad \text{.................. (461)}$$

These formulæ apply to all cases in which a total regulation of 10 per cent., in five steps of 2 per cent. each, is desired. In the present example, the resistance of the series winding, hot, being $r'_{se} = .115$ ohm, and the resistance of the leads $r_1 = .01$ ohm (assuming 4 feet of 4000 circular mil cable, carrying 10 per cent. of the maximum current output, or 4 amperes), we have:

$$\begin{aligned}
r_{\mathrm{I}} &= 45.5 \times .115 - 8.2 \times .01 = \mathbf{5.15} \text{ ohms,} \\
r_{\mathrm{II}} &= 38.2 \times .115 - 6.2 \times .01 = \mathbf{4.32} \quad \text{"} \\
r_{\mathrm{III}} &= 15.67 \times .115 - .01 = \mathbf{1.79} \quad \text{"} \\
r_{\mathrm{IV}} &= 8.33 \times .115 = \mathbf{.96} \quad \text{"} \\
r_{\mathrm{V}} &= 25 \times .115 = \mathbf{2.88} \quad \text{"}
\end{aligned}$$

The currents flowing in the various coils, at the different combinations, are:

First combination:

$$I_{\text{I}} = \frac{r_{\text{II}}\, r_{\text{III}}}{r_{\text{II}}\, r_{\text{III}} + r_{\text{I}}\, r_{\text{III}} + r_{\text{I}}\, r_{\text{II}}} \times .1\, I$$

$$= \frac{4.32 \times 1.79}{4.32 \times 1.79 + 5.15 \times 1.79 + 5.15 \times 4.32} \times .1 \times 40$$

$$= \frac{7.73}{7.73 + 9.22 + 22.35} \times 4 = \frac{7.73}{39.3} \times 4 = .8 \text{ ampere.}$$

$$I_{\text{II}} = \frac{r_{\text{I}}\, r_{\text{III}}}{r_{\text{I}}\, r_{\text{III}} + r_{\text{I}}\, r_{\text{II}} + r_{\text{II}}\, r_{\text{III}}} \times .1\, I = \frac{9.22}{39.3} \times 4 = .95 \text{ amp.}$$

$$I_{\text{III}} = \frac{r_{\text{I}}\, r_{\text{II}}}{r_{\text{I}}\, r_{\text{II}} + r_{\text{I}}\, r_{\text{III}} + r_{\text{II}}\, r_{\text{III}}} \times .1\, I = \frac{22.35}{39.3} \times 4 = 2.3 \text{ amp.}$$

Second combination:

$$I_{\text{II}} = \frac{r_{\text{III}}}{r_{\text{II}} + r_{\text{III}}} \times .08\, I = \frac{1.79}{6.11} \times 3.2 = .95 \text{ ampere.}$$

$$I_{\text{III}} = \frac{r_{\text{II}}}{r_{\text{II}} + r_{\text{III}}} \times .08\, I = \frac{4.32}{6.11} \times 3.2 = 2.3 \text{ amperes.}$$

Third combination:

$I_{\text{III}} = .06\, I = 2.4$ amperes.

Fourth combination:

$I_{\text{III}} = I_{\text{IV}} = .04\, I = 1.6$ amperes.

Fifth combination:

$I_{\text{III}} = I_{\text{IV}} = I_{\text{V}} = .02\, I = .8$ ampere.

By comparison, the maximum current passing through each of the five coils, in the present case of a machine of 40 amps. capacity, is found:

I_{I} = **.8** amp.; or, for the general case of current output I, we have: $I_{\text{I}} = .2 \times .1\, I = .02\, I$, **(462)**
I_{II} = **.95** amp.; or, in general: $I_{\text{II}} = .3 \times .08\, I = .024\, I$, **(463)**
I_{III} = **2.4** amp.; or, in general: $I_{\text{III}} = .06\, I$, ...**(464)**
I_{IV} = **1.6** amp.; or, in general: $I_{\text{IV}} = .04\, I$, ...**(465)**
I_{V} = **.8** amp.; or, in general: $I_{\text{V}} = .02\, I$, ...**(466)**

From the wire gauge table, finally, the size of the wire sufficient to carry the maximum current, and the length and weight of the same, required to make up the necessary resistance, is obtained:

Coil Number.	Carrying Capacity, Amp.	Size of Wire.	Sectional Area, Cir. Mils.	Current Density, C. M. p. Amp.	Resistance Required, Ohms.	Length of Wire, Feet.	Weight, Covered, Lbs.
I	.8	No. 21 B. & S.	810	1012	5.15	400	1.05
II	.95	No. 20 B. & S.	1021	1073	4.32	427	1.29
III	2.4	No. 18 B. W. G.	2401	1000	1.79	414	3.15
IV	1.6	No. 18 B. & S.	1624	1015	.96	150	.76
V	.8	No. 21 B. & S.	810	1012	2.88	224	.58

f. CALCULATION OF EFFICIENCIES.

1. *Electrical Efficiency.*—The electrical efficiency of the above dynamo, by formula (351), p. 405, is:

$$\eta_e = \frac{250 \times 40}{250 \times 40 + 40^2 \times (.275 + .115)}$$

$$= \frac{10,000}{10,624} = .943, \text{ or } \mathbf{94.3\ \%}.$$

2. *Commercial Efficiency and Gross Efficiency.*—The energy losses due to hysteresis, eddy currents, brush contact, and brush friction were found $P_h = 160$, $P_e = 4$, $P_k = .315 \times 746 = 235$, and $P_f = .244 \times 746 = 182$ watts, respectively; assuming that journal friction and air resistance cause a further energy loss of 500 watts, the commercial, or net efficiency of the machine will be, by (359), p. 361:

$$\eta_c = \frac{10,000}{10,624 + 160 + 4 + 235 + 182 + 500}$$

$$= \frac{10,000}{11,705} = .855, \text{ or } \mathbf{85.5\ \%}.$$

In dividing this by the electrical efficiency, the efficiency of conversion, or the gross efficiency, is obtained:

$$\eta_g = \frac{.855}{.943} = .907, \text{ or } \mathbf{90.7\ \%}.$$

3. *Weight Efficiency.*—The weights of the various parts of our dynamo are as follows :

Armature:		
Core, .292 cu. ft. of sheet iron, . .	140 lbs.	
Winding (§ 134, *a*, 7), core insulation, binding, and connecting wires, .	40 lbs.	
Shaft, spiders, pulley, keys, and bolts (estimated), ...	100 lbs.	
Commutator, 7″ dia. × $3\frac{1}{4}$″ length, .	20 lbs.	
Armature, complete, ...		300 lbs.
Frame:		
Magnet core and polepieces (see Fig. 344 and § 134, *c*, 2), (5 × 26 + 54) × $5\frac{5}{8}$ = 1080 cu. ins. of cast steel, .	300 lbs.	
Field winding (§ 134, *c*, 1), core insulation, flanges, etc., . . .	150 lbs.	
Bedplate (cast-iron), bearings, etc., (estimated), ...	250 lbs.	
Frame, complete, ...		700 lbs.
Fittings:		
Brushes, holders, and brush-rocker, (estimated), ...	20 lbs.	
Field regulator (winding, see § 134, *e*, 2), ...	15 lbs.	
Switches, cables, etc. (estimated), .	15 lbs.	
Fittings, complete, ...		50 lbs.
Hence the *total net weight* of the machine, .		**1050** lbs.

The useful output is 10 KW, therefore the weight-efficiency, by § 109:

$$\frac{10{,}000}{1050} = \mathbf{9.5} \text{ watts per pound.}$$

135. Calculation of a Bipolar, Single Magnetic Circuit, Smooth-Drum, High-Speed Shunt Dynamo:

300 KW. Upright Horseshoe Type. Wrought-Iron Cores and Yoke, Cast-Iron Polepieces. 500 Volts. 600 Amps. 400 Revs. per Min.

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—For this machine, since 300 KW is a large output for a bipolar type, we take the upper limit given for the ratio of polar embrace of smooth-

drum armatures, namely, $\beta_1 = .75$. Hence, by Table IV., p. 50: $e = 62.5 \times 10^{-8}$ volt per bifurcation; the number of bifurcations is $n'_p = 1$. The mean conductor velocity, from Table V., p. 52: $v_c = 50$ feet per second; and the field density, from Table VI., p. 54: $\mathcal{H}'' = 30,000$ lines per square inch. The total E. M. F. to be generated, by Table VIII., p. 56: $E' = 1.025 \times 500 = 512.5$ volts.

Consequently, by (26):

$$L_a = \frac{512.5 \times 10^8}{62.5 \times 50 \times 30,000} = \mathbf{547} \text{ feet.}$$

2. *Sectional Area of Armature Conductor, and Selection of Wire.*—By (27), p. 57:

$$\delta_a^2 = 300 \times 600 = \mathbf{180,000} \text{ circular mils.}$$

Taking 3 cables made up of 7 No. 13 B. W. G. wires having .095″ diameter and 9025 circular mils area each, we have a total actual cross-section of

$$3 \times 7 \times 9025 = \mathbf{189,525} \text{ circular mils,}$$

the excess over the calculated area amply allowing for the difference between the current output and the total current generated in the armature, see § 20.

For large drum armatures cables are preferable to thick wires or copper rods, because they can be bent much easier, are much less liable to wasteful eddy currents, and, since air can circulate in the spaces between the single wires, effect a better ventilation of the armatures.

In accordance with § 24, *a*, a single covering of .007″ is selected for the single wires, and an additional double coating of .016″ is chosen for each cable of seven wires, making the total diameter of the insulated cable, see Fig. 345:

$$\delta'_a = 3 \times (.095'' + .007'') + .016'' = \mathbf{.322} \text{ inch.}$$

3. *Diameter of Armature Core.*—
From (30):

$$d'_a = 230 \times \frac{50}{400} = 28\tfrac{3}{4} \text{ inches.}$$

By Table IX.:

$$d_a = .97 \times 28\tfrac{3}{4} = \mathbf{28} \text{ inches.}$$

4. *Length of Armature Core.*—

By (37) and Table XVII, p. 73:

$$n_w = \frac{28 \times \pi \times (1 - .08)}{.322} = 252.$$

By (39), p. 74, and Tables XVIII. and XIX.:

$$n_1 = \frac{.8 - (.090 + .070)}{.322} = 2.$$

By (40), p. 76:

$$l_a = \frac{12 \times 3 \times \left(\frac{547}{1.04}\right)}{252 \times 2} = 37\tfrac{1}{2} \text{ inches.}$$

In this the active length of the armature conductor has been divided by 1.04, taking into consideration the lateral spread of

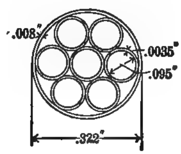


Fig. 345.—Armature Cable, 300-KW Bipolar Horseshoe-Type Generator.

the field in the axial direction, and assuming the same to amount to 4 per cent. of the length of the armature core.

5. *Arrangement of Armature Winding.*—

By (45) p. 89, and Table XXI:

$$(n_c)_{\min} = \frac{500 \times 2}{20} = 50,$$

and

$$(n_c)_{\max} = \frac{500 \times 2}{10} = 100.$$

Two values of n_c between these limits can be obtained, viz.:

$$n_c = \frac{252 \times 2}{3} \div 2 = 84,$$

and

$$n_c = \frac{252 \times 2}{3} \div 3 = 56.$$

For the latter number of divisions, however, there are three conductors per commutator-bar, and since the armature is a

drum, there would be $1\frac{1}{2}$ turn to each coil, which is impossible; therefore, the number of coils employed:

$$n_c = \mathbf{84}.$$

By (47), p. 89, then:

$$n_a = \frac{252 \times 2}{2 \times 84 \times 3} = \mathbf{1};$$

hence, summary of armature winding: **84** coils, each consisting of **1** turn of **3** cables made up of **7** No. **13** B. W. G. wires.

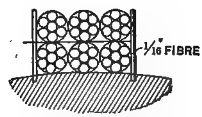


Fig. 346.—Arrangement of Armature Winding, 300-KW Bipolar Horseshoe-Type Generator.

One armature division containing the beginning of one coil and the end of the one diametrically opposite, is shown in Fig. 346.

6. *Weight and Resistance of Armature Winding.*—
By (50), p. 96:

$$L_t = 547 \times \left(1 + 1.3 \times \frac{28}{37\frac{1}{2}}\right) = 1070 \text{ feet.}$$

Here the original value of L_a, without reduction, is used, in regard of the fact that, in a cable, due to the helical arrangement of the wires, the actual length of each strand is greater than the length of the cable itself, and under the assumption that 4 per cent. is the proper allowance for this increase in the present case.

By (58), p. 101, then:

$$wt_a = .00000303 \times 189{,}525 \times 1070 = \mathbf{615} \text{ pounds.}$$

By (59), p. 102, and Table XXVI.:

$$wt'_a = 1.031 \times 615 = \mathbf{634} \text{ pounds.}$$

From (61), p. 105:

$$r_a = \frac{1}{4 \times 21} \times 1070 \times .001144 = \mathbf{.0146} \text{ ohm, at } 15.5\,^\circ \text{ C.}$$

7. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—
By (123), and Table XLVIII., p. 183:

$$d_c = 1.55 \times \sqrt[4]{\frac{300,000}{400}} = \mathbf{8} \text{ inches};$$

see also Table XLIX., p. 185; therefore:

$$b_a = \frac{1}{2}(d_a - d_c) = \frac{28 - 8}{2} = \mathbf{10} \text{ inches},$$

and from (234), p. 342, or Fig. 347:

$$b'_a = 10 \times \sqrt{\frac{28}{10} - 1} = \mathbf{13.4} \text{ inches}.$$

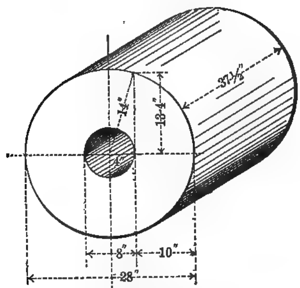


Fig. 347.—Dimensions of Armature Core, 300-KW Bipolar Horseshoe-Type Generator.

Hence by (232), p. 341:

$$S''_{a_1} = 2 \times 37\tfrac{1}{2} \times 10 \times .95 = \mathbf{712} \text{ square inches.}$$

and by (233), p. 341:

$$S''_{a_2} = 2 \times 37\tfrac{1}{2} \times 13.4 \times .95 = \mathbf{956} \text{ square inches.}$$

From (138), p. 202:

$$\Phi = \frac{6 \times 512.5 \times 10^9}{2 \times 84 \times 400} = 45,760,000 \text{ maxwells};$$

consequently:

$$\mathfrak{B}''_{a_1} = \frac{45,760,000}{712} = 64,200 \text{ lines per square inch},$$

$$\mathfrak{B}''_{a_2} = \frac{45,760,000}{956} = 47,800 \text{ lines per square inch.}$$

From Table LXXXVIII., page 336:

$$m''_{a_1} = 15.2 \text{ ampere-turns per inch;}$$
$$m''_{a_2} = \ \ 9.1 \quad `` \quad `` \quad `` \quad ``$$

$$\therefore\ m''_a = \frac{15.2 + 9.1}{2} = 12.15 \text{ ampere-turns per inch.}$$

To the latter corresponds a mean density of:

$$\mathfrak{B}''_a = \mathbf{58,000} \text{ lines per square inch.}$$

8. *Energy Losses in Armature, and Temperature Increase.*—By (68), p. 109:

$$P_a = 1.2 \times 600^2 \times .0146 = \mathbf{6307} \text{ watts.}$$

$$N_1 = \frac{400}{60} = 6.67 \text{ cycles per second;}$$

By (71), p. 112:

$$M = \frac{18 \times \pi \times 37\tfrac{1}{2} \times 10 \times .9}{1728} = 11.05 \text{ cubic feet;}$$

From Table XXIX., ($\mathfrak{B}''_a = 58,000$):

$$\eta = 20.92 \text{ watts per cubic feet;}$$

From XXXIII., ($\delta_i = .010''$):

$$\varepsilon = .0242 \text{ watt per cubic feet.}$$

By (73), p. 112:

$$P_h = 20.92 \times 6.67 \times 11.05 = \mathbf{1540} \text{ watts.}$$

By (76), p. 120:

$$P_e = .0242 \times 6.67^2 \times 11.05 = \mathbf{12} \text{ watts.}$$

By (65), p. 107:

$$P_A = 6307 + 1540 + 12 = \mathbf{7859} \text{ watts.}$$

From Table XXXV., p. 124:

$$l_h = 35 \times 28 + 2 \times .8 = 11\tfrac{1}{2} \text{ inches.}$$

By (78), p. 124:

$$S_A = (28 + 2 \times .8) \times \pi\,(37\tfrac{1}{2} + 1.8 \times 11\tfrac{1}{2}) = \mathbf{5412} \text{ sq. ins.}$$

Ratio of pole area to radiating surface:

$$\frac{30 \times \pi \times 37\frac{1}{2} \times .75}{5412} = .49.$$

For this ratio and a peripheral velocity of

$$\frac{29\frac{5}{8} \times \pi}{12} \times \frac{400}{60} = 51\frac{3}{4} \text{ feet per second.}$$

Table XXXVI., p. 127, gives: $\theta'_a = 44.7°$ C.; consequently by (81):

$$\theta_a = 44.7 \times \frac{7859}{5412} = \mathbf{65°} \text{ Centigrade,}$$

and the resistance of the armature, when hot, is:

$$r'_a = .0146 \times (1 + .004 \times 65) = \mathbf{.0184} \text{ ohm, at } 80.5° \text{ C.}$$

9. *Circumferential Current Density, Safe Capacity, and Running Value of Armature; Relative Efficiency of Magnetic Field.*— By (84), p. 131:

$$i_c = \frac{84 \times 2 \times 300}{28 \times \pi} = \mathbf{572} \text{ amperes per inch.}$$

Corresponding increase of temperature, from Table XXXVII., p. 132: $\theta_a = 60°$ to $80°$ C., which checks the above result.

By (88), p. 134, and Table XXXVIII.:

$$P' = 28^2 \times 37\frac{1}{2} \times .88 \times 400 \times 30{,}000 \times 10^{-9} = \mathbf{310{,}000} \text{ watts.}$$

By (90), p. 135:

$$P'_a = \frac{512.5 \times 600}{615 \times 30{,}000} = \mathbf{.0167} \text{ watt per pound of copper, at unit field density;}$$

this also verifies the calculation, see Table XXXIX., p. 136.

By (155), p. 211:

$$\Phi'_p = \frac{45{,}760{,}000}{512.5 \times 600} \times 50 = \mathbf{7450} \text{ maxwells per watt, at unit velocity;}$$

by Table LXII., p. 212, this is not too high.

10. *Torque, Peripheral Force, and Upward Thrust of Armature.*—
By (93), p. 138:

$$\tau = \frac{11.74}{10^{10}} \times 600 \times 168 \times 45{,}760{,}000 = \mathbf{5420} \text{ foot-pounds.}$$

By (95), p. 138:

$$f_a = .7375 \times \frac{512.5 \times 600}{50 \times 168 \times .88} = \mathbf{30.7} \text{ pounds.}$$

By (103), p. 141:

$$f_t = 11 \times 10^{-9} \times 28 \times 37\tfrac{1}{2} \times (30{,}600^2 - 29{,}400^2) = \mathbf{832} \text{ lbs.},$$

under the assumption that the density of the upper half of the field is 2 per cent. above, and that of the lower half 2 per cent. below, the average.

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Magnetic Flux, and Sectional Areas of Magnet Frame.*—
By (156), p. 214, and Table LXVIII.:

$$\Phi' = 1.20 \times 45{,}760{,}000 = \mathbf{55{,}000{,}000} \text{ maxwells.}$$

By (217), p. 314, wrought-iron cores and yoke being used:

$$S''_{\text{w.i.}} = \frac{55{,}000{,}000}{90{,}000} = \mathbf{611} \text{ square inches.}$$

By (216), p. 313, and Table LXXVI., the minimum section of the cast-iron polepieces:

$$S''_{\text{c.i.}} = \frac{55{,}000{,}000}{50{,}000} = \mathbf{1100} \text{ square inches.}$$

2. *Magnet Cores.*—Selecting the circular form for the cross-section of the magnets, their diameter is:

$$d_m = \sqrt{611 \times \frac{4}{\pi}} = \mathbf{28} \text{ inches.}$$

Length of cores, from Table LXXXI., p. 319, by interpolation:

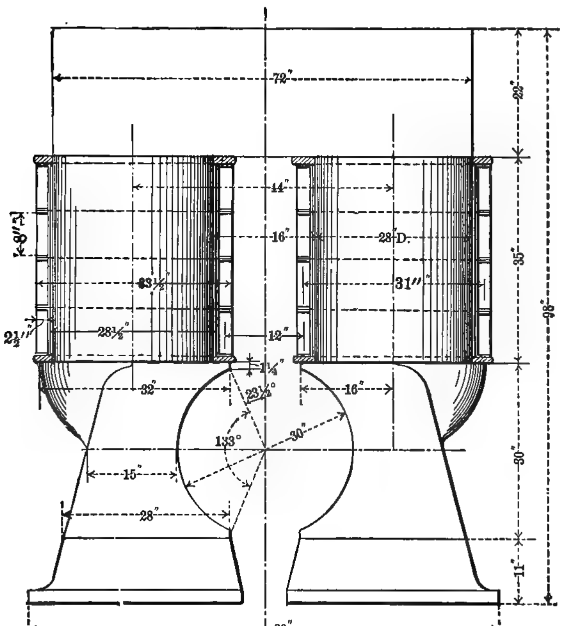
$$l_m = \mathbf{35} \text{ inches.}$$

Distance apart, from Table LXXXV., p. 323,

$$c = \mathbf{16} \text{ inches.}$$

3. *Yoke.*—Making the width of the yoke, parallel to the shaft, equal to the diameter of the cores, its height is found:

$$h_y = \frac{611}{28} = \mathbf{22} \text{ inches.}$$



SCALE: ⅜ INCH = 1 FT.

Fig. 348.—Dimensions of Field Magnet Frame, 300-KW Bipolar Horseshoe-Type Generator.

The length of the yoke is given by the diameter of the cores, and by their distance apart, see Fig. 348:

$$l_y = 2 \times 28 + 16 = \mathbf{72} \text{ inches.}$$

4. *Polepieces.*—The bore of the field is the sum of:

Diameter of armature core,	= 28.000
Winding........4 × .322″,	= 1.288
Insulation and binding, 2 × (.070″ + .070″)	= .280
Clearance (Table LXI., p. 209)............2 × ¼″,	= .500
	30.068 or, say, **30** inches.

Pole distance, by (150), p. 208, and Table LX.:

$$l'_p = 6 \times (30 - 28) = \mathbf{12} \text{ inches.}$$

Length of polepieces equal to length of armature core, or:

$$l_p = \mathbf{37\tfrac{1}{2}} \text{ inches.}$$

Height of polepieces, same as bore:

$$h_p = \mathbf{30} \text{ inches.}$$

Thickness in centre, requiring half of the full area:

$$\frac{1100}{2 \times 37\frac{1}{2}} = 14.7, \text{ say } \mathbf{15} \text{ inches.}$$

Height of pole-tips:

$$\frac{1}{2}\left(30 - \sqrt{30^2 - 12^2}\right) = \mathbf{1\tfrac{1}{4}} \text{ inch.}$$

Height of zinc blocks, from Table LXX., p. 301:

$$h_z = \mathbf{11} \text{ inches.}$$

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap-Spaces.*—

$$\mathcal{H}'' \times v_c = 30{,}000 \times 50 = 1{,}500{,}000,$$

therefore, by (167), p. 226, and Table LXVI.:

$$\mathfrak{P}_1 = \frac{\frac{1}{2}(28 + 30) \times \frac{\pi}{2} \times .88 \times 37\frac{1}{2}}{1.35 \times (30 - 28)} = \frac{1500}{2.7} = \mathbf{536.}$$

2. *Permeance by Stray Paths.*—

By (178), p. 232:

$$\mathfrak{P}_2 = \frac{28 \times \pi \times 35}{2 \times 16 + 1.5 \times 28} = \mathbf{4.16.}$$

By (188), p. 239:

$$\mathfrak{P}_3 = \frac{\frac{1}{2} \times [37\tfrac{1}{2} \times (28 + 15) + 850]}{2 \times 11} = \mathbf{56},$$

the portion of the bed plate opposite one polepiece being estimated to have a surface of $S = 850$ square inches. The projecting area of the polepiece, see Fig. 349, is

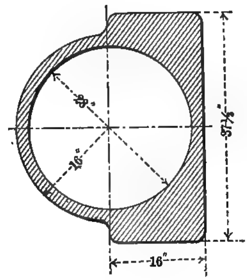


Fig. 349.—Top View of Polepiece, 300 KW Bipolar Horseshoe-Type Generator, Showing Projecting Area.

$$S_3 = 16 \times 37\tfrac{1}{2} + 16^2 \times \frac{\pi}{2} - 28^2 \frac{\pi}{4} = 386 \text{ square inches,}$$

hence by (199), p. 245:

$$\mathfrak{P}_4 = \frac{386}{35} + \frac{37\tfrac{1}{2} \times 30}{2 \times 35 + (30 + 22) \times \frac{\pi}{2}} = 11 + 7.4 = \mathbf{18.4.}$$

3. *Leakage Factor.*—

By (157), p. 218:

$$\lambda = \frac{536 + 41.6 + 56 + 18.4}{536} = \frac{652}{536} = \mathbf{1.215.}$$

Total flux:

$$\Phi' = 1.215 \times 45,760,000 = \mathbf{55,700,000} \text{ maxwells.}$$

d. CALCULATION OF MAGNETIZING FORCES.

1. *Air Gaps.*—

Length, by (166), p. 224:

$$l''_g = 1.35 \times (30 - 28) = 2.7 \text{ inches.}$$

Area, by (141), p. 204:

$$S_g = 29 \times \frac{\pi}{2} \times .88 \times 37\tfrac{1}{2} = 1500 \text{ square inches.}$$

Density, by (142), p. 204:

$$\mathcal{H}'' = \frac{45,760,000}{1500} = 30,500 \text{ lines per square inch.}$$

Magnetizing force required, by (228), p. 339:

$$at_g = .3133 \times 30,500 \times 2.7 = \mathbf{25,800} \text{ ampere-turns.}$$

2. *Armature Core.*—

Length, by (236), p. 343, see Fig. 350:

$$l''_a = 18 \times \pi \times \frac{113\tfrac{1}{2}^\circ}{360} + 10 = 27.85 \text{ inches.}$$

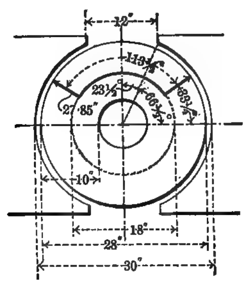


Fig. 350.—Flux Path in Armature, 300-KW Bipolar Horseshoe-Type Generator.

Minimum area, by (232), p. 341:

$$S''_{a_1} = 2 \times 37\tfrac{1}{2} \times 10 \times .95 = 712 \text{ square inches.}$$

$$\therefore \mathcal{B}''_{a_1} = \frac{45,760,000}{712} = 64,200 \text{ lines p. sq. in.}; \; m''_{a_1} = 15.2.$$

Maximum area, by (233), p. 341, and (234), p. 342:

$$S''_{a_2} = 2 \times 37\tfrac{1}{2} \times 13.4 \times .95 = 956 \text{ square inches.}$$

$$\therefore \mathcal{B}''_{a_2} = \frac{45,760,000}{956} = 47,800 \text{ lines p. sq. in.}; \; m''_{a_1} = 9.1.$$

Average specific magnetizing force, by (231), p. 341:

$$m''_a = \frac{15.2 + 9.1}{2} = 12.15 \text{ ampere-turns p. inch.}$$

Corresponding average density: $\mathcal{B}''_a = 58,000$ lines per sq. in.

Magnetizing force required, by (230), p. 340:

$$at_a = 12.15 \times 27.85 = \mathbf{340} \text{ ampere-turns.}$$

3. *Wrought Iron Portion of Frame* (*Cores and Yoke*).—

Length:

$$l''_{w.i.} = 2 \times 35 + 22 + 44 = 136 \text{ inches.}$$

Area:

$$S''_{w.i.} = 28^2 \frac{\pi}{4} = 615.75 \text{ square inches.}$$

Density and corresponding specific magnetizing force:

$$\mathfrak{B}''_{w.i.} = \frac{55,700,000}{615.75} = 90,000 \text{ lines; } m''_{w.i.} = 50.7.$$

Magnetizing force required:

$$at_{w.i.} = 50.7 \times 136 = \mathbf{6900} \text{ ampere-turns.}$$

4. *Cast Iron Portion of Frame* (*Polepieces*).—

Length, by (243), page 348:

$$l''_{c.i.} = 35 + 2 = 37 \text{ inches.}$$

Minimum area (at center):

$$S''_{c.i._1} = 15 \times 37\tfrac{1}{2} = 562.5 \text{ square inches.}$$

Corresponding maximum density and specific magnetizing force:

$$\mathfrak{B}''_{c.i._1} = \frac{\frac{1}{2} \times 55,700,000}{562.5} = 49,500 \text{ lines; } m''_{c.i._1} = 155.$$

Maximum area (at poleface):

$$S''_{c.i._2} = \left(30 \times \pi \times \frac{133}{360} + 2 \times 1\tfrac{1}{4}\right) \times 37\tfrac{1}{2} = 1400 \text{ sq. ins.}$$

Corresponding minimum density and specific magnetizing force:

$$\mathfrak{B}''_{c.i._2} = \frac{45,760,000}{1400} = 32,700 \text{ lines; } m''_{c.i._2} = 57.6.$$

Average specific magnetizing force:

$$m''_{c.i.} = \frac{155 + 57.6}{2} = 106.3 \text{ ampere-turns per inch.}$$

Corresponding average density:

$$\mathfrak{B}''_{c.i.} = 43,500 \text{ lines per square inch.}$$

Magnetizing force required:

$at_{c.i.} = 106.3 \times 37 =$ **3930** ampere-turns.

5. *Armature Reaction.*—

By (250), p. 352, and Table XCI.:

$$at_r = 1.73 \times \frac{84 \times 600}{2} \times \frac{23\frac{1}{2}^\circ}{180} = \mathbf{5700} \text{ ampere-turns.}$$

6. *Total Magnetizing Force Required.*—

By (227), p. 339:

$$AT = 25{,}800 + 340 + 6900 + 3930 + 5700$$
$$= \mathbf{41{,}670} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

Shunt winding to be figured for a temperature increase of 15° C. Regulating resistance to be adjustable for a maximum voltage of 540, and a minimum voltage of 450.

1. *Percentage of Regulating Resistance at Normal Load.*—The maximum output of 540 volts requires a total E. M. F. of

$$512.5 + 40 = 552.5 \text{ volts,}$$

which is 7.8 per cent. in excess of the total E. M. F. generated at normal output; for the maximum voltage, therefore, 1.078 times the normal flux must be produced. The magnetizing forces required for this increased flux are:

Air gaps:

$at'_g = .3133 \times (30{,}500 \times 1.078) \times 2.7 = 27{,}800$ ampere-turns.

Armature core:

$\mathfrak{B}'_a = 58{,}000 \times 1.078 = 62{,}500$ lines; $m'_a = 14.2$.
$at'_a = 14.2 \times 27.85 = 400$ ampere-turns.

Wrought iron:

$\mathfrak{B}'_{w.i.} = 90{,}000 \times 1.078 = 97{,}000$ lines; $m'_{w.i.} = 73.6$.
$at'_{w.i.} = 73.6 \times 136 = 10{,}000$ ampere-turns.

Cast iron:

$\mathfrak{B}'_{c.i.} = 43{,}500 \times 1.078 = 46{,}900$ lines; $m'_{c.i.} = 144$.
$at'_{c.i.} = 144 \times 37 = 5320$ ampere-turns.

Armature reaction:

$$at'_r = 1.77 \times \frac{84 \times 600}{2} \times \frac{23\frac{1}{2}^\circ}{180} = 5820 \text{ ampere-turns.}$$

The total magnetizing force needed for maximum output, consequently, is:

$$AT' = 27{,}820 + 400 + 10{,}000 + 5320 + 5820$$
$$= \mathbf{49{,}340} \text{ ampere-turns.}$$

This surpasses the normal excitation by

$$\frac{100 \times (49{,}340 - 41{,}670)}{41{,}670} = \mathbf{18} \text{ per cent.},$$

that is to say, the extra-resistance in circuit at normal output must be 18 per cent. of the magnet resistance, in order to produce the maximum voltage of 540 with the regulating resistance cut out.

2. *Magnet Winding* (*for* 500 *Volts*).—The mean length of one turn, by (292), p. 375, and Table XCIV., being

$$l_{T} = 3.43 \times 28 = 96 \text{ inches,}$$

the specific length of the required magnet wire is directly obtained by (319), p. 385:

$$\lambda_{sh} = \frac{41{,}670}{500} \times \frac{96}{12} \times 1.18 \times (1 + .004 \times 15) = \mathbf{832} \text{ ft. pr. ohm.}$$

The nearest gauge wires are No. 13 B. W. G. (.095″ + .010″) and No. 11 B. & S. (091″ + .010″), having 874 and 798 feet per ohm, respectively. The former being about 5 per cent. above, and the latter about 4 per cent. below, the required figure, the regulating resistance in circuit at full load, when No. 13 B. W. G. were used, would be about 23 per cent., and for No. 11 B. & S. would be about 14 per cent. of the magnet resistance. In order to obtain the exact amount of regulating resistance desired, the two sizes must be suitably combined. Taking equal weights of each, the resultant specific length is:

$$\lambda_{sh} = \frac{.0419 \times 874 + .0503 \times 798}{.0419 + .0503} = \mathbf{834} \text{ feet per ohm,}$$

where .0419 and .0503 are the resistances per pound of the two wires. This specific length being practically the same as found above, the winding calculated on its basis will, in fact, make $r_x = 18$, which therefore is to be used in the formulæ.

The height of the winding space derived from the above value of l_T is:

$$h_m = \frac{96}{\pi} - 28 = 2\tfrac{1}{2} \text{ inches,}$$

and this into formula (277), page 369, gives the radiating surface of the magnets:

$S_M = (28\tfrac{1}{2} + 2 \times 2\tfrac{1}{2}) \pi \times 2 (35 - 3) = \mathbf{6730}$ square inches, an allowance in length of 3 inches per core being made for flanges, spools, and insulation.

Hence by (312), p. 383:

$$P'_{sh} = \frac{15}{75} \times 6730 \times 1.18 = 1590 \text{ watts,}$$

by (314), p. 384:

$$N_{sh} = \frac{41,670 \times 500}{1590} = 13,100 \text{ shunt-turns.}$$

and by (315), p. 384:

$$L_{sh} = \frac{13,100 \times 96}{12} = 104,800 \text{ feet.}$$

consequently,

$$r_{sh} = \frac{104,800}{834} = \mathbf{125.5} \text{ ohms, resistance of winding, cold } (15.5^\circ \text{ C.});$$

and by (318), p. 385:

$$r'_{sh} = 125.5 \times (1 + .004 \times 15) = \mathbf{133} \text{ ohms, resistance of winding, warm } (30.5^\circ \text{ C}).$$

By (317), p. 384:

$$r''_{sh} = 133 \times 1.18 = \mathbf{157} \text{ ohms, resistance of entire shunt circuit, at normal load;}$$

therefore:

$$I_{sh} = \frac{500}{157} = \mathbf{3.18} \text{ amperes, shunt current, at normal load.}$$

Dividing the magnet resistance (cold) by the average resistance per pound of the two sizes used (equal weights being taken), we obtain the weight of the shunt winding:

$$wt_{sh} = \frac{125.5}{\frac{1}{2}(.0419 + .0503)} = \mathbf{2720} \text{ pounds, bare wire,}$$

or, see Table XXVI., p. 103:

$$wt'_{sh} = 1.03 \times 2720 = \mathbf{2800} \text{ pounds, covered wire.}$$

By (326), p. 389, we receive:

$$wt_{sh} = 31.3 \times 10^{-6} \times 13,100 \times \frac{96}{12} \times 834 = 2730 \text{ lbs.},$$

which checks the above figure.

Formula (257), p. 361, gives:

$$h'_m = \sqrt{\frac{12 \times 104,800 \times \left(\frac{.095 + .091}{2} + .010\right)^2}{65 \times \pi} + 14^2} - 14$$
$$= 16.2 - 14 = 2.2 \text{ inches.}$$

Allowing .3 inch for insulation between the layers, thickness and insulation of bobbins, and clearance, the total height of the magnet winding becomes $h_m = 2.2 + .3 = 2.5$ inches, which is the same as used in calculating the winding. There are, consequently, no errors to be corrected, and the final result of the winding calculation is:

1400 lbs. (covered) of No. **13** B.W.G. wire (.095″ + .010″)

and

1400 lbs. (covered) of No. **11** B. & S. wire (.091″ + .010″),

each wound in 4 spools of 350 pounds, two spools of each size to be placed on each magnet, see Fig. 348. Total weight of magnet wire, 2800 pounds.

3. *Shunt Field Regulator.*—The amount of regulating resistance in circuit at normal load required for the maximum voltage in the preceding was found to be 18 per cent. of the magnet resistance. In order to reduce the voltage from the normal amount to the minimum of 450, the total E. M. F. generated must be decreased from 512.5 to 512.5 − 50 = 462.5 volts, or by 9¾ per cent.; hence the minimum flux is .9025 of the normal flux, and the magnetizing forces for the minimum voltage are:

Air gaps:

$$at'_g = .3133 \times (30{,}500 \times .9025) \times 2.64^* = 22{,}800 \text{ ampere-turns.}$$

Armature core:

$$\mathcal{B}''_a = 58{,}000 \times .9025 = 52{,}350 \text{ lines; } m''_a = 10.3.$$
$$\therefore\ at'_a = 10.3 \times 27.85 = 270 \text{ ampere-turns.}$$

Wrought iron:

$$\mathcal{B}''_{w.i.} = 90{,}000 \times .9025 = 8120 \text{ lines; } m''_{w.i.} = 33.2.$$
$$\therefore\ at'_{w.i.} = 33.2 \times 136 = 4520 \text{ ampere-turns.}$$

Cast iron:

$$\mathcal{B}''_{c.i.} = 43{,}500 \times .9025 = 39{,}260 \text{ lines; } m''_{c.i.} = 86.8.$$
$$\therefore\ at'_{c.i.} = 86.8 \times 37 = 3210 \text{ ampere-turns.}$$

Armature reaction:

$$at'_r = 1.7 \times \frac{84 \times 600}{2} \times \frac{23\frac{1}{2}}{180} = 5600 \text{ ampere-turns.}$$

The total excitation required for minimum voltage is the sum of the above magnetizing forces:

$$AT' = 22{,}800 + 270 + 4520 + 3210 \times 5600$$
$$= \mathbf{36{,}400} \text{ ampere-turns.}$$

This minimum excitation being

$$\frac{100 \times (41{,}670 - 36{,}400)}{41{,}670} = \mathbf{12.7} \text{ per cent.}$$

smaller than the normal excitation, the normal resistance of the shunt circuit, in order to effect the corresponding increase in the exciting current, must be increased by 12.7 per cent., or the magnet resistance by $1.18 \times 12.7 = 15$ per cent.

The total resistance of the regulator, therefore, by formula (331), p. 393, is:

$$r_r = (.18 + .15) \times r'_{sh} = .33 \times 133 = \mathbf{44} \text{ ohms.}$$

By (332), p. 393:

$$(I_{sh})_{max} = \frac{540}{133} = 4.06 \text{ amperes.}$$

* For the minimum density the product $\mathcal{H}'' \times v_c$ being $1{,}500{,}000 \times .9025 = 1{,}353{,}750$, Table LXVI. gives a coefficient of field-deflection $k_{13} = 1.32$, which makes the length of the magnetic circuit in the gaps $l''_g = 1.32 \times (30 - 28) = 2.64$ inches.

By (333), p. 393:

$$(I_{sh})_{min} = \frac{450}{133 + 44} = 2.54 \text{ amperes.}$$

Supposing that the regulator is to have 60 contact-steps, so as to give an average regulation of $1\frac{1}{2}$ volt per step, the resistance of each coil of the rheostat will be

$$\frac{44}{60} = .733 \text{ ohm};$$

and if iron wire at 6500 circular mils per ampere is employed, the area of the wires for the various coils ranges between $4.06 \times 6500 = 26,390$ and $2.54 \times 6500 = 16,510$ circular mils. The data for the gauge numbers lying between these limits are:

Gauge Number.	Diameter (inch).	Sectional Area (Cir. Mils).	Carrying Capacity, Amps. (6500 Cir. Mils p. A.)
No. 6. B. & S.	.162	26,251	4.04
No. 9 B. W. G.	.148	21,904	3.38
No. 7 B. & S.	.144	20,817	3.21
No. 10 B. W. G.	.134	17,956	2.76
No. 8 B. & S.	.1285	16,510	2.54

Inserting the above values of the current capacities into formula (335), p. 394, we obtain:

$$n_{x_1} = \frac{4.06 - 4.04}{4.06 - 2.54} \times 60 = 1,$$

$$n_{x_2} = \frac{4.06 - 3.38}{4.06 - 2.54} \times 60 = 27,$$

$$n_{x_3} = \frac{4.06 - 3.21}{4.06 - 2.54} \times 60 = 33,$$

$$n_{x_4} = \frac{4.06 - 2.76}{4.06 - 2.54} \times 60 = 51,$$

and

$$n_{x_5} = \frac{4.06 - 2.54}{4.06 - 2.54} \times 60 = 60;$$

from which follows that coils 1 to 26 are to consist of No. 6 B. & S. wire, of which about 300 feet are needed for the required resistance of .733 ohm; that coils 27 to 32 are to be of No. 9 B. W. G., length per coil about 250 feet; coils 33 to 50 of No. 7 B. & S., length about 240 feet; coils 51 to 59 of No. 10

B. W. G., length about 205 feet; and coil 60 of No. 8 B. & S. wire, about 190 feet in length.

f. CALCULATION OF EFFICIENCIES.

1. *Electrical Efficiency.*—

By (352), p. 406:

$$\eta_e = \frac{500 \times 600}{500 \times 600 + 603.18^2 \times .0184 + 3.18^2 \times 157} = \frac{300,000}{308,280}$$
$$= .975, \text{ or } \mathbf{97.5}\ \%.$$

2. *Commercial Efficiency and Gross Efficiency.*—The energy lost by hysteresis and eddy currents was found $P_h + P_e = 1552$ watts; energy losses by commutation and friction estimated at 12,000 watts; hence the commercial efficiency, by (360), p. 407:

$$\eta_c = \frac{300,000}{308,280 + 1552 + 12,000} = \frac{300,000}{321,832} = .932, \text{ or } \mathbf{93.2}\ \%;$$

and the gross efficiency:

$$\eta_g = \frac{\eta_c}{\eta_e} = \frac{.932}{.975} = .957, \text{ or } \mathbf{95.7}\ \%.$$

3. *Weight Efficiency.*—The net weight of the machine is estimated as follows:

Armature:		
Core, 11.05 cu. ft. of wrought iron,	5,300 lbs.	
Winding, insulation, binding, etc., .	700 "	
Shaft, commutator, pulley, etc., .	3,000 "	
Armature, complete, ...		9,000 lbs.
Frame:		
Magnet cores, $28^2 \frac{\pi}{4} \times 70 =$ 43,100 cu. ins. of wrought iron,	12,075 lbs.	
Keeper, $28 \times 22 \times 72 = 44,350$ cu. ins. of wrought iron .	12,325 "	
Polepieces, about $(18 \times 30 \times 37\frac{1}{2}) \times 2 = 40,500$ cu. ins. of cast iron . .	10,600 "	
Field winding, core insulation, spools, flanges, etc., .	3,000 "	
Bedplate, bearings, zinc blocks, etc., ...	10,000 "	
Frame, complete, ...		48,000 lbs.

Fittings:

Brushes, holders, and brush rocker, ...	400 lbs.	
Switches, cables, etc., . . .	300 "	
Fittings, complete, ...		700 lbs.
Total net weight of dynamo, . .		**57,700** lbs.

Hence, the weight efficiency:

$$\frac{300,000}{57,700} = \mathbf{5.2} \text{ watts per lb.}$$

136. Calculation of a Bipolar, Single Magnetic Circuit, Smooth-Drum, High-Speed Compound Dynamo:

300 KW. Upright Horseshoe Type. Wrought-Iron Cores and Yoke. Cast-Iron Polepieces. 500 Volts. 600 Amps. 400 Revs. per Min.

The armature and field frame calculated in § 135 are given; the machine is to be overcompounded for a line loss of 5 per cent.; temperature of magnet winding to rise $22\frac{1}{2}°$ C.; extra-resistance in shunt circuit to be not less than 18 per cent. at normal load.

a. CALCULATION OF MAGNETIZING FORCES.

1. *Determination of Number of Shunt Ampere-Turns.*—Useful flux required on open circuit:

$$\Phi_0 = \frac{6 \times 500 \times 10^9}{168 \times 400} = \mathbf{44,700,000} \text{ maxwells;}$$

hence by §§ 104 and 135:

$$at_{g_0} = .3133 \times \frac{44,700,000}{1500} \times 1.32\,(30 - 28)$$

$$= .3133 \times 29,800 \times 2.64 = 24,700 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{a_{1_0}} = \frac{44,700,000}{712} = 62,700; \quad \mathfrak{B}''_{a_{2_0}} = \frac{44,700,000}{956} = 46,700.$$

$$\therefore at_{a_0} = \frac{14.3 + 8.8}{2} \times 27.85 = 11.55 \times 27.85 = 320 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{w.i._o} = \frac{1.215 \times 44,700,000}{615.75} = 88,200 \text{ lines per sq. inch;}$$

$$\therefore\ at_{w.i._o} = 46.4 \times 136 = 6300 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{c.i._{1_o}} = \frac{1.215 \times 44,700,000}{2 \times 562.5} = 48,250 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{c.i._{2_o}} = \frac{44,700,000}{1400} = 32,000 \text{ lines per square inch;}$$

$$\therefore\ at''_{c.i._o} = \frac{143 \times 56.2}{2} \times 37 = 99.6 \times 37 = 3680 \text{ amp.-turns.}$$

$$AT_{sh} = AT_o = 24,700 + 320 + 6300 + 3680$$
$$= \mathbf{35,000} \text{ ampere-turns.}$$

2. *Determination of Number of Series Ampere-Turns.*—Total E. M. F. at normal output, by (333), p. 393:

$$E' = 1.05 \times 500 + 1.25 \times 603 \times .0184 = \mathbf{539} \text{ volts;}$$

and therefore:

$$\Phi = \frac{6 \times 539 \times 10^9}{168 \times 400} = \mathbf{48,200,000} \text{ maxwells.}$$

$$at_g = .3133 \times \frac{48,200,000}{1500} \times 1.35\,(30 - 28)$$
$$= .3133 \times 32,100 \times 2.7 = 27,100 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{a_1} = \frac{48,200,000}{712} = 67,700 \text{ lines,}$$

$$\mathfrak{B}''_{a_2} = \frac{48,200,000}{956} = 50,500 \text{ lines;}$$

$$\therefore\ at_a = \frac{17.7 + 9.7}{2} \times 27.85 = 13.7 \times 27.85 = 380 \text{ amp.-turns.}$$

$$\mathfrak{B}''_{w.i.} = \frac{1.215 \times 48,200,000}{615.75} = 95,200 \text{ lines per square inch.}$$

$$at_{w.i.} = 67.8 \times 136 = 9220 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{c.i._1} = \frac{1.215 \times 48,200,000}{2 \times 562.5} = 52,200 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{c.i._2} = \frac{48,200,000}{1400} = 34,400 \text{ lines per square inch;}$$

$$at_{c.i.} = \frac{184.7 + 63.1}{2} \times 37 = 123.9 \times 37 = 4580 \text{ amp.-turns.}$$

$$at_r = 1.76 \times \frac{84 \times 603}{2} \times \frac{23\frac{1}{2}^\circ}{180} = 5820 \text{ ampere-turns.}$$

$$AT = 27,100 + 380 + 9220 + 4580 + 5820$$
$$= \mathbf{47,100} \text{ ampere-turns.}$$

Consequently by (339), p. 397:

$$AT_{se} = 47,100 - 35,000 = \mathbf{12,100} \text{ ampere-turns.}$$

b. CALCULATION OF MAGNET WINDING.

1. *Series Winding.*—

By (343), p. 400:

$$\delta_{se}^2 = 65 \times \frac{47,100 \times 600 \times 96}{6730 \times 22\frac{1}{2}} \times (1 + .004 \times 22\tfrac{1}{2})$$

$$= \mathbf{1,267,000} \text{ circular mils.}$$

Taking 5 cables of 19 No. 9 B. & S. wires each, the actual area is:

$$5 \times 19 \times 13,094 = \mathbf{1,243,930} \text{ circular mils.}$$

The number of turns required is:

$$N_{se} = \frac{12,120}{600} = \mathbf{20}, \text{ or } \mathbf{10} \text{ turns per core;}$$

hence the series field resistance, at 15.5° C., by (344), p. 400:

$$r_{se} = .875 \times \frac{20 \times 96}{5 \times 19 \times 13,094} = \mathbf{.00135} \text{ ohm,}$$

and the weight:

$$wt_{se} = 20 \times \frac{96}{12} \times (5 \times 19 \times 13,094) = \mathbf{603} \text{ lbs., bare wire;}$$

or,

$$wt'_{se} = 1.028 \times 603 = \mathbf{620} \text{ lbs., covered wire.}$$

2. *Shunt Winding.*—The potential across the shunt field being 1.05 × 500 = 525 volts, the specific length of the shunt wire, for 18 per cent. extra-resistance, and 22½° C. rise in temperature, is, by (319), p. 385:

$$\lambda_{sh} = \frac{35,000}{525} \times \frac{96}{12} \times 1.18 \times (1 + .004 \times 22\tfrac{1}{2})$$

$$= \mathbf{687} \text{ feet per ohm.}$$

The two nearest gauge numbers are No. 11 B. & S. (798 feet per ohm) and No. 14 B. W. G. (667 feet per ohm); taking two parts, by weight, of No. 14 B. W. G. to one part of No. 11 B. & S., we obtain:

$$\lambda_{sh} = \frac{.0503 \times 798 + 2 \times .0718 \times 667}{.0503 + 2 \times .0718} = \mathbf{702} \text{ feet per ohm,}$$

which is a trifle more than 2 per cent. in excess of the required specific length. By increasing the percentage of extra

resistance in the same ratio, that is, by making $r_x = 20$ per cent., formula (319) will give the specific length actually possessed by the combination of shunt wires selected. Hence: by (346), p. 400:

$$P_{sh} = \frac{22\frac{1}{2}^\circ}{75} \times 6730 - 600^2 \times .00135 \times (1 + .004 \times 22\frac{1}{2}^\circ)$$
$$= 2020 - 530 = 1490 \text{ watts;}$$

by (312), p. 383:

$$P'_{sh} = 1490 \times 1.20 = 1788 \text{ watts;}$$

by (314), p. 384:

$$N_{sh} = \frac{34,980 \times 525}{1788} = \mathbf{10,270} \text{ turns;}$$

by (315), p. 384:

$$L_{sh} = 10,270 \times \frac{96}{12} = 82,160 \text{ feet;}$$

Weight:

$$wt_{sh} = 82,160 \times \frac{2 \times .02085 + .02493}{3} = \mathbf{1825} \text{ lbs., bare wire,}$$

$$wt'_{sh} = 1.035 \times 1825 = \mathbf{1890} \text{ lbs., covered wire;}$$

Resistance:

$$r_{sh} = \frac{82,160}{702} = \mathbf{117} \text{ ohms, resistance of shunt winding, } 15.5^\circ \text{ C.;}$$

by (318), p. 385:

$r'_{sh} = 117 \times (1 + .004 \times 22\frac{1}{2}) = \mathbf{127.5}$ ohms, resistance of shunt winding, 38° C.;

by (317), p. 384:

$r''_{sh} = 127.5 \times 1.20 = \mathbf{153}$ ohms, resistance of entire shunt circuit, normal load.

$$\therefore \quad I_{sh} = \frac{525}{153} = \mathbf{3.43} \text{ amperes, shunt current, normal load.}$$

3. *Arrangement of Winding on Cores.*—

Total weight of series winding: .	$wt'_{se} =$ 620 lbs.
Total weight of shunt winding: .	$wt'_{sh} =$ 1890 lbs.
Total weight of magnet winding: . .	**2510** lbs.

The weight of the series wire being just about one-quarter of the total weight, the winding is with advantage placed upon 8 spools, 4 per core, the lower one of each being used for the series wire, one of the upper three being wound with No. 11 B. & S., and the remaining two with No. 14 B. W. G. wire; weight of wire per series spool, 310 pounds, per shunt spool, 315 pounds.

Each series spool has $5 \times 10 = 50$ cables which are arranged in 4 layers, two of which contain 12, and two 13 cables. The diameter of each series cable, consisting of 19 No. 9 B. & S. wires, is $5 \times (.1144'' + .010'') = .622$ inch, hence the winding depth in the series spools, $4 \times .622'' = 2.488$ inches, and the length of one layer (13 cables) $= 13 \times .622'' = 8.086$ inches. Since the available height of each spool is

$$\frac{35 - 2\frac{1}{2}}{4} = 8\tfrac{1}{8} \text{ inches,}$$

by this arrangement the spool will be just filled.

In the shunt bobbins the total 10,270 turns are divided in the ratio of the quantities used and of the specific lengths (feet per pound) of the two sizes of wire, *i. e.*, in the ratio of $2 \times 48 : 40.1$; hence there are

$$10{,}270 \times \frac{2 \times 48}{2 \times 48 + 40.1} = \mathbf{7240} \text{ turns of No. } \mathbf{14} \text{ B. W. G.}$$

and

$$10{,}270 \times \frac{40.1}{2 \times 48 + 40.1} = \mathbf{3030} \text{ turns of No. } \mathbf{11} \text{ B. \& S.}$$

Each No. 14 B. W. G. spool, therefore, contains

$$\frac{7240}{4} = \mathbf{1810} \text{ turns,}$$

and, the number of turns per layer being

$$\frac{8.125}{.083 + .010} = \mathbf{87},$$

has a net winding depth of

$$\frac{1810}{87} \times .093'' = \mathbf{1.95} \text{ inch.}$$

Each of the No. 11 B. & S. spools has

$$\frac{3030}{2} = \mathbf{1515} \text{ turns;}$$

the number of turns per layer is:

$$\frac{8.125}{.091 + .010} = \mathbf{80},$$

and consequently, the net winding depth:

$$\frac{1515}{80} \times .101'' = \mathbf{1.92} \text{ inch.}$$

Actual magnetizing force at full load:

		AMPERE-TURNS.
Series magnetizing force, AT_{se} =	20×600 =	12,000
Shunt magnetizing force, AT_{sh} =	$10,270 \times 3.43$ =	35,226
Total magnetizing force, ...		**47,226**

137. Calculation of a Bipolar, Double Magnetic Circuit, Toothed-Ring, Low-Speed Compound Dynamo:

50 KW. Double Magnet Type. Wrought-Iron Cores. Cast-Iron Yokes and Polepieces. 125 Volts. 400 Amps. 200 Revs. per Min.

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—For $\beta_1 = .70$ ($\alpha = 27°$), Table IV., p. 50, gives $e = 60 \times 10^{-8}$ volt per foot; from Table V., p. 52*a*, $v_c = 32$ feet per second; from Table VI., p. 54, $\mathcal{H}'' = 20,000$ lines per square inch; and from Table VIII., p. 56, $E' = 1.064 \times 125 = 133$ volts; hence by (26), p. 55:

$$L_a = \frac{133 \times 10^8}{60 \times 32 \times 20,000} = \mathbf{346} \text{ feet.}$$

2. *Sectional Area of Armature Conductor, and Selection of Wire.*—

By (27), p. 57:

$$\delta_a^2 = 300 \times 400 = 120,000 \text{ circular mils.}$$

For 20 No. 14 B. W. G. wires ($.083'' + .016''$), the actual area is:

$$20 \times 6889 = \mathbf{137,980} \text{ circular mils.}$$

The subdivision of the armature conductor into a large number of wires has the particular advantage in toothed armatures, that by a simple regrouping of the wires, the same slot will answer for a number of different voltages. Thus, in the present case, for instance, the same number of wires arranged in groups of 10 will give 250 volts at 200 amperes, and arranged 5 in parallel will furnish 500 volts at 100 amperes.

3. *Diameter of Armature Core and Dimensions of Slots.*—
By (30), p. 58:

$$d'_a = 230 \times \frac{32}{200} = 36.8 \text{ inches.}$$

From Table XV., p. 70, the approximate size of the slot is $1\frac{5}{8}'' \times \frac{7}{16}''$. The width of this slot will accommodate 4 No. 14 B. W. G. wires, thus:

$b_s = (.083 + .016'') \times 4 + 2 \times .020'' = .436$, or $\frac{7}{16}$ inch,
the slot insulation, $e = .020''$, being taken from Table XIX., p. 82.

Each conductor being made up of 20 wires, the number of layers in each slot must, therefore, be a multiple of 5. The nearest number of layers thus qualified is 15, hence the actual depth of the slots, if .010″ is allowed for separating the conductors, and .035″ for binding:

$$\begin{aligned} h_a &= (.083'' + .016'') \times 15 + .020'' + 2 \times .010'' + .035'' \\ &= 1.6'', \text{ or } \mathbf{1\tfrac{9}{16}} \text{ inch.} \end{aligned}$$

External diameter of armature:

$$d''_a = 36.8 + 1\tfrac{9}{16} = \mathbf{38\tfrac{1}{2}} \text{ inches.}$$

Diameter at bottom of slots:

$$d_a = 38\tfrac{1}{2} - 2 \times 1\tfrac{9}{16} = \mathbf{35\tfrac{3}{8}} \text{ inches.}$$

Number of slots, by (34), p. 70:

$$n'_c = \frac{38\frac{1}{2} \times \pi}{2 \times \frac{7}{16}} = \mathbf{138.}$$

4. *Length of Armature Core.*—
By (40), p. 76:

$$l_a = \frac{12 \times 20 \times 346}{138 \times 4 \times 15} = \mathbf{10} \text{ inches.}$$

5. *Arrangement of Armature Winding.*—The number of commutator divisions must be between 40 and 60, and must be a divisor of the number of slots, 138, taking 3 slots per commutator section, we have

$$n_c = \frac{138}{3} = \mathbf{46};$$

therefore, by (46), p. 89:

$$n_a = \frac{138 \times 4 \times 15}{46 \times 20} = \mathbf{9}.$$

The armature winding, consequently, consists in **46** coils of **9** turns of **20** No. **14** B. W. G. wires, each coil occupying 3

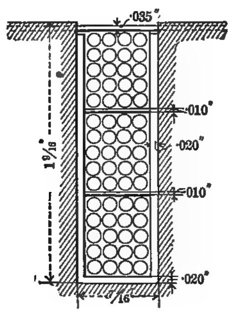


Fig. 351.—Arrangement of Armature Winding, 50-KW Double-Magnet Type, Low-Speed Generator.

slots. One slot, containing 3 turns, or one-third of an armature coil, is shown in Fig. 351.

6. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—

By (138), p. 202:

$$\Phi = \frac{6 \times 133 \times 10^9}{414 \times 200} = \mathbf{9{,}630{,}000} \text{ maxwells.}$$

By (48), p. 92, and Table XXII.:

$$b_a = \frac{9{,}630{,}000}{2 \times 80{,}000 \times 10 \times .9} = \mathbf{6\tfrac{3}{4}} \text{ inches.}$$

Internal diameter of armature core, Fig. 352:

$$35\tfrac{3}{8} - 2 \times 6\tfrac{3}{4} = \mathbf{21\tfrac{7}{8}} \text{ inches.}$$

Mean diameter of core:

$$d'''_a = 21\tfrac{7}{8} + 6\tfrac{3}{4} + 1\tfrac{9}{16} = \mathbf{30\tfrac{3}{16}} \text{ inches.}$$

Maximum depth of core, from (234), p. 342:

$$b'_a = 6\tfrac{3}{4} \times \sqrt{\frac{38\tfrac{1}{2}}{6\tfrac{3}{4}} - 1} = 14.8 \text{ inches.}$$

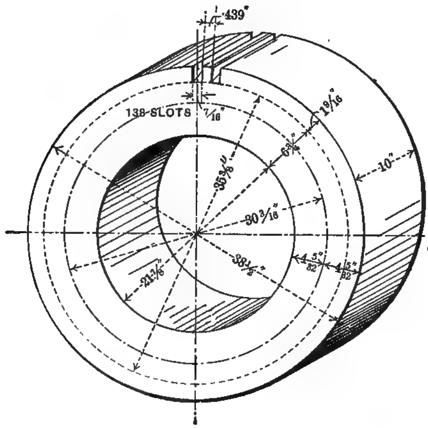


Fig. 352.—Dimensions of Armature Core, 50-KW Double-Magnet Type, Low-Speed Generator.

By (232), p. 341:

$$S_{a_1} = 2 \times 10 \times 6\tfrac{3}{4} \times .90 = \mathbf{121} \text{ square inches.}$$

By (233), p. 341:

$$S_{a_2} = 2 \times 10 \times 14.8 \times .90 = \mathbf{266} \text{ square inches.}$$

Therefore:

$$\mathfrak{B}''_{a_1} = \frac{9,630,000}{121} = 79,600 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{a_2} = \frac{9,630,000}{266} = 36,200 \text{ lines per square inch.}$$

$m''_{a_1} = 30.7$ ampere-turns; $m''_{a_2} = 6.7$ ampere-turns p. inch.

By (231), p. 341:

$$m''_a = \frac{30.7 + 6.7}{2} = \mathbf{18.7} \text{ ampere-turns per inch.}$$

Corresponding average density:

$$\mathfrak{B}''_a = \mathbf{69,000} \text{ lines per square inch.}$$

7. *Weight and Resistance of Armature Winding.*—

By (53) p. 99:

$$L_t = \frac{2 \times (10 + 6\tfrac{3}{4}) + 1\tfrac{9}{16} \times \pi}{10} \times 346 = 1360 \text{ feet.}$$

By (58), p. 101:

$$wt_a = .00000303 \times 137{,}980 \times 1360 = \mathbf{568} \text{ lbs., bare wire.}$$

By (59), p. 102:

$$wt'_a = 1.066 \times 568 = \mathbf{605} \text{ lbs., covered wire.}$$

By (61), p. 105:

$$r_a = \frac{1}{4 \times 20} \times 1360 \times .0015 = \mathbf{.0256} \text{ ohm, at } 15.5^\circ \text{ C.}$$

8. *Energy Losses in Armature, and Temperature Increase.*—

By (68), p. 109:

$$P_a = 1.2 \times 400^2 \times .0256 = \mathbf{4950} \text{ watts.}$$

From Fig. 352:

$$M = \frac{\left(30\tfrac{3}{16} \times \pi \times 8\tfrac{5}{16} - 138 \times 1\tfrac{9}{16} \times \frac{7}{16}\right) \times 10 \times .90}{1728}$$

$$= 3.61 \text{ cubic feet;}$$

$$N_1 = \frac{200}{60} = 3.33 \text{ cycles per second;}$$

from Table XXIX. ($\mathfrak{B}''_a = 69{,}000$):

$$\eta = 27.61 \text{ watts per cubic foot;}$$

from Table XXXI. ($\delta_i = .020''$):

$$\varepsilon = .138 \text{ watts per cubic foot.}$$

By (73), p. 112:

$$P_h = 27.61 \times 3.33 \times 3.61 = \mathbf{320} \text{ watts;}$$

By (76), p. 120:

$$P_e = .138 \times 3.33^2 \times 3.61 = \mathbf{6} \text{ watts.}$$

By (65), p. 107:

$$P_A = 4950 + 320 + 6 = \mathbf{5276} \text{ watts.}$$

By (79), p. 125:

$$S_A = 2 \times 30\tfrac{3}{16} \times \pi \times (10 + 6\tfrac{3}{4} + 4 \times 1\tfrac{9}{16})$$
$$= \mathbf{4360} \text{ square inches.}$$

Ratio of pole-area to radiating surface:

$$\frac{38\frac{3}{4} \times \pi \times 10 \times .70}{4360} = .195.$$

From Table XXXVI., p. 127, by interpolation:

$$\theta'_a = 44^\circ \text{ C.}$$

By (81), p. 127:

$$\theta_a = 44 \times \frac{5276}{4360} = \mathbf{53\tfrac{1}{2}}^\circ \text{ C.}$$

Armature resistance, hot:

$$r'_a = .0256 \times (1 + .004 \times 53\tfrac{1}{2}^\circ) = \mathbf{.0314} \text{ ohm, at } 69^\circ \text{ C.}$$

9. *Circumferential Current Density, Safe Capacity and Running Value of Armature; Relative Efficiency of Magnetic Field.*—

By (84), p. 131:

$$i_c = \frac{414 \times 200}{38\frac{1}{2} \times \pi} = \mathbf{685} \text{ amperes per inch circumference.}$$

Table XXXVII., p. 132: $\theta_a = 40$ to 60° C.

By (88), p. 134:

$$P' = 1.33 \times 38\tfrac{1}{2}^2 \times 10 \times .85 \times 200 \times 20{,}000 \times 10^{-6}$$
$$= \mathbf{67{,}000} \text{ watts.}$$

By (90), p. 135:

$$P'_a = \frac{133 \times 400}{568 \times 20{,}000} = \mathbf{.0047} \text{ watt per pound of copper, at unit field density.}$$

By (155), p. 211:

$$\Phi'_p = \frac{9{,}630{,}000}{133 \times 400} \times 32 = \mathbf{5800} \text{ maxwells per watt, at unit velocity.}$$

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Magnetic Flux, and Sectional Areas of Frame.*—

By (156), p. 214, and Table LXVIII.:

$$\Phi' = 1.25 \times 9{,}630{,}000 = \mathbf{12{,}000{,}000} \text{ maxwells.}$$

By (217), p. 314:

$$S''_{\text{w.i.}} = \frac{12,000,000}{90,000} = \mathbf{133.3} \text{ square inches.}$$

By (220), p. 314:

$$S''_{\text{c.i.}} = \frac{12,000,000}{45,000} = \mathbf{266.7} \text{ square inches.}$$

2. *Magnet Cores.*—The two cores being magnetically in parallel, each must have one-half the area $S''_{\text{w.i.}}$ found above for wrought iron, and making their breadth equal to that of the armature core, their thickness is found:

$$\frac{133.3}{2 \times 10} = 6.67, \text{ or say } \mathbf{6\tfrac{3}{4}} \text{ inches.}$$

3. *Polepieces.*—Thickness at ends joining cores:

$$2 \times 6\tfrac{3}{4} = \mathbf{13\tfrac{1}{2}} \text{ inches.}$$

Bore, by Table LXI., p. 209:

$$d_p = 38\tfrac{1}{2} + 2 \times \tfrac{1}{4} = \mathbf{39} \text{ inches.}$$

Length of centre portion (equal to diameter of armature core):

38½ inches.

Depth of magnet winding (Table LXXX., p. 317):

$$h_m = 2\tfrac{3}{4} \text{ inches.}$$

Allowing ½ inch clearance between the magnet winding and the pole-tips, the total length of the polepieces is:

$$38\tfrac{1}{2} + 2 \times (2\tfrac{3}{4} + \tfrac{1}{2}) = \mathbf{45} \text{ inches.}$$

Pole-distance:

$$l'_p = 39 \times \sin 27^\circ = \mathbf{15} \text{ inches,}$$

which is 4.45 times the total length of the gap space (compare with Table LX., p. 208).

Thickness in centre, required for mechanical strength only:

3 inches.

Thickness of pole-tips:

$$\frac{1}{2}\left(38\tfrac{1}{2} - \sqrt{39^2 - 15^2}\right) = \mathbf{1\tfrac{1}{4}} \text{ inch.}$$

All other dimensions of the frame can be directly derived from Fig. 353.

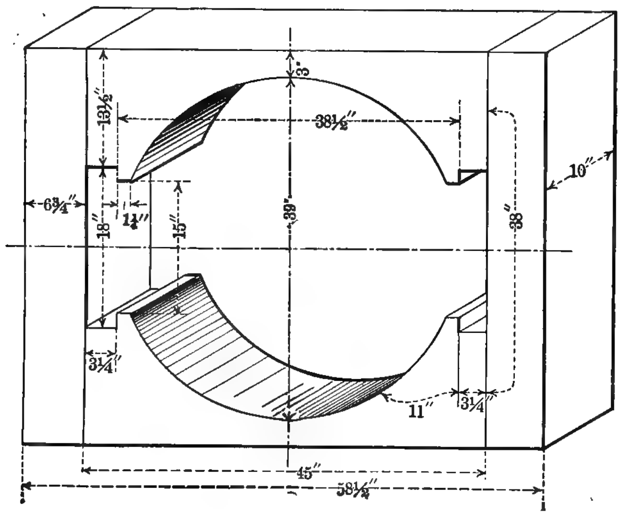


Fig. 353.—Dimensions of Field-Magnet Frame, 50 KW Double-Magnet Type, Low-Speed Generator.

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap Spaces.*—

$$b_t = \frac{38\frac{1}{2} \times \pi}{138} - \frac{7}{16} = .8765 - .4375 = .439 \text{ inch;}$$

Ratio of radial clearance to pitch:

$$\frac{.25}{.8765} = .286;$$

Product of field density and conductor velocity:

$$20,000 \times 32 = 640,000,$$

hence by Table LXVII., p. 230, the factor of field deflection,

$$k_{12} = 1.5;$$

and by (174), p. 230:

$$\mathcal{P}_1 = \frac{\frac{1}{4}\left[39 \times \pi \times .70 + (.439 + .219) \times 138 \times .85\right] \times 10}{1.5 \times (39 - 38\frac{1}{2})}$$

$$= \frac{408}{.75} = \mathbf{544}.$$

2. *Permeance of Stray Paths.*—

By (194), p. 242:

$$\mathfrak{P}_s = 2\left\{\frac{(58\frac{1}{2} + 10) \times 13\frac{1}{2} + 3\frac{1}{4} \times 10}{18} + \frac{10 \times 1\frac{1}{2}}{15 + 1\frac{1}{2}}\right\}$$

$$= 2\,(53.1 + .9) = \mathbf{108.}$$

3. *Leakage Factor.*—

By (157), p. 218:

$$\lambda = \frac{544 + 108}{544} = 1.20.$$

Ratio of width of slot to pitch:

$$\frac{.4375}{.8765} = .5,$$

therefore, by (158), p. 218, and Table LXV.:

$$\lambda' = 1.03 \times 1.20 = \mathbf{1.24.}$$

d. CALCULATION OF MAGNETIZING FORCES.

1. *Shunt Magnetizing Force.*

$$\Phi_0 = \frac{6 \times 125 \times 10^9}{414 \times 200} = \mathbf{9{,}060{,}000} \text{ maxwells.}$$

Air gaps:

$$at_{g_0} = .3133 \times \frac{9{,}060{,}000}{408} \times .75 = .3133 \times 22{,}200 \times .75$$

$$= 5216 \text{ ampere-turns.}$$

Armature core:

$$\mathfrak{B}''_{a_{1_0}} = \frac{9{,}060{,}000}{121} = 74{,}800 \text{ lines; } \mathfrak{B}''_{a_{2_0}} = \frac{9{,}060{,}000}{266} = 34{,}000 \text{ lines;}$$

$$l''_a = 28\tfrac{5}{8} \times \pi \times \frac{90^\circ + 27^\circ}{360} + 6\tfrac{3}{4} + 3\tfrac{1}{8} = 39 \text{ inches;}$$

$$\therefore at_{a_0} = \frac{24.7 + 6.3}{2} \times 39 = 15.5 \times 39 = 605 \text{ ampere-turns.}$$

Magnet cores (wrought iron):

$$\mathfrak{B}''_{w.i._0} = \frac{1.24 \times 9{,}060{,}000}{2 \times 6\frac{3}{4} \times 10} = 83{,}300 \text{ lines per square inch;}$$

$$\therefore at_{w.i._0} = 36.5 \times 38 = 1387 \text{ ampere-turns.}$$

Polepieces (cast iron with admixture of aluminum): The polepieces consist of two end portions of uniform cross-section and of a centre portion of varying cross-section. The combined length of the uniform portions is, from Fig. 353:

$$l''_1 = 2 \times 3\tfrac{1}{4} = 6\tfrac{1}{2} \text{ inches},$$

and the mean length of the varying cross-section, by (243), p. 348:

$$l''_2 = \frac{38\frac{1}{2}}{2} + 1\tfrac{1}{2} + 1\tfrac{1}{4} = 22 \text{ inches}.$$

The flux-densities and the corresponding magnetizing forces are:

$$\mathfrak{B}''_{c.i._{1_0}} = \frac{1.24 \times 9{,}060{,}000}{2 \times 13\frac{1}{2} \times 10} = \frac{11{,}225{,}000}{270} = 41{,}700 \text{ lines per sq. inch};$$

$$\mathfrak{B}''_{c.i._{2_0}} = \frac{9{,}060{,}000}{\frac{1}{2}(38\frac{3}{4}\pi \times .7) \times 10 + 2 \times 1\frac{3}{8}} = \frac{9{,}060{,}000}{428.8} = 21{,}100 \text{ lines p. sq. inch};$$

$$m''_{c.i._{1_0}} = 79 \text{ amp.-turns p. inch}; \quad m''_{c.i._{2_0}} = 25.4 \text{ amp.-turns. p. inch.}$$

Hence, according to formula (242), page 346:

$$at_{c.i._0} = 79 \times 6\tfrac{1}{2} + \frac{79 + 25.4}{2} \times 22 = 1662 \text{ ampere-turns.}$$

The shunt magnetizing force is, therefore:

$$AT_{sh} = 5216 + 605 + 1387 + 1662 = \mathbf{8870} \text{ ampere-turns.}$$

2. *Series Magnetizing Force.*—

$$E' = 125 + 1.25 \times 400 \times .0256 = 137 \text{ volts.}$$

$$\Phi = \frac{6 \times 137 \times 10^9}{414 \times 200} = \mathbf{9{,}930{,}000} \text{ maxwells.}$$

Air gaps:

$$at_g = .3133 \times \frac{9{,}930{,}000}{408} \times 75 = .3133 \times 24{,}300 \times .75$$
$$= 5710 \text{ ampere-turns.}$$

Armature core:

$$\mathfrak{B}''_{a_1} = \frac{9{,}930{,}000}{121} = 82{,}100 \text{ lines per square inch};$$

$$\mathfrak{B}''_{a_2} = \frac{9{,}930{,}000}{266} = 37{,}400 \text{ lines per square inch.}$$

$$\therefore \; at_a = \frac{35 + 7}{2} \times 39 = 820 \text{ ampere-turns.}$$

Magnet cores:

$$\mathfrak{B}''_{\text{w.i.}} = \frac{1.24 \times 9,930,000}{135} = 91,200 \text{ lines}; \; m''_{\text{w.i.}} = 54.2;$$

$$\therefore at_{\text{w.i.}} = 54.2 \times 38 = 2060 \text{ ampere-turns.}$$

Polepieces:

$$\mathfrak{B}''_{\text{c.i.}_1} = \frac{12,300,000}{270} = 45,600 \text{ lines}; \; m''_{\text{c.i.}_1} = 98.6;$$

$$\mathfrak{B}''_{\text{c.i.}_2} = \frac{9,930,000}{428.8} = 23,200 \text{ lines}; \; m''_{\text{c.i.}_2} = 28.2;$$

$$\therefore at_{\text{c.i.}} = 98.6 \times 6\tfrac{1}{2} + \frac{98.6 + 28.2}{2} \times 22 = 640 + 1400$$

$$= 2040 \text{ ampere-turns.}$$

The average specific magnetizing force of the variable section,

$$\frac{1}{2}(98.6 + 28.2) = 63.4,$$

corresponds to an average density of $\mathfrak{B}''_p = 41,000$ lines per square inch, from which Table XCI., p. 352, gives $k_{14} = 1.71$.

The maximum density in the armature teeth, at normal load, is:

$$\frac{9,930,000}{\frac{1}{2} \times .7 \times (35\tfrac{3}{8} \times \pi - 138 \times \tfrac{7}{16}) \times 10 \times .90}$$

$$= \frac{9,930,000}{160} = 62,000 \text{ lines per square inch,}$$

and for this, Table XC., p. 350, gives $k_{13} = .36$. Hence by (250), p. 352:

$$at_r = 1.71 \times \frac{414 \times 200}{2} \times \frac{.36 \times 27^\circ}{180} = 3830 \text{ amp.-turns.}$$

$$\therefore AT = 5710 + 820 + 2060 + 2040 + 3830$$

$$= \mathbf{14,460} \text{ ampere-turns.}$$

$$AT_{se} = 14,460 - 8870 = \mathbf{5590} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

Temperature-increase permitted, $\theta_m = 19^\circ$ C. Percentage of extra-resistance in circuit at normal load, $r_x = 35\%$.

1. *Series Winding.*—Apportioning one-third of the total winding depth, $h_m = 2\frac{3}{4}''$, to the series winding (AT_{se} being about one-third of AT), about 1 inch will be taken up by the latter, hence, if the series coil is wound next to the core, the mean length of a series turn:

$$l'_T = 2\,(10 + 6\tfrac{3}{4}) + 1 \times \pi = 36.64 \text{ inches,}$$

and the mean length of a shunt turn:

$$l''_T = 2\,(12 + 8\tfrac{3}{4}) + 1\tfrac{3}{4} \times \pi = 47 \text{ inches.}$$

The radiating surface of each magnet is:

$$S_M = 2\,(10 + 6\tfrac{3}{4} + 2\tfrac{3}{4}\,\pi) \times (18'' - 1'') = 860 \text{ square inches.}$$

By (343), p. 400, thus:

$$\delta_{se}^2 = 65 \times \frac{14{,}460 \times 400 \times 36.64}{860 \times 19} \times (1 + .004 \times 19^\circ)$$

$$= \mathbf{910{,}000} \text{ circular mils.}$$

For 22 No. 4 B. & S. wires (.204″ + .012″) the actual area is:

$$22 \times 41{,}743 = \mathbf{918{,}346} \text{ circular mils.}$$

Number of turns required per magnetic circuit, if both coils are in series:

$$N_{se} = \frac{5590}{400} = \mathbf{14.}$$

By (344), p. 400, for the two series coils:

$$r_{se} = .875 \times \frac{2 \times 14 \times 36.64}{918{,}346} = \mathbf{.00098} \text{ ohm, at } 15.5^\circ \text{ C.}$$

$$r'_{se} = 1.078 \times .00098 = \mathbf{.00106} \text{ ohm, at } 34.5^\circ \text{ C.}$$

and the total weight:

$$wt_{se} = 2 \times 14 \times \frac{36.64}{12} \times 22 \times .1264 = \mathbf{238} \text{ lbs., bare wire;}$$

$wt'_{se} = 1.029 \times 238 = \mathbf{245}$ lbs., covered, or $\mathbf{122\frac{1}{2}}$ lbs. per magnet.

2. *Shunt Winding.*—The two shunt coils to be connected in parallel.

By (318), p. 385:

$$\lambda_{sh} = \frac{8870}{125} \times \frac{47}{12} \times 1.35 \times (1 + .004 \times 19) = \mathbf{397} \text{ ft. per ohm.}$$

The nearest gauge wire is No. 14 B. and S. (.064" + .007") with a specific length of 398 feet per ohm.

By (346), p. 400:

$$P_{sh} = \frac{19}{75} \times 860 - 400^2 \times \frac{.00106}{2} = 218 - 85 = 133 \text{ watts.}$$

By (312), p. 383:

$$P'_{sh} = 133 \times 1.35 = 180 \text{ watts.}$$

By (314), p. 384:

$$N_{sh} = \frac{8870 \times 125}{180} = \mathbf{6170} \text{ turns per magnet.}$$

By (315), p. 384:

$$L_{sh} = 6170 \times \frac{47}{12} = 24,200 \text{ feet per core.}$$

Total weight:

$wt_{sh} = 2 \times 24,200 \times .01243 =$ **604** lbs., bare wire.

$wt'_{sh} = 1.0325 \times 604 =$ **624** lbs., covered, or **312** lbs. per magnet.

Shunt resistance per core:

$$r_{sh} = \frac{24,200}{398} = \mathbf{60.8} \text{ ohms, at } 15.5^\circ \text{ C.}$$

$r'_{sh} = 60.8 \times 1.076 =$ **65.5** ohms, at 34.5° C.

$r''_{sh} = 65.5 \times 1.35 =$ **88.4** ohms, each shunt circuit.

Exciting current:

$$I_{sh} = \frac{125}{88.4} = \mathbf{1.42} \text{ amperes, at normal load.}$$

3. *Arrangement of Magnet Winding on Cores.*—
Number of series wires per layer:

$$\frac{17}{.216} = \mathbf{78.}$$

Number of layers of series wire:

$$\frac{14 \times 22}{78} = \mathbf{4.}$$

Height of series winding:

$$4 \times .216 = \mathbf{.864} \text{ inch.}$$

Number of shunt wires per layer:

$$\frac{17}{.071} = \mathbf{240.}$$

Number of layers of shunt wire:

$$\frac{6170}{240} = \mathbf{26.}$$

Height of shunt winding:

$$26 \times .071 = \mathbf{1.846} \text{ inch.}$$

Allowing .1 inch for core covering and insulation between layers, the actual total depth of magnet winding is:

$$h_m = .864 + 1.846 + .1 = \mathbf{2\tfrac{13}{16}} \text{ inches.}$$

Actual magnetizing force at full load:

		AMPERE-TURNS.
Series magnetizing force, $AT_{se} = 14 \times 400$	=	5600
Shunt magnetizing force, $AT_{sh} = 26 \times 240 \times 1.42$	=	8850
Total magnetizing force, ... AT	=	**14,450**

f. CALCULATION OF EFFICIENCIES.

1. *Electrical Efficiency.*—By (353), p. 406:

$$\eta_e = \frac{125 \times 400}{125 \times 400 + (400 + 2 \times 1.42)^2 \times .0314 + 400^2 \times .00106 + (2 \times 1.42)^2 \times \frac{88.4}{2}}$$

$$= \frac{50,000}{55,630} = .898, \text{ or } \mathbf{89.8}\,\%.$$

2. *Commercial Efficiency.*—Allowing 2500 watts for commutator- and friction-losses, we have by (361), p. 408:

$$\eta_c = \frac{50,000}{55,630 + 332 + 2500} = \frac{50,000}{58,462} = .856, \text{ or } \mathbf{85.6}\,\%.$$

3. *Weight Efficiency.*—The estimated weights of the different parts of our dynamo are:

Armature:

Core, 3.56 cubic feet of wrought iron, .	1710 lbs.
Winding, insulation, binding, etc., . .	640 "
Shaft, commutator, spiders, etc., . .	500 "
Armature complete, . . .	2850 lbs.

Frame:	
Magnet cores, $2 \times 45 \times 10 \times 6\frac{3}{4} = 6075$ cubic inches of wrought iron, . .	1700 lbs.
Polepieces, $[45 \times 45 - (39^2 \times \frac{\pi}{4} + 2 \times 18 \times 3\frac{1}{4} + 2 \times 15 \times 1\frac{1}{4})] \times 10 = 6700$ cubic inches of cast iron, ...	1750 "
Field-winding and insulation, $250 + 650 =$	900 "
Dynamo portion of bed, bearings, etc., .	800 "
Frame, complete, ...	5150 lbs.
Fittings:	
Brushes, holders, and brush-rocker, . .	100 lbs.
Switches, series field regulator, cables, etc.,	100 "
Fittings, complete, ...	200 lbs.
Total net weight of dynamo, . . .	**8200** lbs.

The specific output, therefore, is:

$$\frac{50,000}{8200} = \mathbf{6.1} \text{ watts per pound.}$$

138. Calculation of a Multipolar, Multiple Magnet, Smooth Ring, High-Speed Shunt Dynamo:

1200 Kilowatts. Radial Innerpole Type. 10 Poles. Cast Steel Frame. 150 Volts. 8000 Amps. 232 Revs. per min.

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—Assuming

$$\beta_1 = .78 \qquad \left(\alpha = \frac{180 \times (1 - .78)}{10} = 4^\circ\right),$$

we find, from Table IV., p. 50:

$$e = 60 \times 10^{-8} \text{ volt per foot.}$$

Table V., p. 52*a*, gives an average conductor speed of 90 ft. p. sec. for a 1000-KW high-speed ring armature; we will take in the present case:

$$v_c = 96 \text{ feet per second;}$$

From Table VIII., p. 56, we obtain:

$$E' = 1.02 \times 150 = 153 \text{ volts.}$$

This machine being of comparatively low voltage and high current strength, the field-density obtained from Table VI. is reduced according to the rule given on page 54, thus:

$$\mathcal{H}'' = \tfrac{2}{3} \times 60,000 = 40,000 \text{ lines per square inch.}$$

Consequently, by (26), p. 55:

$$L_a = \frac{5 \times 153 \times 10^8}{60 \times 96 \times 40,000} = \mathbf{332} \text{ feet.}$$

2. *Area and Shape of Armature Conductor.*—By § 20:

$$\delta_a^2 = 600 \times \frac{8000}{5} = \mathbf{961,000} \text{ circular mils.}$$

In this case we will employ a wedge-shaped conductor, the external surface of the armature being used as a commutator. The height of the winding space, by Table XVIII., p. 75, is $h_a = .75$ inch, from which is to be deducted .100 inch for core insulation (column *a*, Table XIX., p. 82), and .025 inch for thickness of bar covering (half of the .050 inch insulation between two bars, column *e*, Table XIX.), leaving .625 inch for the height of the armature conductor, whose mean width on the internal periphery, therefore, is:

$$\frac{960,000 \times \frac{\pi}{4}}{.625 \times 10^6} = 1.2 \text{ inch.}$$

Since this would make too massive a single conductor, we divide it into 4 bars of .3 inch average width.

3. *Diameter of Armature Core, Number of Conductors.*—
By (30), p. 58:

$$d_a = 230 \times \frac{96}{232} = \mathbf{96} \text{ inches,}$$

being rounded off to the next *higher* even dimension, since in this case d_a is the *internal* diameter of the armature. The mean winding diameter, therefore:

$$d'_a = 96 - 2 \times .125 - .625 = 95\tfrac{1}{8} \text{ inches,}$$

and the number of armature conductors:

$$N_c = \frac{95\frac{1}{8} \times \pi}{4 \times (.3 + .05)} = \mathbf{200.}$$

4. *Length of Armature Core.*—
By (40), p. 76:

$$l_a = \frac{12 \times 332}{200} = \mathbf{20} \text{ inches.}$$

5. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—
By (137), p. 201:

$$\Phi = \frac{6 \times 5 \times 153 \times 10^9}{200 \times 232} = \mathbf{99{,}000{,}000} \text{ maxwells.}$$

By (48), p. 92, and Table XXII.:

$$b_a = \frac{99{,}000{,}000}{10 \times 70{,}000 \times 20 \times .90} = \mathbf{8} \text{ inches.}$$

External diameter of armature core, Fig. 354,

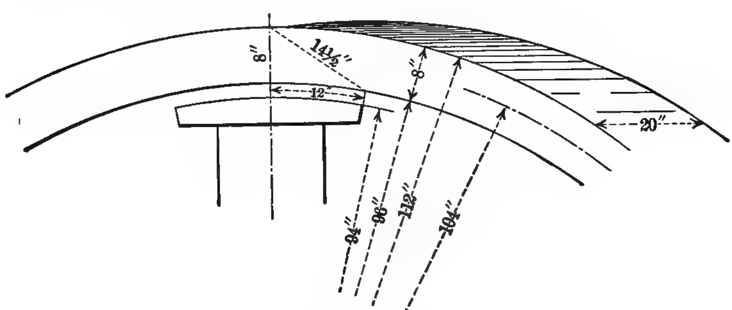


Fig. 354.—Dimensions of Armature Core, 1200-KW 10-Pole Radial Innerpole-Type Generator.

$$d''_a = 96 + 2 \times 8 = 112 \text{ inches.}$$

Mean diameter of armature core,

$$d'''_a = 96 + 8 = 104 \text{ inches.}$$

The width of one-half field space is

$$\frac{95 \times \pi}{2 \times 10} \times .78 = 12 \text{ inches,}$$

hence, by Fig. 354:

$$b'_a = \sqrt{12^2 + 8^2} = 14\tfrac{1}{2} \text{ inches.}$$

By (232), p. 341:

$$S_{a_1} = 10 \times 20 \times 8 \times .9 = \mathbf{14{,}400} \text{ square inches.}$$

By (233), p. 341:

$$S_{a_2} = 10 \times 20 \times 14\tfrac{1}{2} \times .9 = \mathbf{28{,}100} \text{ square inches.}$$

Hence: $\mathfrak{B}''_{a_1} = \dfrac{99{,}000{,}000}{14{,}400} = 68{,}800$ lines per square inch;

$$\mathfrak{B}''_{a_2} = \frac{99{,}000{,}000}{28{,}100} = 34{,}000 \text{ lines per square inch.}$$

By (231), p. 341:

$$m''_a = \frac{18.5 + 6.5}{2} = \mathbf{12.5} \text{ ampere-turns per inch.}$$

Corresponding average density:

$$\mathfrak{B}''_a = \mathbf{58{,}750} \text{ lines per square inch.}$$

7. *Size of Armature Conductor; Weight and Resistance of Armature Winding.*—

From Fig. 355 the exact size of the armature bars is obtained as follows:

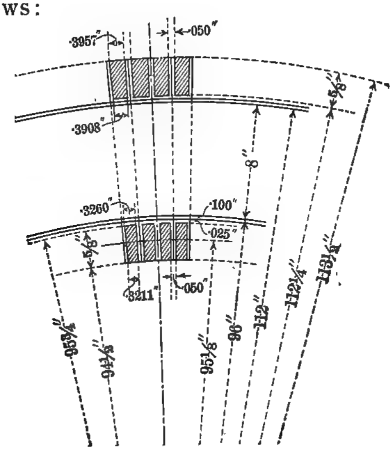


Fig. 355.—Dimensions of Armature Conductor, 1200-KW 10-Pole Radial Innerpole-Type Generator.

Minimum thickness of bar on inner circumference:

$$\frac{94\tfrac{1}{2} \times \pi}{4 \times 200} - .050'' = \mathbf{.3211} \text{ inch.}$$

Maximum thickness of bar on inner circumference:

$$\frac{95\frac{3}{4} \times \pi}{4 \times 200} - .050'' = \mathbf{.3260} \text{ inch.}$$

Minimum thickness of bar on outer circumference:

$$\frac{112\frac{1}{4} \times \pi}{4 \times 200} - .050'' = \mathbf{.3908} \text{ inch.}$$

Maximum thickness of bar on outer circumference:

$$\frac{113\frac{1}{2} \times \pi}{4 \times 200} - .050'' = \mathbf{.3957} \text{ inch.}$$

Area of conductor on inner circumference:

$$(\delta_a)^2 = 4 \times 625 \times \frac{321.1 + 326.0}{2} = \mathbf{808{,}875} \text{ square mils.}$$

Area of conductor on outer circumference:

$$(\delta'_a)^2 = 4 \times 625 \times \frac{390.8 + 395.7}{2} = \mathbf{983{,}125} \text{ square mils.}$$

Mean length of armature turn:

$$2 \times (20 + 8 + 2 \times \tfrac{3}{8}) = 59\tfrac{1}{2} \text{ inches.}$$

Total length of armature winding:

$$L_t = \frac{200 \times 59\frac{1}{2}}{12} = 992 \text{ feet.}$$

Weight:

$$wt_a = .00000303 \times \frac{(808{,}875 + 983{,}125)\frac{4}{\pi}}{2} \times 992 = \mathbf{3440} \text{ lbs.}$$

Armature resistance:

$$r_a = \frac{1}{4 \times 5^2} \times 992 \times \frac{10.5}{896{,}000 \times \frac{4}{\pi}} = \mathbf{.000091} \text{ ohm, at } 15.5^\circ \text{ C.}$$

8. *Energy Losses in Armature, and Temperature Increase.*—By (68), p. 109:

$$P_a = 1.2 \times 8000^2 \times .000091 = \mathbf{7000} \text{ watts.}$$

By (71), p. 112:

$$M = \frac{104 \times \pi \times 20 \times 8 \times .9}{1728} = 27.2 \text{ cubic feet.}$$

$$N_1 = \frac{232}{60} \times 5 = 19.33 \text{ cycles per second.}$$

From Table XXIX., p. 113, (for $\mathfrak{B}''_a = 58,750$):

$$\eta = 21.35.$$

From Table XXXI., p. 116, (for $\delta_i = .015''$):

$$\epsilon = .0258 \times 1.5^2 = .058.$$

By (73), p. 112:

$$P_h = 21.35 \times 19.33 \times 27.2 = \mathbf{11,220} \text{ watts.}$$

By (76), p. 120:

$$P_e = .058 \times 19.33^2 \times 27.2 = \mathbf{580} \text{ watts.}$$

By (65), p. 107:

$$P_A = 7000 + 11,220 + 580 = \mathbf{18,800} \text{ watts,}$$

By (79), p. 125:

$$S_A = 2 \times 104 \times \pi \times (20 + 8 + 4 \times \tfrac{3}{4}) = \mathbf{20,250} \text{ sq. ins.}$$

Ratio of pole area to radiating surface:

$$\frac{94 \times \pi \times 20 \times .78}{20,250} = .227.$$

From Table XXXVI., p. 127:

$$\theta'_a = 40\tfrac{1}{4}^\circ \text{ C.}$$

By (81), p. 127:

$$\theta_a = 40\tfrac{1}{4} \times \frac{18,800}{20,250} = \mathbf{37\tfrac{1}{2}}^\circ \text{ C.}$$

Armature resistance, warm:

$$r'_a = .000091 \times (1 + .004 \times 37\tfrac{1}{2}^\circ) = \mathbf{.000105} \text{ ohm, at } 53^\circ \text{ C.}$$

9. *Circumferential Current Density, Safe Capacity and Running Value of Armature; Relative Efficiency of Magnetic Field.*—By (84), p. 131:

$$i_c = \frac{200 \times \frac{8000}{10}}{104 \times \pi} = \mathbf{490} \text{ amperes per inch circumference.}$$

By (88), p. 134:

$$P' = 96^2 \times 20 \times .85 \times 232 \times 40{,}000 \times 10^{-6} = \mathbf{1{,}510{,}000} \text{ watts.}$$

By (90), p. 135:

$$P'_a = \frac{153 \times 8000}{3440 \times 40{,}000} = \mathbf{.0089} \text{ watt per lb. of copper, at unit density.}$$

By (155), p 211:

$$\Phi'_p = \frac{99{,}000{,}000}{153 \times 8000} \times 96 = \mathbf{7770} \text{ maxwells per watt at unit velocity.}$$

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Magnetic Flux, and Sectional Area of Frame.*—By (156), p. 214, and Table LXVIII.:

$$\Phi' = 1.12 \times 99{,}000{,}000 = \mathbf{111{,}000{,}000} \text{ maxwells.}$$

By (218), p. 314:

$$S''_{c.s.} = \frac{111{,}000{,}000}{85{,}000} = \mathbf{1310} \text{ square inches.}$$

2. *Magnet Cores.*—There being 10 magnetic circuits through the 10 cores, each circuit containing two of the magnets in series, the sectional area of one core must be one-fifth of the total frame area obtained; making the breadth of the cores **19½** inches, that is, ½ inch narrower than armature and polepieces, their thickness is found:

$$\frac{1310}{5 \times 19\frac{1}{2}} = \mathbf{13\tfrac{1}{2}} \text{ inches.}$$

The length of the cores is obtained from Table LXXXIII., p. 321, the nearest cross-section being 12 × 24 inches, for which

$$l_m = \mathbf{16} \text{ inches.}$$

3. *Polepieces.*—External diameter of field frame, by Table LXI., p. 209:

$$d_p = 96 - 2 \times (\tfrac{3}{4} + \tfrac{3}{8}) = \mathbf{93\tfrac{3}{4}} \text{ inches.}$$

Distance between pole-corners:

$$l'_p = 93\tfrac{3}{4} \times \sin 4^\circ = \mathbf{6\tfrac{1}{2}} \text{ inches.}$$

This is not as large as given by Table LX., p. 208, but is sufficient for the radial innerpole type. Taking $2\frac{7}{8}$ inches for the centre thickness of the polepieces, their dimensions are derived as follows:

Width of plane face:

$$2 \times \left(\frac{93\frac{3}{4}}{2} - 2\tfrac{7}{8}\right) \times \tan 14^\circ = \mathbf{22} \text{ inches.}$$

Width of curved face:

$$93\tfrac{3}{4} \times \sin 14^\circ = \mathbf{22\tfrac{3}{4}} \text{ inches.}$$

Thickness of pole-tips:

$$\frac{93\frac{3}{4}}{2} - \frac{\frac{93\frac{3}{4}}{2} - 2\frac{7}{8}}{\cos 14^\circ} = \mathbf{1\tfrac{1}{2}} \text{ inch.}$$

4. *Yoke.*—Making the width of the yoke

$$19\tfrac{1}{2} + 2 \times \tfrac{1}{2} = \mathbf{20\tfrac{1}{2}} \text{ inches,}$$

its radial thickness must be:

$$\frac{1310}{10 \times 20\frac{1}{2}} = \mathbf{6\tfrac{1}{2}} \text{ inches.}$$

From Fig. 356, the diameter across flats is:

$$93\tfrac{3}{4} - 2 \times (2\tfrac{7}{8} + 16) = \mathbf{56} \text{ inches.}$$

Diameter across corners:

$$\frac{56}{\cos 18^\circ} = \mathbf{59} \text{ inches.}$$

Length of side of decagon:

$$56 \times \sin 18^\circ = \mathbf{17\tfrac{1}{4}} \text{ inches.}$$

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap Spaces.*—

$$\mathcal{H}'' \times v_c = 40{,}000 \times 96 = 3{,}840{,}000;$$

by Table LXVI., p. 225, $k_{12} = 1.25$; hence, by (167), p. 226:

$$\mathfrak{P}_1 = \frac{\frac{1}{4}(93\frac{3}{4} + 96) \times \pi \times .85 \times 20}{1.25 \times (96 - 93\frac{3}{4})} = \frac{2540}{2.8} = \mathbf{907.}$$

2. *Permeance of Stray Paths.*—Distance apart of cores, at yoke-end:

$$c_1 = (17\tfrac{1}{4} - 13\tfrac{1}{2}) \times \cos 18^\circ = 3.6 \text{ inches.}$$

Distance apart of cores, at pole-end:

$$c_2 = \frac{3.6 \times \left(16 + \dfrac{1\frac{7}{8}}{\tan 18}\right)}{\dfrac{1\frac{7}{8}}{\tan 18^\circ}} = 13.6 \text{ inches.}$$

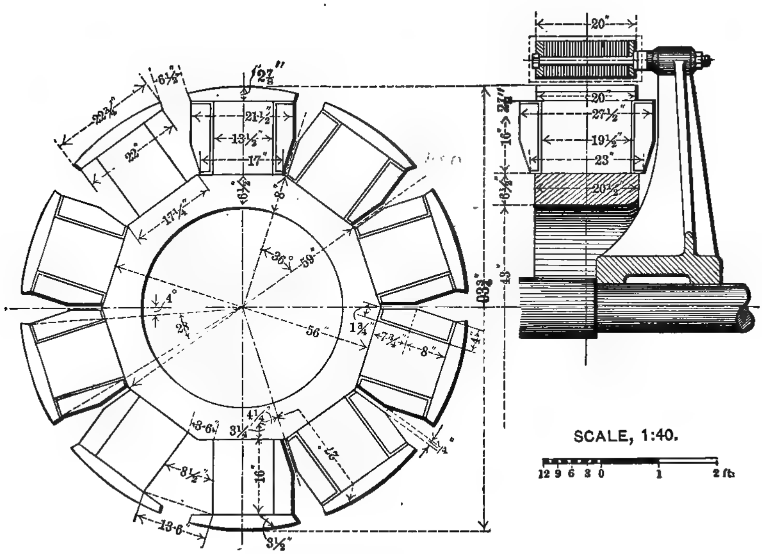


Fig. 356.—Dimensions of Field-Magnet Frame, 1200-KW 10-Pole Radial Innerpole-Type Generator.

Projecting area of polepiece:

$$S_1 = 22 \times 20 - 19\tfrac{1}{2} \times 13\tfrac{1}{2} = 177 \text{ square inches.}$$

Projecting area of yoke:

$$S_2 = 20\tfrac{1}{2} \times 17\tfrac{1}{4} - 19\tfrac{1}{2} \times 13\tfrac{1}{2} = 91 \text{ square inches.}$$

Total stray permeance, from Fig. 356:

$$\mathfrak{P}_2 = 10 \times \left[\frac{19\frac{1}{2} \times 16}{13.6 + 3.6} + \frac{20 \times 1\frac{1}{2}}{6\frac{1}{2}} + \frac{\frac{1}{2}(177 + 91)}{2 \times 16}\right]$$
$$= 10 \times (18.1 + 4.6 + 4.2) = \mathbf{269.}$$

3. *Leakage Factor, and Total Flux.*—
By (157), p. 218:

$$\lambda = \frac{907 + 269}{907} = \frac{1176}{907} = \mathbf{1.295.}$$

This is considerably higher than the value taken from Table LXVIII. and employed in the calculation of the frame area (see p. 572). The corrected total flux of

$$\Phi' = 1.295 \times 99,000,000 = \mathbf{128,000,000} \text{ maxwells}$$

brings the density in the frame up to

$$\mathfrak{B}''_{\text{c.s.}} = \frac{128,000,000}{5 \times 19\frac{1}{2} \times 13\frac{1}{2}} = \mathbf{97,500} \text{ lines per square inch,}$$

which, however, is within the practical limits of magnetization for cast steel (see Table LXXVI., p. 313), making a re-dimensioning of the frame unnecessary.

d. CALCULATION OF MAGNETIZING FORCES.

1. *Air Gaps.*—Actual density:

$$\mathfrak{H}'' = \frac{99,000,000}{2540} = \mathbf{39,000} \text{ lines per square inch.}$$

By (228), p. 339:

$$at_g = .3133 \times 39,000 \times 2.8 = \mathbf{34,200} \text{ ampere-turns.}$$

2. *Armature Core.*—
By (236), p. 343:

$$l''_a = 104 \times \pi \times \frac{\frac{90}{5} + 4}{360} + 8 = 28 \text{ inches;}$$

$$m''_a = 12.5 \text{ ampere-turns per inch (p. 569):}$$
$$\therefore at_a = 12.5 \times 38 = \mathbf{350} \text{ ampere-turns.}$$

3. *Magnet Frame.*—Length of path (see Fig. 356):

$$2 \times (3\tfrac{1}{2} + 16 + 3\tfrac{1}{4} + 4\tfrac{1}{4}) = 54 \text{ inches.}$$

The specific magnetizing force correspondiug to the above flux density $\mathfrak{B}''_{\text{c.s.}}$ of 97,500 lines, for cast steel, is:

$$m''_{\text{c.s.}} = 86 \text{ ampere-turns per inch.}$$
$$\therefore at_m = 86 \times 54 = \mathbf{4650} \text{ ampere-turns.}$$

4. *Armature Reaction.*—Mean density in polepieces:

$$\frac{128,000,000}{5 \times 22\frac{3}{8} \times 20} = 57,200 \text{ lines per square inch.}$$

hence by (250), p. 352, and Table XCI.:

$$at_r = 1.25 \times \frac{200 \times 8000}{10} \times \frac{4^\circ}{180} = \mathbf{4450} \text{ ampere-turns.}$$

5. *Total Magnetizing Force Required.*—
By (227), p. 339:

$$AT = 34,200 + 350 + 4650 + 4450 = \mathbf{43,650} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

In the present machine the winding space is limited by the shape of the frame, the height available at the pole end of the core being 4 inches, and at the yoke end only $1\frac{3}{4}$ inch, see Fig. 356. The larger depth can be employed until the distance between two adjoining coils becomes the same as that allowed at the yoke end; leaving $\frac{1}{4}$ inch for the bobbin flanges, and for insulation and clearance, it is thus found that $8\frac{3}{4}$ inches of the available length of each core can be wound 4 inches deep, and that for the remaining 7 inches the winding depth tapers from 4 inches to $1\frac{3}{4}$ inch. This gives a mean winding depth of

$$h_m = \frac{4 \times 8\frac{3}{4} + \frac{1}{2}(4 + 1\frac{3}{4}) \times 7}{15\frac{3}{4}} = \mathbf{3\frac{1}{2}} \text{ inches.}$$

Mean length of one turn:

$$l_T = 2(19\frac{1}{2} + 13\frac{1}{2}) + 3\frac{1}{2} \times \pi = \mathbf{77} \text{ inches.}$$

Radiating surface of each magnet:

$$S_M = 2\,(19\frac{1}{2} + 13\frac{1}{2} + 3\frac{1}{2} \times \pi) \times 15\frac{3}{4} = \mathbf{1585} \text{ square inches.}$$

By means of formula (328), p. 390, we can now determine the minimum temperature increase that can be obtained with the present design (by entirely filling the given winding space). The weight of bare copper wire filling one bobbin is, by (330), p. 390:

$$wt_m = 77 \times 15\frac{3}{4} \times 3\frac{1}{2} \times .21 = 890 \text{ pounds.}$$

hence by (329), p. 390:

$$\theta_m = \frac{\left[31.3 \times \left(\frac{43.65}{2} \times \frac{77}{12}\right)^2 \times \frac{75}{1385}\right]}{890 - .004 \times \left[31.3 \times 140^2 \times \frac{75}{1385}\right]} = \mathbf{44^\circ}\text{ C.}$$

Although this is rather high, especially for so large a machine, it is yet within practical limits, and we therefore base the winding calculation on the above dimensions of the winding space.

Connecting the 10 coils in 5 groups of 2 each, the terminal voltage of 150 volts will correspond to the total magnetizing force of one circuit, and formula (318), p. 385, gives the specific length of the wire required, for 20 per cent. extra-resistance:

$$\lambda_{sh} = \frac{43{,}650}{150} \times \frac{77}{12} \times 1.20 \times (1 + .004 \times 44^\circ) = \mathbf{2635}$$
feet per ohm.

No. **8** B. W. G. wire (.165″ + .010″) has a specific length of 2637 feet per ohm.

By 312, p. 383:

$$P'_{sh} = \frac{44}{75} \times 2 \times 1480 \times 1.20 = 2080 \text{ watts per magnetic circuit.}$$

By (314), p. 384:

$$N_{sh} = \frac{43{,}650 \times 150}{2080} = \mathbf{3150} \text{ turns per circuit.}$$

By (315), p. 384:

$$L_{sh} = \frac{3150 \times 2080}{12} = 20{,}200 \text{ feet, per pair of magnets.}$$

$$\therefore \quad r_{sh} = \frac{20{,}200}{2637} = \mathbf{7.67} \text{ ohms, 2 coils in series, at } 15.5^\circ \text{ C.}$$

By (318), p. 385:

$$r'_{sh} = 7.67 \times (1 + .004 \times 44) = \mathbf{9.0} \text{ ohms, one group, at } 59.5^\circ \text{ C.}$$

By (317) p. 384:

$r''_{sh} = 9.0 \times 1.20 = \mathbf{10.8}$ ohms, one shunt branch, at normal load.

$\therefore\ I_{sh} = \dfrac{150}{10.8} = \mathbf{13.9}$ amperes, current in each branch.

There being 5 magnetic circuits with their magnetizing coils in parallel, the total exciting current is:

$$13.9 \times 5 = \mathbf{69.5} \text{ amperes,}$$

while the joint shunt resistance of the 10 coils is:

$$\frac{9.0}{5} = \mathbf{1.8} \text{ ohm, at } 59.5^\circ \text{ C.}$$

Total weight:

$$wt_{sh} = \frac{5 \times 7.67}{.0046} = \mathbf{8330} \text{ pounds, bare wire.}$$

$$wt'_{sh} = 8330 \times 1.0221 = \mathbf{8530} \text{ pounds, covered wire,}$$

or **853** pounds of No. **8** B. W. G. wire per core.

Actual magnetizing force at full load:

$$AT' = 3150 \times 13.9 = \mathbf{43{,}800} \text{ ampere-turns.}$$

Since in this example the dimensioning of the winding space was the starting point of the winding calculation, no checking of the result with reference to the length of mean turn, radiating surfaces, etc., is necessary.

f. CALCULATION OF EFFICIENCIES.

1. *Electrical Efficiency.*—

By (352), p. 406:

$$\eta_e = \frac{150 \times 8000}{150 \times 8000 + 8069.5^2 \times .000105 + 5 \times 13.9^2 \times 10.8}$$

$$= \frac{1{,}200{,}000}{1{,}217{,}200} = .987, \text{ or } \mathbf{98.7}\ \%.$$

2. *Commercial Efficiency.*—Taking the commutation- and friction-losses at 40,000 watts, we obtain by (360), p. 407:

$$\eta_c = \frac{1{,}200{,}000}{1{,}217{,}200 + 11{,}800 + 40{,}000} = \frac{1{,}200{,}000}{1{,}269{,}000} = .947,$$

or **94.7** %.

3. *Weight-Efficiency.*—The weight of the machine is obtained as follows:

Armature:

Core, 27.2 cu. ft. of wrought iron,	13,000 lbs.	
Winding and insulation, etc., .	4,000 "	
Armature spider, shaft, etc., .	8,000 "	
Armature, complete, ...		25,000 lbs.

Frame:

Magnet cores, $10 \times 19\frac{1}{2} \times 13\frac{1}{2} \times 16 = 42,100$ cu. ins. of cast steel, ...	11,500 lbs.	
Polepieces, $10 \times 22\frac{3}{8} \times 20 \times 2\frac{1}{4} = 10,050$ cu. ins. of cast steel,	2,800 "	
Yoke, $\left(.735 \times 59^2 - 43^2 \frac{\pi}{4}\right) \times 20\frac{1}{2} = 20,500$ cu. ins. of cast steel,	5,700 "	
Field winding, spools, and insulation), ...	10,000 "	
Flange for fastening yoke to engine frame, outboard bearing, etc., ...	12,000 "	
Frame, complete, ...		42,000 lbs.

Fittings:

Brush shifting and raising devices, brushes, studs, etc., .	3,000 lbs.	
Switches, cables, etc., . .	1,000 "	
Fittings, complete, ...		4,000 lbs.
Total net weight of dynamo, . .		**71,000** lbs.

Weight efficiency:

$$\frac{1,200,000}{71,000} = \mathbf{16.9} \text{ watts per lb.}$$

139. Calculation of a Multipolar, Single Magnet, Smooth Ring, Moderate Speed Series Dynamo:

30 KW. Single Magnet Innerpole Type.
6 Poles. Wrought-Iron Core. Cast Steel Polepieces.
600 Volts. 50 Amps. 400 Revs. per Min.

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—

$$\beta_1 = .75; \ \alpha = \frac{180 \ (1 - .75)}{6} = 7\tfrac{1}{2}^\circ; e = 57.5 \times 10^{-8} \text{ v. p. ft.}$$

$v_c = 60$ feet per second; $\mathcal{H}'' = 15{,}000$ lines per square inch;

$$E' = 1.10 \times 600 = 660 \text{ volts.}$$

By (26), p. 55:

$$L_a = \frac{3 \times 660 \times 10^8}{57.5 \times 60 \times 15{,}000} = \mathbf{3870} \text{ feet.}$$

2. *Sectional Area of Armature Conductor.*—

By (27), p. 57:

$$\delta_a^2 = 300 \times \frac{50}{3} = \mathbf{5000} \text{ circular mils.}$$

No. 15 B. W. G. (.072″ + .016″) has a cross-section of 5184 circular mils.

3. *Diameter of Armature Core, and Number of Conductors.*—

By (30), p. 58:

$$d_a = 230 \times \frac{60}{400} = \mathbf{35} \text{ inches.}$$

The diameter over the winding on the internal circumference being about 34 inches, and 3 layers with its insulations making a well-proportioned winding space for the case in question, the total number of conductors on the armature is:

$$N_c = \frac{34 \times \pi \times 3}{.072 + .016} = \mathbf{3600.}$$

Actual depth of winding:

$$h_a = 3 \times (.072'' + .016'') + .060'' = \mathbf{.324} \text{ inch.}$$

4. *Length of Armature Core.*—

By (48), p. 92:

$$l_a = \frac{3870 \times 12}{3600} = \mathbf{13} \text{ inches.}$$

5. *Arrangement of Armature Winding.*—

By (45), p. 89:

$$(n_c)_{\min} = \frac{660 \times 3}{11.5} = 172.$$

Taking **180** commutator divisions, we have **30** coils of **20** convolutions per pole.

6. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—

By (138), p. 202:

$$\Phi = \frac{6 \times 3 \times 660 \times 10^8}{3600 \times 400} = \mathbf{8{,}250{,}000} \text{ maxwells.}$$

By (48), p. 92:

$$b_a = \frac{8{,}250{,}000}{6 \times 50{,}000 \times 13 \times .85} = \mathbf{2\tfrac{1}{2}} \text{ inches.}$$

External diameter of armature core:

$$35 + 2 \times 2\tfrac{1}{2} = 40 \text{ inches.}$$

Mean diameter of armature core:

$$d'''_a = 35 + 2\tfrac{1}{2} = 37\tfrac{1}{2} \text{ inches.}$$

Maximum depth:

$$b'_a = \sqrt{\left(\frac{35 \times \pi}{2 \times 6} \times .75\right)^2 + 2\tfrac{1}{2}^2}$$

$$= \sqrt{6.9^2 + 2\tfrac{1}{2}^2} = \mathbf{7\tfrac{3}{8}} \text{ inches.}$$

Deducting $\frac{3}{4}$ inch taken up by armature bolt and insulation, the minimum core depth is reduced to $2\frac{1}{2} - \frac{3}{4} = 1\frac{3}{4}$ inch; hence

$$S''_{a_1} = 6 \times 13 \times 1\tfrac{3}{4} \times .85 = \mathbf{116} \text{ square inches.}$$
$$S''_{a_2} = 6 \times 13 \times 7\tfrac{3}{8} \times .85 = \mathbf{514} \text{ square inches.}$$

Maximum and minimum densities:

$$\mathfrak{B}''_{a_1} = \frac{8,250,000}{116} = 71,000 \text{ lines}; \mathfrak{B}''_{a_2} = \frac{8,250,000}{514} = 16,000 \text{ lines}.$$

Mean specific magnetizing force and corresponding average flux density:

$$m''_a = \frac{20.5 + 2.9}{2} = \mathbf{11.7} \text{ ampere-turns per inch.}$$

$$\mathfrak{B}''_a = \mathbf{57,000} \text{ lines per square inch.}$$

7. *Weight and Resistance of Armature Winding.*—

By (53), p. 99:

$$L_t = \frac{2 \times (13 + 2\tfrac{1}{2}) + .324 \times \pi}{13} \times 3870 = 9500 \text{ feet.}$$

By (58), p. 101:

$$wt_a = .00000303 \times 5184 \times 9500 = \mathbf{149} \text{ lbs.}$$

By (59), p. 102:

$$wt'_a = 1.078 \times 149 = \mathbf{161} \text{ lbs.}$$

By (61), p. 105:

$$r_a = \frac{1}{4 \times 3^2} \times 9500 = .002 = \mathbf{.528} \text{ ohm, at } 15.5° \text{ C.}$$

8. *Energy Losses in Armature, and Temperature Increase.*—

$$M = \frac{37\tfrac{1}{2} \times \pi \times 13 \times 2\tfrac{1}{2} \times .85}{1728} = 1.89 \text{ cubic feet.}$$

$$N_1 = \frac{400}{60} \times 3 = 20 \text{ cycles per second.}$$

By (68), p. 109:

$$P_a = 1.2 \times 50^2 \times .528 = \mathbf{1585} \text{ watts.}$$

By (73), p. 112:

$$P_h = 20.35 \times 20 \times 1.89 = \mathbf{780} \text{ watts.}$$

By (76), p. 120:

$$P_e = .094 \times 20^2 \times 1.89 = \mathbf{70} \text{ watts.}$$

By (65), p. 107:

$$P_A = 1585 + 780 + 70 = \mathbf{2435} \text{ watts.}$$

By (79), p. 125:

$$S_A = 2 \times 37\tfrac{1}{2}\,\pi \times (13 + 2\tfrac{1}{2} + 4 \times \tfrac{3}{8}) = \mathbf{4000} \text{ sq. ins.}$$

Ratio of pole area to radiating surface:

$$\frac{34 \times \pi \times 13 \times .75}{4000} = .26.$$

By (81), p. 127:

$$\theta_a = 42 \times \frac{2435}{4000} = \mathbf{25\tfrac{1}{2}}^\circ \text{ C.}$$

$$r'_a = (1 + .004 \times 25\tfrac{1}{2}) \times .528 = \mathbf{.583} \text{ ohm, at } 41^\circ \text{ C.}$$

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Magnetic Flux, and Sectional Areas of Frame.*—By (156), p. 214:

$$\Phi' = 1.30 \times 8,250,000 = \mathbf{10,700,000} \text{ maxwells.}$$

By (217), p. 314:

$$S_{w.i.} = \frac{10,700,000}{90,000} = \mathbf{119} \text{ square inches.}$$

By (218), p. 314:

$$S_{c.s.} = \frac{10,700,000}{85,000} = \mathbf{126} \text{ square inches.}$$

2. *Magnet Core.*—The magnet being hollow, its internal diameter must be determined first.

Diameter of shaft, by (123), p. 185:

$$d_c = 1.3 \times \sqrt[4]{\frac{30,000}{400}} = 4 \text{ inches.}$$

Making the hole in the core $4\frac{1}{2}$ inches in diameter, the external core diameter becomes:

$$d_m = \sqrt{(119 + 15.9) \times \frac{4}{\pi}} = \mathbf{13} \text{ inches.}$$

3. *Polepieces.*—

$$d_p = 35 - 2 \times (.324 + \tfrac{7}{32}) = \mathbf{33\tfrac{29}{32}} \text{ inches.}$$

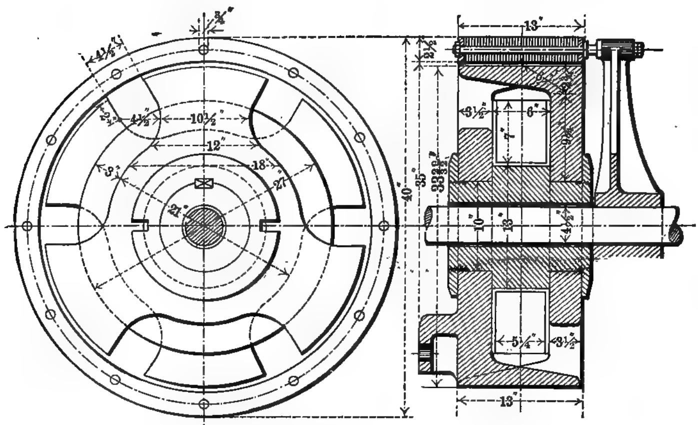
$$l'_p = 33\tfrac{29}{32} \times \sin 7\tfrac{1}{2}^\circ = \mathbf{4\tfrac{1}{2}} \text{ inches.}$$

Providing the same distance between all projecting portions of opposite polarity, the shape shown in Fig. 357 is obtained, having a mean width of about 12 inches per magnetic circuit. The axial thickness of the polepieces, therefore, must be:

$$\frac{126}{3 \times 12} = \mathbf{3\tfrac{1}{2}} \text{ inches,}$$

leaving the length of the magnet core:

$$l_m = 13 - 2 \times 3\tfrac{1}{2} = \mathbf{6} \text{ inches.}$$



SCALE, 1:20.

Fig. 357.—Dimensions of Armature Core and Field Magnet Frame, 30-KW 6-Pole, Single-Magnet Innerpole Type, Moderate-Speed Generator.

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap Spaces.*—

$$\mathcal{H}'' \times v_c = 15{,}000 \times 60 = 900{,}000.$$

By (167), p. 226:

$$\mathcal{P}_1 = \frac{\frac{1}{4}\left(33\frac{29}{32} + 35\right) \times \pi \times .835 \times 13}{1.15 \times \left(35 - 33\frac{29}{32}\right)} = \frac{590}{1.25} = \mathbf{520.}$$

2. *Permeance of Stray Paths.*—

From Fig. 357:

$$\mathcal{P}_3 = \frac{(24^2 - 13^2)\frac{\pi}{4}}{6} + \frac{6 \times (2 \times 13 + 12 \times 3\frac{1}{2})}{4\frac{1}{2}}$$

$$= 53.3 + 90.7 = \mathbf{144.}$$

3. *Leakage Factor.*—

$$\lambda = \frac{520 + 144}{520} = \mathbf{1.28}.$$

d. CALCULATION OF MAGNETIZING FORCES.

1. *Air Gaps.*—Actual density:

$$\mathcal{H}'' = \frac{8,250,000}{590} = 14,000 \text{ lines per square inch.}$$

By (228), p. 339:

$$at_g = .3133 \times 14,000 \times 1.25 = \mathbf{5480} \text{ ampere-turns.}$$

2. *Armature Core.*—

By (236), p. 343:

$$l''_a = 37\tfrac{1}{2}\pi \times \frac{\frac{90}{3} + 7\tfrac{1}{2}^\circ}{360} + 2\tfrac{1}{2} = 14\tfrac{3}{4} \text{ inches.}$$

$$m''_a = 11.7 \text{ ampere-turns per inch, see p. 582;}$$

$$\therefore at_a = 11.7 \times 14\tfrac{3}{4} = \mathbf{170} \text{ ampere-turns.}$$

3. *Magnet Frame.*—Wrought iron portion:

Length, $l''_{w.i.} = 6 + 3\tfrac{1}{2} + \tfrac{1}{2} = 10$ inches.

Area, $S''_{w.i.} = (13^2 - 4\tfrac{1}{2}^2)\dfrac{\pi}{4} = 116.8$ square inches.

Density, $\mathcal{B}''_{w.i.} = \dfrac{10,700,000}{116.8} = 92,000$ lines per square inch.

Specific magnetizing force:

$$m''_{w.i.} = 56.5 \text{ ampere-turns per inch.}$$

Magnetizing force required for wrought iron portion:

$$at_{w.i.} = 56.5 \times 10 = \mathbf{565} \text{ ampere-turns.}$$

Cast steel portion:

Length, $l''_{c.s.} = 2 \times (9\tfrac{1}{4} + 6) = 30\tfrac{1}{2}$ inches.

Minimum cross-section,

$$S_{c.s_1} = 3 \times 10\tfrac{1}{2} \times 3\tfrac{1}{2} = 110 \text{ square inches.}$$

Maximum cross-section,

$$S''_{c.s._2} = 3 \times 18 \times 3\tfrac{1}{2} = 189 \text{ square inches.}$$

Maximum and minimum flux densities:

$$\mathfrak{B}''_{c.s._1} = \frac{10,700,000}{110} = 97,300; \quad \mathfrak{B}''_{c.s._2} = \frac{10,700,000}{189} = 56,700.$$

Average specific magnetizing force,

$$m''_{c.s.} = \frac{86 + 15}{2} = 50.5 \text{ ampere-turns per inch.}$$

Magnetizing force required for cast steel portion:

$$at_{c.s.} = 50.5 \times 30\tfrac{1}{2} = \mathbf{1540} \text{ ampere-turns.}$$

4. *Armature Reaction.*—Density in polepieces:

$$\mathfrak{B}''_p = \frac{10,700,000}{\tfrac{1}{2} \times 34 \times \pi \times .75 \times 13} = 20,500 \text{ lines per square inch.}$$

By (250), p. 352, and Table XCI.:

$$at_r = 1.25 \times \frac{3600 \times 50}{3} \times \frac{7\tfrac{1}{2}^\circ}{180} = \mathbf{3130} \text{ ampere-turns.}$$

5. *Total Magnetizing Force Required.*—

$$AT = 5480 + 170 + 565 + 1540 + 3130 = \mathbf{10,885} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

Limit of temperature increase, $\theta_m = 22^\circ$ C.

By (287), p. 374:

$$N_{se} = \frac{10,885}{50} = \mathbf{218} \text{ turns.}$$

Allowing a winding depth of 7 inches, the mean length of one turn is

$$l_T = (13 + 7) \times \pi = 62.8 \text{ inches;}$$

hence, by (288), p. 374:

$$L_{se} = \frac{218 \times 62.8}{12} = \mathbf{1140} \text{ feet.}$$

$$S_M = 27 \times \pi \times 6 + 6 \times 14 \times 3 = \mathbf{760} \text{ square inches.}$$

$$r_{se} = \frac{22}{75} \times \frac{760}{50^2} \times \frac{1}{1 + .004 \times 22} = \mathbf{.082} \text{ ohm, at } 15.5^\circ \text{ C.}$$

$$\lambda_{se} = \frac{1140}{.082} = \mathbf{13,900} \text{ feet per ohm.}$$

The coil being round and of comparatively large diameter, a single wire, No. **00** B. W. G. (.380" + .020") can be employed without difficulty.

Number of turns per layer:

$$\frac{5\frac{1}{4}}{.38 + .02} = \mathbf{13.}$$

Number of layers:

$$\frac{218}{13} = \mathbf{17.}$$

Net depth of winding space:

$$h'_m = 17 \times (.38 + .02) = 6.8 \text{ inches.}$$

Adding to this the thickness of the bobbin and insulation, we have $h_m = 7$ inches, as above.

Weight of winding:

$$wt_{se} = 1140 \times .437 = \mathbf{500} \text{ lbs., bare wire;}$$
$$wt'_{se} = 1.022 \times 500 = \mathbf{510} \text{ lbs., covered wire.}$$

Resistance:

$$r_{se} = 500 \times .00016 = \mathbf{.08} \text{ ohm, at } 15.5^\circ \text{ C.}$$

Actual magnetizing force:

$$AT_{se} = 13 \times 17 \times 50 = \mathbf{11{,}050} \text{ ampere-turns.}$$

140. Calculation of a Multipolar, Multiple Magnet, Toothed-Ring, Low-Speed Compound Dynamo: 2000 KW. Radial Outerpole Type. 16 Poles. Cast-Steel Frame. Drum-Wound Ring Armature. 540 Volts. 3700 Amps. 70 Revs. per Min.

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—

For $\beta_1 = .70$, we have

$$\alpha = \frac{180\,(1 - .70)}{16} = 3\tfrac{3}{8}^\circ,$$

and

$$\beta = \frac{360}{16} - 2 \times 3\tfrac{3}{8} = 15\tfrac{3}{4}^\circ.$$

From Table IV., p. 50: $e = 55 \times 10^{-8}$ volt per foot;

$$\frac{e}{n'_p} = \frac{55}{8} \times 10^{-8} = 6.875 \times 10^{-8} \text{ volt per foot.}$$

From Table V., p. 52*a*, the average conductor velocity for this case is 45 ft. p. sec.; in order to obtain an external armature diameter of exactly 12 feet, however, we will here take:

$$v_c = 42.8 \text{ feet per second.}$$

From Table VI., p. 54:

$$\mathcal{H}'' = 35{,}000 \text{ lines per square inch;}$$

and from Table VIII., p. 56:

$$E' = 1.02 \times 540 = 551 \text{ volts.}$$

Hence, by (26), p. 55:

$$L_a = \frac{551 \times 10^8}{6.875 \times 42.8 \times 35{,}000} = \mathbf{5350} \text{ feet.}$$

2. *Mean Winding Diameter of Armature.*—

By (30), p. 58: $d'_a = 230 \times \frac{42.8}{70} = \mathbf{140\tfrac{3}{4}}$ inches.

3. *Area and Shape of Armature Conductor; Size and Number of Slots.*—

By § 20: $\delta_a^2 = 600 \times \frac{3700}{8} = \mathbf{278{,}000}$ circular mils,

which is equivalent to

$$278{,}000 \times \frac{\pi}{4} = \mathbf{219{,}000} \text{ square mils.}$$

A bar, $\frac{7}{8}$ inch high by $\frac{1}{4}$ inch wide, has a cross-section of 218,750 square mils. Arranging 6 such bars in each slot, as shown in Fig. 358, the width of the slot is found $\frac{11}{16}$ inch, and its total depth, $3\frac{1}{8}$ inches. The distance between mean winding diameter and external circumference is $1\frac{5}{8}$ inches, hence the external diameter:

$$d''_a = 140\tfrac{3}{4} + 2 \times 1\tfrac{5}{8} = \mathbf{144} \text{ inches,}$$

and by (34), p. 70:

$$n'_c = \frac{144 \times \pi}{2 \times \frac{11}{16}} = 329, \text{ or, say, } \mathbf{336} \text{ slots,}$$

this being the nearest number divisible by 16.

4. *Length of Armature Core.*—

By (48), p. 92:

$$l_a = \frac{5350 \times 12}{336 \times 6} = \mathbf{32} \text{ inches.}$$

5. *Arrangement of Armature Winding.*—

By (45), p. 89:

$$(n_c)_{\min} = \frac{551 \times 8}{10} = 440.$$

One commutator-division per slot making the number of commutator-bars smaller than this minimum, we have to take

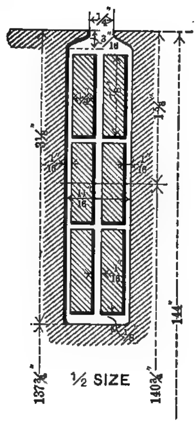


Fig. 358.—Dimensions of Slot and Armature Conductors, 2000-KW 16-Pole, Radial Outerpole Type, Low-Speed Generator.

two per slot, and the winding must be arranged in **772** coils of **3** turns each.

6. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—

By (138), p. 201:

$$\Phi = \frac{6 \times 8 \times 551 \times 10^9}{336 \times 6 \times 70} = \mathbf{188{,}000{,}000} \text{ maxwells.}$$

By (48), p. 92:

$$b_a = \frac{188{,}000{,}000}{16 \times 65{,}000 \times (32 - 2\tfrac{1}{2}) \times .90} = \mathbf{6\tfrac{7}{8}} \text{ inches,}$$

allowance being made for 6 air-ducts of $\frac{1}{4}$ inch width, and for 2 phosphor-bronze end-frames of $\frac{1}{2}$ inch thickness, thus:

$$6 \times \tfrac{1}{4} + 2 \times \tfrac{1}{2} = 2\tfrac{1}{2} \text{ inches.}$$

Total radial depth of armature core:

$$6\tfrac{7}{8} + 3\tfrac{1}{8} = \mathbf{10} \text{ inches.}$$

Maximum depth of armature core:

$$b'_a = \sqrt{\left(144 \times \sin\frac{15\frac{3}{4}}{4}\right)^2 + 10^2} = \sqrt{10^2 + 10^2} = \mathbf{14} \text{ inches.}$$

Minimum and maximum cross-sections:

$$S''_{a_1} = 16 \times 29\tfrac{1}{2} \times 6\tfrac{7}{8} \times .9 = \mathbf{2920} \text{ square inches.}$$

$$S''_{a_2} = 16 \times 29\tfrac{1}{2} \times 14 \times .9 = \mathbf{5950} \text{ square inches.}$$

Maximum and minimum flux densities:

$$\mathfrak{B}''_{a_1} = \frac{188,000,000}{2920} = 64,400; \quad \mathfrak{B}''_{a_2} = \frac{188,000,000}{5950} = 31,600.$$

Mean specific magnetizing force and corresponding average density:

$$m''_a = \frac{15.0 + 5.8}{2} = 10.4 \text{ ampere-turns per inch.}$$

$$\mathfrak{B}''_a = \mathbf{53,000} \text{ lines per square inch.}$$

7. *Weight and Resistance of Armature Winding.*—

By (57), p. 100:

$$L_t = \left(1 + .293 \times \frac{144}{32}\right) \times 5350 = 12,400 \text{ feet.}$$

By (58), p. 101:

$$wt_a = .0000303 \times 278,000 \times 12,400 = \mathbf{10,425} \text{ lbs.}$$

By (61), p. 105:

$$r_a = \frac{1}{4 \times 8^2} \times 12,400 \times \frac{10.5}{278,000} = \mathbf{.00183} \text{ ohm, at } 15.5° \text{ C.}$$

8. *Energy Losses in Armature, and Temperature Increase.*—

By (72), p. 112:

$$M = \frac{134 \times \pi \times 10 - 336 \times 3 \times \frac{11}{16} \times 29\frac{1}{2} \times .9}{1728}$$

$$= 55 \text{ cubic feet.}$$

In this the depth of the slot is taken 3 inches only, in order to allow for the volume of the lateral projections of the teeth.

Frequency:

$$N_1 = \frac{70}{60} \times 8 = 9.33 \text{ cycles per second.}$$

By (68), p. 109:

$$P_a = 1.2 \times 3700^2 \times .00183 = \mathbf{30,000} \text{ watts.}$$

By (73), p. 112:

$$P_h = 18.1 \times 9.33 \times 55 = \mathbf{9300} \text{ watts.}$$

By (76), p. 120:

$$P_e = .081 \times 9.33^2 \times 55 = \mathbf{400} \text{ watts.}$$

By (65), p. 107:

$$P_A = 30,000 + 9300 + 400 = \mathbf{39,700} \text{ watts.}$$

By (79), p. 125:

$$S_A = 134 \times \pi \times 2 \times (36 + 10) = \mathbf{38,700} \text{ sq. inches.}$$

Ratio of pole area to radiating surface:

$$\frac{144\tfrac{5}{8} \times \pi \times 32 \times .70}{38,700} = \frac{10,200}{38,700} = .264;$$

hence by (81), with the use of Table XXXVI., p. 127:

$$\theta_a = 43\tfrac{1}{2} \times \frac{39,700}{38,700} = \mathbf{45°} \text{ C.}$$

and by (63), p. 106:

$$r'_a = (1 + .004 \times 45°) \times .00183 = \mathbf{.00216} \text{ ohm, at } 60\tfrac{1}{2}° \text{ C.}$$

[NOTE.—For the calculation of the hysteresis loss in toothed armatures, Dr. Max Breslauer[1] gives a more accurate expression, consisting of two terms, $P'_h + P''_h$; the former, P'_h, representing the loss in the solid portion of the core, and the latter, P''_h, the loss in the teeth only. While P'_h is obtained from (73) by inserting for M the weight of the solid portion, the second term, P''_h, is the hysteresis loss in the teeth, due to

[1] "On the Calculation of the Energy Loss in Toothed Armatures," by Dr. Max Breslauer, *Elektrotechn. Zeitschr.*, vol. xviii. p. 80 (February 11, 1897); *Electrical World*, vol. xxix. p. 325 (March 6, 1897).

the smallest density (in the largest section, at the periphery of the armature) multiplied by a factor, a, which depends upon the ratio,

$$\frac{b''_t}{b_t},$$

of minimum to maximum width of tooth, and upon the shape of the slot, ranging as follows:

TABLE CVI.—FACTOR OF HYSTERESIS LOSS IN ARMATURE TEETH.

RATIO OF MINIMUM TO MAXIMUM WIDTH OF TOOTH, $\frac{b''_t}{b_t}$	FACTOR a OF HYSTERESIS LOSS IN ARMATURE TEETH.	
	Rectangular Slot.	Circular Slot.
0	5.00	21.00
0.05	3.75	13.00
.1	3.04	8.75
.2	2.47	5.34
.3	2.10	3.77
.4	1.83	2.90
.5	1.61	2.25
.6	1.44	1.81
.7	1.30	1.51
.8	1.19	1.30
.9	1.09	1.14
1.0	1.00	1.00

The hysteresis loss in the mass of the teeth, however, ordinarily is only a small fraction of the total hysteresis loss, P_h, of the armature, and the total hysteresis loss in well-designed machines is so small compared with the C^2R-loss that the difference in the total energy loss due to the use of the above method amounts to but a few per cent., and that, therefore, in the majority of practical cases such a refinement in the calculation is unnecessary.

Thus, in the present example, which is chosen to illustrate the above statement, because in it the difference between the approximate and the exact methods, on account of the great

mass of the teeth—about 11 cubic feet—is near its maximum amount, the hysteresis loss in the solid portion of the armature core is:

$$P'_h = 18.1 \times 9.33 \times (65 - 11) = 7450 \text{ watts.}$$

Minimum density in teeth:

$$\frac{188{,}000{,}000}{2 \times .70 \times \left(144\,\pi - \frac{336}{4}\right) \times 29\tfrac{1}{2} \times .9} = \frac{188{,}000{,}000}{3425}$$

$$= 55{,}000 \text{ lines per square inch;}$$

Hysteresis factor for this density:

$$\eta = 19.21 \text{ watts per cubic foot.}$$

Ratio of minimum to maximum width of tooth:

$$\frac{b''_t}{b_t} = \frac{\frac{137\frac{3}{4} \times \pi}{336} - \frac{11}{16}}{\frac{144 \times \pi}{336} - \frac{1}{4}} = \frac{.6}{1.1} = .545.$$

Tooth-factor, by interpolation from the above table:

$$a = 1.53;$$

hence, hysteresis loss in teeth:

$$\therefore\ P''_h = 19.21 \times 9.33 \times 11 \times 1.53 = 3000 \text{ watts.}$$

The total hysteresis loss, therefore, theoretically accurate, is

$$P_h = 7450 + 3000 = 10{,}450 \text{ watts.}$$

This is about $12\frac{1}{2}$ per cent. greater than the value found on p. 591 ($P_h = 9300$ watts), while the increase in the value of P_A due to this difference amounts to about 3 per cent. only.]

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Magnetic Flux and Sectional Area of Frame.*—By (156), p. 214:

$$\Phi' = 1.15 \times 188{,}000{,}000 = \mathbf{216{,}000{,}000} \text{ maxwells.}$$

By (218), p. 339:

$$S_{c.s.} = \frac{216{,}000{,}000}{85{,}000} = \mathbf{2540} \text{ square inches.}$$

2. *Cores.*—The length of the polepieces being 32 inches (equal to length of armature core), and their circumferential width being

$$144\tfrac{7}{8} \times \sin \frac{15\tfrac{3}{4}^\circ}{2} = 20 \text{ inches,}$$

the core section must be so dimensioned that the projecting strip of the polepiece has the same width both in the lateral

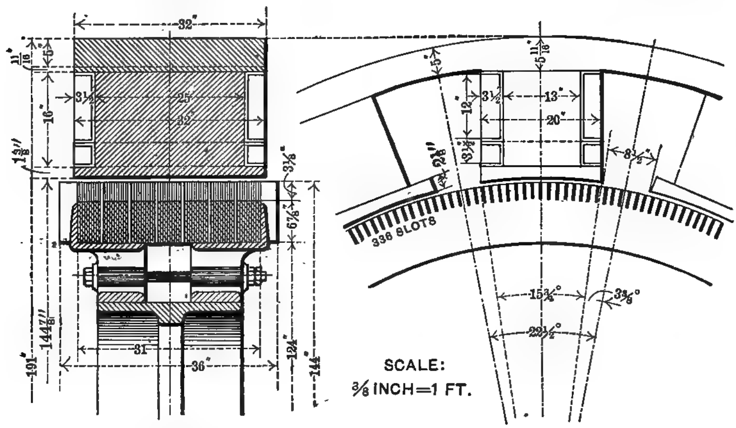


Fig. 359.—Dimensions of Armature and Field Magnet Frame, 2000 KW, 16-Pole, Radial Outerpole Type, Low-Speed Generator.

and in the circumferential directions; making this uniform width of the polepiece-shoulder $3\tfrac{1}{2}$ inches, see Fig. 359, the total actual cross-section of the cores becomes:

$$S''_{\text{c.s.}} = 8 \times 25 \times 13 = \mathbf{2600} \text{ square inches.}$$

Length of cores, by Table LXXXIII., p. 321:

$$l_m = \mathbf{16} \text{ inches.}$$

3. *Polepieces.*—

Bore:

$$d_p \times 144 + 2 \times \frac{7}{16} = \mathbf{144\tfrac{7}{8}} \text{ inches.}$$

Distance between pole-corners:

$$l'_p = 144\tfrac{7}{8} \times \sin 3\tfrac{3}{8}^\circ = \mathbf{8\tfrac{1}{2}} \text{ inches.}$$

Radial thickness, in centre, $1\frac{3}{8}$ inch;

$$\text{at ends, } 1\tfrac{1}{2} + \left(72\tfrac{7}{16} - \sqrt{75\tfrac{7}{16}^2 - 10^2}\right) = 2\tfrac{1}{8} \text{ inches.}$$

4. *Yoke.*—Making the yoke of same width as the armature core, its radial thickness is:

$$h_y = \frac{2540}{16 \times 32} = 5 \text{ inches.}$$

In order to secure a straight seat for the cores and to allow room for the flanges of the magnet-coils, bosses of $\frac{11}{16}$ inch radial height must be provided at the internal periphery of the yoke, making the external diameter of the frame, Fig. 359,

$$144\tfrac{7}{8} + 2 \times \left(1\tfrac{3}{8} + 16 + \tfrac{11}{16} + 5\right) = \mathbf{191} \text{ inches.}$$

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap-Spaces.*—

$$b_t = \frac{144 \times \pi}{336} - \tfrac{1}{4} = 1.35 - .25 = 1.1 \text{ inch; } b'_t = \tfrac{3}{16} \text{ inch.}$$

Ratio of radial clearance to pitch:

$$\frac{.4375}{1.35} = .324.$$

Product of field-density and conductor-velocity:

$$35{,}000 \times 42.8 = 1{,}500{,}000.$$

By Table LXVII., p. 230:

$$k_{12} = 1.60.$$

From Table LXVI., p. 225, for a corresponding perforated armature,

$$k_{12} = 1.20.$$

Average factor of field deflection:

$$\frac{1.60 + 1.20}{2} = 1.40.$$

By (175), p. 230:

$$\mathfrak{P}_1 = \frac{\frac{1}{4}(144\frac{7}{8} \times \pi \times .70 + 1.29 \times 336 \times .76) \times \frac{1}{2}(32 + 31)}{1.40 \times (144\frac{7}{8} - 144)}$$

$$= \frac{5100}{1.6} = \mathbf{3190.}$$

2. *Permeance of Stray Paths.*—

By (181), p. 233:

$$\mathfrak{P}_2 = 8 \times \left[\frac{25 \times 16}{19} + \frac{2 \times 13 \times 16}{19 + 13 \times \frac{\pi}{4}}\right] = 8 \times (16.8 + 14.2)$$

$$= \mathbf{248.}$$

From Fig. 359:

$$\mathfrak{P}_3 = 16 \times \frac{2\frac{1}{4} \times 32}{8\frac{1}{2}} = 16 \times 9 = \mathbf{144.}$$

From Fig. 359:

$$\mathfrak{P}_4 = 8 \times \frac{(32 \times 20) - (25 \times 13)}{16} = 8 \times 19.7 = \mathbf{158.}$$

3. *Leakage Factor; Total Flux.*—

By (157), p. 218:

$$\lambda = \frac{3190 + 248 + 144 + 158}{3190} = \frac{3740}{3190} = 1.17.$$

By (158), p. 218:

$$\lambda' = 1.025 \times 1.17 = \mathbf{1.20.}$$

$$\therefore \quad \Phi' = 1.20 \times 188{,}000{,}000 = \mathbf{260{,}000{,}000} \text{ maxwells.}$$

d. CALCULATION OF MAGNETIZING FORCES.

1. *Shunt Magnetizing Force.*—

No-load flux:

$$\Phi_0 = \frac{6 \times 8 \times 540 \times 10^9}{336 \times 6 \times 70} = \mathbf{184{,}000{,}000} \text{ maxwells.}$$

Air gap ampere-turns:

$$at_{g_0} = .3133 \times \frac{184{,}000{,}000}{5100} \times 2.55 = 28{,}800 \text{ ampere-turns.}$$

Armature ampere-turns:

$$l''_a = 130\tfrac{7}{8} \times \pi \times \frac{\frac{90}{8} + 3\frac{3}{8}}{360} + 6\tfrac{7}{8} + 2 \times 3\tfrac{1}{8} = 30 \text{ inches.}$$

$$\mathfrak{B}''_{a_{1_0}} = \frac{184{,}000{,}000}{2920} = 63{,}000; \quad \mathfrak{B}''_{a_{2_0}} = \frac{184{,}000{,}000}{5950} = 31{,}000;$$

$$\therefore at_{a_0} = \frac{14.5 + 5.6}{2} \times 30 = 300 \text{ ampere-turns.}$$

Magnet core ampere-turns:

$$\mathfrak{B}''_{m_0} = \frac{1.20 \times 184{,}000{,}000}{2600} = 85{,}000 \text{ lines per square inch;}$$

$$\therefore at_{m_0} = 44 \times 16 \times 2 = 1410 \text{ ampere-turns.}$$

Polepiece ampere-turns:

$$\mathfrak{B}''_{p_{1_0}} = \mathfrak{B}_{m_0} = 85{,}000 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{p_{2_0}} = \frac{184{,}000{,}000}{10{,}200} = 18{,}000 \text{ lines per square inch;}$$

$$\therefore at_{p_0} = \frac{44 + 4.7}{2} \times 2\tfrac{3}{4} \times 2 = 130 \text{ ampere-turns.}$$

Yoke ampere-turns:

Maximum depth $= \sqrt{5\frac{11}{16}^2 + 6\frac{1}{2}^2} = 8.7$ inches (see Fig. 359);

$$\mathfrak{B}''_{y_{1_0}} = \frac{1.20 \times 184{,}000{,}000}{16 \times 5 \times 32} = 86{,}300 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{y_2} = \frac{1.20 \times 184{,}000{,}000}{16 \times 8.7 \times 32} = 49{,}600 \text{ lines per square inch.}$$

Length of magnetic circuit in yoke (Fig. 359):

$$l''_y = \frac{186 \times \pi}{16} - 6\tfrac{1}{2} + 5\tfrac{11}{16} = 36 \text{ inches;}$$

$$\therefore at_{y_0} = \frac{47 + 11.8}{2} \times 36 = 1060 \text{ ampere-turns.}$$

Total shunt magnetizing force:

$$AT_{sh} = 28{,}800 + 300 + 1410 + 130 + 1060$$
$$= \mathbf{31{,}700} \text{ ampere-turns.}$$

2. *Series Magnetizing Force.*—

Full-load flux: $\Phi = 188{,}000{,}000$ maxwells.

Air gap ampere-turns:

$$at_g = .3133 \times \frac{188{,}000{,}000}{5100} \times 2.55 = 29{,}600 \text{ ampere-turns.}$$

Armature ampere-turns, see pp. 590 and 597:

$$at_a = 10.4 \times 30 = 310 \text{ ampere-turns.}$$

Magnet core ampere-turns:

$$\mathfrak{B}''_m = \frac{1.20 \times 188,000,000}{2600} = 87,000 \text{ lines per square inch;}$$

$$\therefore at_m = 49 \times 32 = 1570 \text{ ampere-turns.}$$

Polepiece ampere-turns:

$$\mathfrak{B}''_{p_1} = \frac{1.20 \times 188,000,000}{2600} = 87,000 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{p_2} = \frac{1.20 \times 188,000,000}{10,200} = 22,100 \text{ lines per square inch;}$$

$$\therefore at_p = \frac{49 + 5.4}{2} \times 5\tfrac{1}{2} = 27.2 \times 5\tfrac{1}{2} = 150 \text{ ampere-turns.}$$

Yoke ampere-turns:

$$\mathfrak{B}''_{y_1} = \frac{1.20 \times 188,000,000}{16 \times 5 \times 32} = 88,200 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{y_2} = \frac{1.20 \times 188,000,000}{16 \times 8.7 \times 32} = 50,700 \text{ lines per square inch;}$$

$$\therefore at_y = \frac{52 + 12}{2} \times 36 = 32 \times 36 = 1150 \text{ ampere-turns.}$$

Compensating ampere-turns: For $\mathfrak{B}''_p = 75,000$ (corresponding to $m''_p = 27.2$), Table XCI., p. 352, gives

$$k_{14} = 1.25.$$

Maximum density in teeth:

$$\mathfrak{B}''_t = \frac{188,000,000}{\frac{1}{2} \times .70 \times (137\tfrac{3}{4} \times \pi - 136 \times \tfrac{11}{16}) \times 29\tfrac{1}{2} \times .9}$$

$$= \frac{188,000,000}{1880} = 100,000 \text{ lines per square inch.}$$

For this density, the brush-lead coefficient is found, from Table XC., p. 350:

$$k_{13} = .55,$$

the value being taken near the upper limit, on account of the low conductor velocity. By (250), p. 352, therefore:

$$at_r = 1.25 \times \frac{336 \times 6 \times 3700}{16} \times \frac{.55 \times 3\tfrac{3}{8}}{180} = 6000 \text{ amp.-turns.}$$

Total magnetizing force required at full load:

$$AT = 29,600 + 310 + 1570 + 150 + 1150 + 6000$$
$$= \mathbf{38,780} \text{ ampere-turns.}$$

And the required series excitation:

$$AT_{se} = 38{,}780 - 31{,}700 = \mathbf{7080} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

Rise of temperature, $\theta_m = 37\frac{1}{2}°$ C. Percentage of Regulating Resistance, $r_x = 20\%$.

1. *Series Winding.*—

$$l_T = 2 \times (25 + 13) + 3\tfrac{1}{2} \times \pi = 87 \text{ inches.}$$

$$S_M = 2 \times (25 \times + 13 + 3\tfrac{1}{2} \times \pi) \times (16 - \tfrac{1}{2})$$
$$= 1460 \text{ square inches per core.}$$

Connecting all the 16 series coils in parallel, the current flowing in each will be:

$$I_{se} = \frac{3700}{16} = 231.25 \text{ amperes,}$$

and the number of series turns required on each core, two magnets being in series in each magnetic circuit,

$$N_{se} = \frac{\frac{1}{2} \times 7080}{231.25} = \mathbf{16} \text{ turns.}$$

By (343), p. 400:

$$\delta_{se}^2 = 65 \times \frac{\frac{1}{2} \times 38{,}780 \times 231.25 \times 87}{1460 \times 37\frac{1}{2}} \times (1 + .004 \times 37\tfrac{1}{2})$$
$$= 532{,}000 \text{ circular mils.}$$

Using a 19-wire cable, the area of the wire required is:

$$\frac{532{,}000}{19} = 28{,}000 \text{ circular mils.}$$

The nearest gauge wire is No. 8 B. W. G. (.165″ + .010″), making a cable-diameter of

$$5 \times (.165 + .010) = .875 \text{ inch.}$$

The winding depth available accommodates

$$\frac{3\frac{1}{2}}{.875} = 4 \text{ layers}$$

of this cable; hence there are required:

$$\frac{16}{4} = 4 \text{ turns per layer,}$$

and the axial length of the series coil is

$$4 \times .875 = 3\tfrac{1}{2} \text{ inches,}$$

leaving for the shunt coil a length of

$$16 - \tfrac{1}{2} - 3\tfrac{1}{2} = 12 \text{ inches.}$$

By (344), p. 400:

$$r_{se} = .875 \times \frac{16 \times 87}{19 \times 27{,}225} = \mathbf{.00235} \text{ ohm per core.}$$

Joint resistance of all series coils:

$$\frac{.00235}{16} = \mathbf{.000147} \text{ ohm at } 15.5^\circ \text{ C.}$$

Total weight, bare:

$$wt_{se} = 16 \times 16 \times \frac{87}{12} \times 19 \times .0824$$

$$= \mathbf{2910} \text{ pounds, or } \mathbf{182} \text{ pounds per core.}$$

2. *Shunt Winding.*—Grouping all the 16 shunt coils in series, the gauge of the shunt wire must be:

$$\lambda_{sh} = \frac{\frac{1}{2}\,(31{,}700)}{\frac{1}{16} \times 540} \times \frac{87}{12} \times 1.20 \times (1 + .004 \times 37\tfrac{1}{2})$$

$$= \mathbf{4690} \text{ feet per ohm.}$$

No. 5 B. W. G. wire (.220″ + .012″) has 4688 feet per ohm, and therefore gives the required resistance.

By (346), p. 400:

$$P_{sh} = \frac{37\frac{1}{2}}{75} \times 1460 - 231.25^2 \times .00235 \times (1 + .004 \times 37\tfrac{1}{2})$$

$$= 730 - 143 = 587 \text{ watts.}$$

By (312), p. 383:

$$P'_{sh} = 587 \times 1.20 = \mathbf{705} \text{ watts per magnet.}$$

By (314), p. 383:

$$N_{sh} = \frac{\frac{1}{2}(31,700) \times \frac{1}{16} \times 540}{705} = \mathbf{760} \text{ turns per core.}$$

Number of turns in one layer:

$$\frac{12}{.232} = \mathbf{51};$$

Number of layers required:

$$\frac{760}{51} = \mathbf{15}.$$

Winding space taken up:

$$15 \times .232 = 3\tfrac{1}{2} \text{ inches.}$$

By (315), p. 384:

$$L_{sh} = 51 \times 15 \times \frac{87}{12} = \mathbf{5540} \text{ feet per core.}$$

Total weight, bare:

$$wt_{sh} = 16 \times 5540 \times .1465 = \mathbf{13,000} \text{ lbs., or } \mathbf{812} \text{ lbs. per core.}$$

Total resistance:

$$r_{sh} = 16 \times 5540 \times .0002128 = \mathbf{18.9} \text{ ohms, at } 15.5^\circ \text{ C.}$$

By (318), p. 385:

$$r'_{sh} = 18.9 \times (1.004 \times 37\tfrac{1}{2}) = \mathbf{21.7} \text{ ohms, at } 53^\circ \text{ C.}$$

By (317), p. 384:

$$r''_{sh} = 21.7 \times 1.20 = \mathbf{26} \text{ ohms, entire shunt circuit.}$$

$$\therefore \quad I_{sh} = \frac{540}{26} = \mathbf{20.8} \text{ amperes, shunt current, at normal load.}$$

Actual magnetizing force:

$$AT_{se} = 2 \times 16 \times 231.25 = 7,500 \text{ ampere-turns.}$$
$$AT_{sh} = 2 \times 51 \times 15 \times 20.8 = 31,800 \text{ " "}$$

Total exciting power: $AT = \mathbf{39,300}$ ampere-turns.

e. CALCULATION OF EFFICIENCIES.

1. *Electrical Efficiency.*—

By (353), p. 406:

$$\eta_e = \frac{540 \times 3700}{540 \times 3700 + (3720.8)^2 \times .00216 + 3700^2 \times .000147 + 20.8^2 \times 26}$$

$$= \frac{2,000,000}{2,043,300} = .978, \text{ or } \mathbf{97.8\ \%}.$$

2. *Commercial Efficiency.*—

By (361), p. 408:

$$\eta_c = \frac{2,000,000}{2,043,300 + 9700 + 60,000} = \frac{2,000,000}{2,113,000} = .947, \text{ or } \mathbf{94.7\ \%}.$$

3. *Weight-Efficiency.*—

The weight of this machine is estimated as follows:

Armature:

Core, 55 cubic feet of wrought iron, .	26,500 lbs.	
Winding and insulation, connections, etc., ...	12,000 "	
Commutator, ...	15,000 "	
Skeleton pulley, spider frames, shaft, etc.,	16,500 "	
Armature, complete, ...		70,000 lbs.

Frame:

Magnet-cores, 16 × 13 × 25 × 16 = 83,200 cubic inches of cast steel, .	23,000 lbs.	
Yoke 186 × π × 32 × 5 = 93,500 cubic inches of cast steel, ...	26,000 "	
Polepieces, 16 × 20 × 32 × $1\frac{3}{4}$ = 18,000 cubic inches of cast steel, . . .	5,000 "	
Field-winding, spools, and insulation, .	20,000 "	
Supporting lugs, flanges and bosses on frame, outboard bearing, etc., . .	16,000 "	
Frame, complete, ...		90,000 lbs.

Fittings:

Brush-shifting and raising devices, brushes and holders, etc., ...	4,000 lbs.
Switches, connections, cables, etc., .	1,000 "
Fittings, complete, ...	5,000 lbs.
Total net weight of dynamo, . .	**165,000** lbs.

Weight efficiency:

$$\frac{2,000,000}{165,000} = \mathbf{12.1} \text{ watts per pound.}$$

141. Calculation of a Multipolar, Consequent Pole, Perforated Ring, High-Speed Shunt Dynamo:

100 KW. Fourpolar Iron Clad Type. Wrought-Iron Cores, Cast-Steel Yoke and Polepieces. 200 Volts. 500 Amps. 600 Revs. per Min.

(Calculation in Metric Units.)

a. CALCULATION OF ARMATURE.

1. *Length of Armature Conductor.*—

From § 15: $\beta_1 = .70$,

$$\alpha = \frac{180 \times (1 - .70)}{4} = 13\tfrac{1}{2}^\circ;$$

From Table IV., p. 50:

$$e_1 = 3.8 \times 10^{-5} \text{ volt per metre per bifurcation.}$$

From Table V., p. 52:

$$v_c = 24 \text{ metres per second;}$$

From Table VII., p. 54:

$$\mathcal{H} = 3850 \text{ gausses;}$$

From Table VIII., p. 56:

$$E' = 1.04 \times 200 = 208 \text{ volts.}$$

By (26), p. 55:

$$L_a = \frac{2 \times 208 \times 10^{-5}}{3.8 \times 24 \times 3850} = \mathbf{118} \text{ metres.}$$

2. *Sectional Area of Armature Conductor.*—

By (28), p. 57:

$$(\delta_a)^2_{\min} = .2 \times \frac{500}{2} = \mathbf{50} \text{ mm.}^2,$$

or by (29) p. 57:

$$(\delta_a)_{\min} = .5 \times \sqrt{\frac{500}{2}} = \mathbf{8} \text{ mm.}$$

3. *Mean Winding Diameter of Armature, and Number of Perforations.*—

By (31), p. 58:

$$d'_a = 1900 \times \frac{24}{600} = \mathbf{76} \text{ cm.}$$

Adding to the diameter of the armature wire 2 mm. radially for slot-lining and clearance, the size of the perforation will be 12 mm. per conductor. By Table XVI., p. 71, the depth of the slots, for a machine of the size under consideration, may reach 5 cm., hence the conductors can be placed 4 layers deep. The number of the perforations, by Table XIII., p. 66, should be between 100 and 150, and the thickness of the projections, by Fig. 52, p. 72, should be from .5 to .9 times the width of the channels; and these two conditions are fulfilled by making the slots of a width sufficient for one conductor. The number of perforations, then, is:

$$n'_c = \frac{76 \times \pi}{1.2 + .65} = \mathbf{128.} \quad (=4 \times 32).$$

4. *Length of Armature Core.*—By § 23, p. 76:

$$l_a = \frac{100 \times 118}{128 \times 4} = \mathbf{23} \text{ cm.}$$

5. *Arrangement of Armature Winding.*—

By (45), p. 89:

$$(n_c)_{\min} = \frac{208 \times 2}{10} = 42 \text{ divisions.}$$

The next larger divisor of 128 being 64, the winding consists of **64** coils of **8** conductors each.

6. *Radial Depth, Minimum and Maximum Cross-Section, and Average Magnetic Density of Armature Core.*—

By (138), p. 202:

$$\Phi = \frac{6 \times 2 \times 208 \times 10^9}{512 \times 600} = \mathbf{8,120,000} \text{ maxwells.}$$

By (48), p. 92:

$$b_a = \frac{8,120,000}{4 \times 10,000 \times 23 \times .88} = \mathbf{10} \text{ cm.},$$

the maximum flux-density in the armature core, $\mathfrak{B}_a = 10,000$ gausses, being taken from Table XXII., p. 91, and the ratio

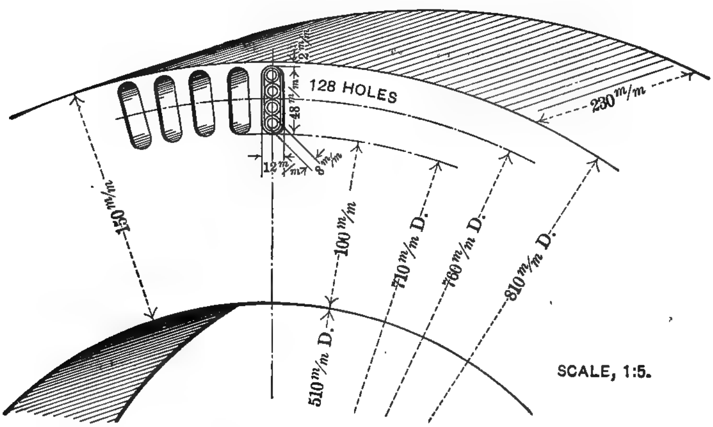


Fig. 360.—Dimensions of Armature, 100-KW Fourpolar Iron Clad Generator.

of magnetic to total armature section, $k_2 = .88$, from Table XXIII., p. 94.

External diameter of armature, see Fig. 360:

$$d''_a = 76 + 5 = \mathbf{81} \text{ cm.}$$

Diameter at bottom of channels:

$$76 - 5 = 71 \text{ cm.}$$

Internal diameter of core:

$$71 - 2 \times 10 = 51 \text{ cm.}$$

Maximum depth: $b'_a = \dfrac{81 \times \pi \times .70}{8} = 22.3$ cm.

Minimum and maximum cross-section:

$$S_{a_1} = 4 \times 23 \times 10 \times .88 = \mathbf{810} \text{ cm}^2.$$

$$S_{a_2} = 4 \times 23 \times 22.3 \times .88 = \mathbf{1810} \text{ cm}^2.$$

Maximum and minimum densities:

$$\mathfrak{B}_{a_1} = \frac{8,120,000}{810} = 10,000 \text{ lines per cm}^2;$$

$$\mathfrak{B}_{a_2} = \frac{8,120,000}{1810} = 4500 \text{ lines per cm}^2.$$

Mean specific magnetizing force:

$$m_a = \frac{6.1 + 2.1}{2} = 4.1 \text{ ampere-turns per cm.}$$

Average density corresponding to $m_a = 4.1$, from Table LXXXIX., p. 337:

$$\mathfrak{B}_a = \mathbf{8100} \text{ gausses.}$$

7. *Weight and Resistance of Armature Winding.*—By (53), p. 99:

$$L_t = \frac{2\,(23 + 10) + 5 \times \pi}{23} \times 118 = 420 \text{ m.}$$

By § 28, p. 101:

$$wt_a = .0089 \times 8^2 \times \frac{\pi}{4} \times 420 = \mathbf{188} \text{ kg.}$$

By (62), p. 105:

$$r_a = \frac{1}{4 \times 2^2} \times 420 \times \left(\frac{.017}{8^2 \times \frac{\pi}{4}}\right) = \mathbf{.0089} \text{ ohm, at } 15.5^\circ \text{ C.}$$

8. *Energy-Losses in Armature, and Temperature Increase.*—By (74), p. 114:

$$M_1 = \frac{65.5 \times \pi \times 23 \times 15 \times .88 - 128 \times \left(1.2 \times 3.6 + 12^2 \frac{\pi}{4}\right)}{1,000,000}$$

$$= .0615 \text{ cbm.}$$

Frequency:

$$N_1 = \frac{600}{60} \times 2 = 20 \text{ cycles per second.}$$

By Table XXX., p. 115 ($\mathfrak{B}_a = 8100$):

$$\eta' = 627.6 \text{ watts per cbm.}$$

By Table XXXIV., p. 122 ($\delta_i = 0.5$ mm.):

$$\varepsilon' = 2.7 \text{ watts per cbm.}$$

By (68), p. 109:

$$P_a = 1.2 \times 500^2 \times .0089 = \mathbf{2670} \text{ watts.}$$

By (73), p. 112:

$$P_h = 627.6 \times 20 \times .0615 = \mathbf{773} \text{ watts.}$$

By (76), p. 120:

$$P_e = 2.7 \times 20^2 \times .0615 = \mathbf{67} \text{ watts.}$$

By (65), p. 107:

$$P_A = 2670 + 773 + 67 = \mathbf{3510} \text{ watts.}$$

By (79), p. 125:

$$S_A = \frac{81 \times 51 - 2 \times 5}{2} \times \pi \times 2 \times (23 + 15 + 3 \times 5)$$
$$= \mathbf{20{,}000} \text{ cm}^2.$$

Ratio of pole area to radiating surface:

$$\frac{81 \times \pi \times 23 \times .70}{20{,}000} = .205.$$

From Table XXXVI, p. 127:

$$\theta'_a = 41^\circ \text{ C.}$$

By (81), p. 127:

$$\theta_a = 6.45 \times 41 \times \frac{3510}{20{,}000} = \mathbf{42^\circ} \text{ C.}$$

Armature resistance, warm, by (63), p. 106:

$$r'_a = .0089 \times (1 + .004 \times 42) = \mathbf{.0104} \text{ ohm, at } 57.5^\circ \text{ C.}$$

b. DIMENSIONING OF MAGNET FRAME.

1. *Total Flux through Magnetic Circuit, and Sectional Areas of Frame.*—

By (156) and Table LXVIII.:

$$\Phi' = 1.30 \times 8{,}120{,}000 = \mathbf{10{,}500{,}000} \text{ maxwells.}$$

By Table LXXVI., p. 313:

$$S_{\text{w.i.}} = \frac{10{,}500{,}000}{14{,}000} = \mathbf{750} \text{ cm}^2.$$

and

$$S_{\text{c.s.}} = \frac{10{,}500{,}000}{13{,}000} = \mathbf{810} \text{ cm}^2.$$

2. *Magnet Cores.*—Each of the two magnet cores carries two

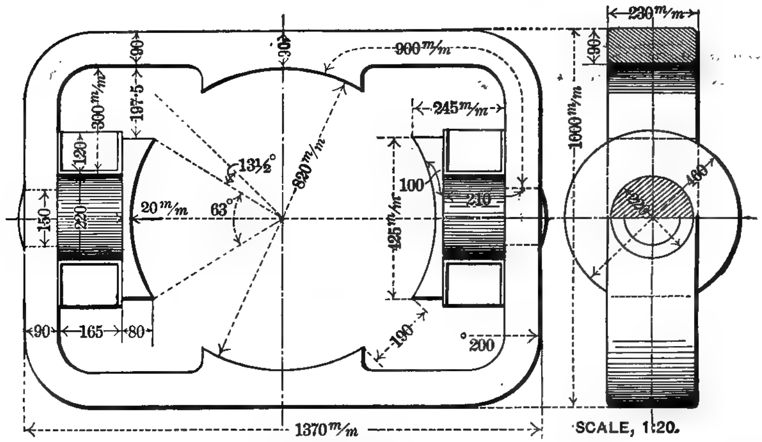


Fig. 361.—Dimensions of Field-Magnet Frame, 100-KW Fourpolar Iron-Clad Generator.

of the four magnetic circuits, Fig. 361, hence the magnet diameter:

$$d_{\text{m}} = \sqrt{\frac{750}{2} \times \frac{4}{\pi}} = \mathbf{22} \text{ cm.}$$

For a flux of 5,250,000 maxwells passing through each core, Table LXXXII., p. 320, gives .75 as the ratio of length to diameter, consequently

$$l_{\text{m}} = .75 \times 22 = \mathbf{16.5} \text{ cm.}$$

3. *Polepieces.*—The radial clearance, from Table LXI., p. 209, being about 5 mm., the bore is:

$$d_{\text{p}} = 810 + 2 \times 5 = \mathbf{820} \text{ mm.}$$

Pole distance:

$$l'_{\text{p}} = 820 \times \sin 13\tfrac{1}{2}° = \mathbf{190} \text{ mm.}$$

Pole chord:

$$h_p = 820 \times \sin 31\tfrac{1}{2}^\circ = \mathbf{425} \text{ mm.}$$

Thickness in centre, 22 mm.;

$$\text{at ends, } 20 + \frac{820}{2} - \sqrt{\left(\frac{820}{2}\right)^2 - \left(\frac{425}{2}\right)^2} = \mathbf{80} \text{ mm.}$$

4. *Yoke.*—Only one magnetic circuit passes through the yoke-section; for a breadth of 23 cm., equal to length of armature core, therefore, the thickness of the yoke is:

$$h_y = \frac{810}{4 \times 23} = \mathbf{9} \text{ cm.}$$

Length over all (Fig. 361):

$$820 + 2 \times (20 + 165 + 90) = \mathbf{1370} \text{ mm.}$$

Height of frame:

$$820 + 2 \times 90 = \mathbf{1000} \text{ mm.}$$

c. CALCULATION OF MAGNETIC LEAKAGE.

1. *Permeance of Gap-Spaces.*—

For $v_c \times \mathfrak{H} = 24 \times 3850 = 92{,}500,$

Table LXVI., p. 225, gives

$$k_{12} = 1.22;$$

therefore, by (176), p. 230:

$$\mathfrak{P}_1 = \frac{\frac{\pi}{4}(82 \times .7 + 81 \times .8) \times 23}{1.95 \times (82 - 81)} = \frac{2200}{1.22} = \mathbf{1800.}$$

2. *Permeance of Stray Paths.*—

By (165), p. 223, and (185), p. 237:

$$\mathfrak{P}_2 = \frac{(16.5 + 38.5) \times (22\,\pi + 23 + 2 \times 9)}{30 + .3 \times 22} = \mathbf{166.}$$

By (196), p. 243:

$$\mathfrak{P}_3 = 2 \times \frac{42.5 \times (8 + 2.2)}{19 + 42.5 \times \frac{\pi}{4}} = \mathbf{17.}$$

By (204), p. 247:

$$\mathfrak{P}_4 = \frac{4 \times (8 \times 23)}{19.75} + \frac{2 \times \left(42.5 \times 23 - 22^2 \frac{\pi}{4}\right)}{16.5} = \mathbf{110.}$$

3. *Probable Leakage Coefficient, and Total Flux.*—

By (157), p. 218:

$$\lambda = \frac{1800 + 166 + 17 + 110}{1800} = \frac{2093}{1800} = \mathbf{1.16.}$$

By (158) p. 218, and Table LXV.:

$$\lambda' = 1.04 \times 1.16 = \mathbf{1.20},$$

$$\therefore \Phi' = 1.20 \times 8,120,000 = \mathbf{9,750,000} \text{ maxwells.}$$

d. CALCULATION OF MAGNETIZING FORCES.

1. *Air Gaps.*—

Actual density:

$$\mathfrak{H} = \frac{8,120,000}{2200} = 3690 \text{ gausses.}$$

Magnetizing force:

$$at_g = .8 \times 3690 \times 1.22 = \mathbf{3600} \text{ ampere-turns.}$$

2. *Armature Core.*—

By (237), p. 343,

$$l''_a = 61 \times \pi \times \frac{\frac{90}{2} + 13\tfrac{1}{2}^\circ}{360} + 10 + 2 \times 5 = 51 \text{ cm.}$$

Magnetizing force:

$$at_a = 4.1 \times 51 = \mathbf{210} \text{ ampere-turns.}$$

3. *Magnet Cores.*—

$$\mathfrak{B}_{w.i.} = \frac{9,750,000}{2 \times 22^2 \frac{\pi}{4}} = 12,800 \text{ gausses.}$$

Magnetizing force:

$$at_{w.i.} = 13.8 \times 21 = \mathbf{290} \text{ ampere-turns.}$$

4. *Polepieces.*—

Density at junction with cores:

$$\mathcal{B}_{p_1} = \frac{9,750,000}{2 \times 22^2 \frac{\pi}{4}} = 12,800 \text{ gausses;}$$

Density at poleface:

$$\mathcal{B}_{p_2} = \frac{8,120,000}{\frac{1}{2} \times 81.6 \times \pi \times 23 \times .70} = 3940 \text{ gausses,}$$

By (241), p. 346, and Table LXXXIX.:

$$m_p = \frac{15.2 + 2.34}{2} = 8.77 \text{ ampere-turns per cm.}$$

Corresponding average density:

$$\mathcal{B}_p = 10,750 \text{ gausses.}$$

Length of circuit in polepieces, see Fig. 361:

$$l_p = 10 \text{ cm.}$$

Magnetizing force:

$$at_p = 8.77 \times 10 = \mathbf{90} \text{ ampere-turns.}$$

5. *Yoke.*—

$$\mathcal{B}_{c.s.} = \frac{9,750,000}{4 \times 23 \times 9} = 11,800 \text{ gausses;}$$

$$m_{c.s.} = 11.1 \text{ ampere-turns per cm.;}$$

$$l_{c.s.} = 90 \text{ cm. (Fig. 361).}$$

Magnetizing force:

$$\therefore \; at_{c.s.} = 11.1 \times 90 = \mathbf{1000} \text{ ampere-turns.}$$

6. *Armature Reaction.*—

For $\mathcal{B}_p = 10,750$ gausses, Table XCI., p. 352, gives $k_{14} = 1.25$.

Maximum density in iron projections:

$$\frac{8,120,000}{\frac{1}{2} \times .70 \times (72.2 \times \pi - 128 \times 1.2) \times 23 \times .88} = 15,700 \text{ gausses,}$$

for which Table XC., p. 350, gives an average coefficient of brush lead of $k_{13} = .4$.

Hence by (250), p. 352:

$$at_r = 1.25 \times \frac{512 \times 500}{4} \times \frac{.4 \times 13\frac{1}{2}}{180} = \mathbf{2400} \text{ ampere-turns.}$$

7. *Total Magnetizing Force Required.*—Summing up we have:

$$AT = 3600 + 210 + 290 + 90 + 1000 + 2400$$
$$= \mathbf{7590} \text{ ampere-turns.}$$

e. CALCULATION OF MAGNET WINDING.

Temperature increase desired, $\theta_m = 40°$ C.; percentage of regulating resistance, at normal load, $r_x = 50$ per cent. of magnet resistance.

Table LXXX., p. 317, gives for a 20 cm. multipolar type magnet core a ratio of winding height to core diameter of .36, which makes the winding depth for the present case:

$$h_m = .36 \times 22 = \mathbf{8} \text{ cm.,}$$

and therefore the mean length of one turn:

$$l_T = (22 + 8) \times \pi = 94.25 \text{ cm.}$$

Hence by (318), p. 385, if the two coils are connected in series, each taking 100 volts,

$$\lambda_{sh} = \frac{7590}{100} \times \frac{94.25}{100} \times 1.50 \times (1 + .004 \times 40)$$
$$= \mathbf{124.5} \text{ metres per ohm.}$$

According to the common millimetre wire gauge, a wire of a specific length of 122 metres per ohm has a diameter of $\delta_m = 1.6$ mm., bare, or $\delta'_m = 1.6 + .25 = 1.85$ mm., covered. This wire will give the required temperature increase with

$$r_x = \frac{50 \times 122}{124.5} = 49 \text{ per cent.}$$

extra-resistance in circuit.

Radiating surface:

$$S_M = (22 + 2 \times 8) \times \pi \times (16.5 - .5) = \mathbf{1910} \text{ cm}^2.$$

$$\therefore \; P'_{sh} = \frac{40}{75} \times \frac{1910}{6.45} \times 1.49 = \mathbf{236} \text{ watts.}$$

By (314), p. 384:

$$N_{sh} = \frac{7590 \times 100}{236} = \mathbf{3220} \text{ turns.}$$

Number of turns possible per layer:

$$\frac{165 - 5}{1.85} = \mathbf{86};$$

Number of layers required:

$$\frac{3220}{86} = \mathbf{38};$$

Net winding depth needed:

$$h'_m = 38 \times 1.85 = \mathbf{70} \text{ mm.}$$

By (315), p. 384:

$$L_{sh} = 86 \times 38 \times \frac{94.25}{100} = \mathbf{2980} \text{ m.}$$

$$\therefore \quad r_{sh} = \frac{2980}{122} = \mathbf{24.5} \text{ ohms, per coil, at } 15.5^\circ \text{ C.}$$

By (318), p. 385:

$$r'_{sh} = 24.5 \times (1 + .004 \times 40) = \mathbf{28.5} \text{ ohms, at } 55.5^\circ \text{ C.}$$

By (317), p. 384:

$$r''_{sh} = 2 \times 28.5 \times 1.49 = \mathbf{85} \text{ ohms, total resistance of shunt circuit.}$$

$$\therefore \quad I_{sh} = \frac{200}{85} = \mathbf{2.35} \text{ amperes.}$$

Actual magnetizing force:

$$AT = 86 \times 38 \times 2.35 = \mathbf{7670} \text{ ampere-turns.}$$

Weight per coil, bare:

$$wt_{sh} = \frac{2980}{1000} \times 17.8 = \mathbf{53} \text{ kg.,}$$

17.8 being the weight, in kilogrammes, of 1000 metres of copper wire, of 1.6 mm. diameter.

CHAPTER XXX.

EXAMPLES OF LEAKAGE CALCULATIONS, ELECTRIC MOTOR DESIGN, ETC.

142. Leakage Calculation for a Smooth Ring, One-Material Frame, Inverted Horseshoe Type Dynamo:

9.5 KW "Phœnix" Dynamo.[1]
105 Volts. 90 Amps. 1420 Revs. per Min.

a. PROBABLE LEAKAGE FACTOR (FROM DIMENSIONS OF MACHINE).

1. *Permeance of Air Gaps.*—From Fig. 362, which shows the principal dimensions of this machine, its gap area is obtained:

$$S_g = \frac{1}{2}\left(\frac{10\frac{5}{8} \times \pi}{2} + 11\frac{3}{8} \times \pi \times \frac{112^\circ}{360}\right) \times 9 = 125 \text{ square ins.}$$

The useful flux (see below, § 142, *b.*, 1, p. 616):

$$\Phi = 2{,}600{,}000 \text{ maxwells,}$$

therefore the field density:

$$\mathcal{H}'' = \frac{2{,}600{,}000}{125} = 20{,}800 \text{ lines per square inch.}$$

The conductor velocity being

$$v_c = \frac{11 \times \pi}{12} \times \frac{1420}{60} = 68 \text{ feet per second,}$$

the product of density and speed is

$$\mathcal{H}'' \times v_c = 20{,}800 \times 68 = 1{,}415{,}000,$$

for which Table LXVI., p. 225, gives a factor of field deflection: $k_{12} = 1.30$.

[1] Silvanus P. Thompson, "Dynamo-Electric Machinery," fourth edition, p. 416 and Plate V.

Hence, by (167), p. 226:

$$\mathfrak{P}_1 = \frac{125}{1.30 \times (11\frac{3}{8} - 10\frac{5}{8})} = \frac{125}{.975} = \mathbf{128.}$$

2. *Stray Permeances.*—

By (177), p. 232:

$$\mathfrak{P}_2 = \frac{9 \times 8\frac{3}{4}}{2 \times 6\frac{1}{2}} + \frac{7 \times 8\frac{3}{4}}{6\frac{1}{2} + 7 \times \frac{\pi}{2}} = 6.1 + 3.5 = \mathbf{9.6.}$$

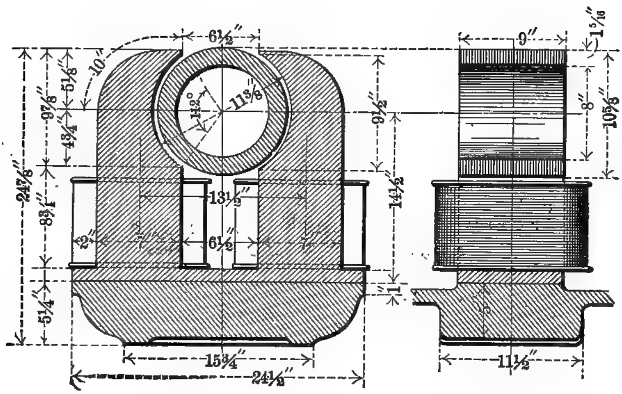


Fig. 362.—9.5 KW Phœnix Dynamo.

By (192), p. 241:

$$\mathfrak{P}_3 = \frac{10 \times 9}{6\frac{1}{2} + 10 \times \frac{\pi}{2}} + \frac{2 \times 9\frac{7}{8} \times \frac{1}{2}\,(7 + 4\frac{9}{16})}{6\frac{1}{2} + 7 \times \frac{\pi}{2}} = 4.1 + 6.5 = \mathbf{10.6.}$$

The projecting area of the yoke, at each core is:

$$S = \left(\frac{6\frac{1}{2}}{2} + 2\right) \times 9 = 47.25 \text{ square inches,}$$

hence, by (202), p. 246,

$$\mathfrak{P}_4 = \frac{47.25}{8\frac{3}{4}} + \frac{9 \times 4\frac{3}{4}}{8\frac{3}{4} + (6 + 4\frac{3}{4}) \times \frac{\pi}{4}} = 5.4 + 2.5 = \mathbf{7.9.}$$

3. *Probable Leakage Factor.*—

By (157), p. 218:

$$\lambda = \frac{128 + 9.6 + 10.6 + 7.9}{128} = \frac{156.1}{128} = \mathbf{1.22}.$$

b. ACTUAL LEAKAGE FACTOR (FROM MACHINE TEST).

1. *Total Magnetizing Force of Machine.*—The dynamo is compound-wound, having a series resistance of .021 ohm, and a shunt resistance of 39.76 ohms: its armature resistance is .04 ohm. Therefore, the total current generated:

$$I' = 90 + \frac{105}{39.76} = 92.65 \text{ amperes,}$$

and the total E. M. F.:

$$E' = 105 + 92.65 \times .04 + 90 \times .021 = 110.6 \text{ volts.}$$

There are 180 conductors on the periphery of the armature, hence by (138), p. 202:

$$\Phi = \frac{6 \times 110.6 \times 10^9}{180 \times 1420} = 2,600,000 \text{ maxwells.}$$

The magnet winding consists of 108 series and 3454 shunt turns, and the two series coils are connected in parallel, the two shunt coils in series to each other, consequently:

$$AT_{se} = 108 \times \frac{90}{2} = 4860 \text{ ampere-turns,}$$

and

$$AT_{sh} = 3454 \times 2.65 = 9140 \text{ ampere-turns;}$$

making the total actual exciting power:

$$AT = 4860 + 9140 = \mathbf{14,000} \text{ ampere-turns.}$$

2. *Magnetizing Force Required for Magnet Frame.*—

$\mathcal{H}'' = 20,800$ (p. 614); $l''_g = .975$ (p. 615);

$\therefore at_g = .3133 \times 20,800 \times .975 = 6350$ ampere-turns.

$l_a = 9$ inches; $b_a = 1\frac{9}{16}$ inches; $d_a = 10\frac{5}{8}$ inches;

$$b'_a = 1\tfrac{9}{16} \times \sqrt{\frac{10\frac{5}{8}}{1\frac{9}{16}} - 1} = 3.77 \text{ inches; } k_2 = .865;$$

$$S''_{a_1} = 2 \times 9 \times 1\tfrac{9}{16} \times .865 = 24.35 \text{ square inches;}$$

$$S''_{a_2} = 2 \times 9 \times 3.77 \times .865 = 58.7 \text{ square inches;}$$

$$\mathfrak{B}''_{a_1} = \frac{2,600,000}{24.35} = 106,750 \text{ lines; } m''_{a_1} = 174 \text{ ampere-turns;}$$

$$\mathfrak{B}''_{a_2} = \frac{2,600,000}{58.7} = 44,000 \text{ lines; } m''_{a_2} = 8.4 \text{ ampere-turns;}$$

$$l''_a = 9\tfrac{5}{16} \times \pi \times \frac{90 + 34}{360} + 1\tfrac{5}{16} = 11\tfrac{1}{2} \text{ inches;}$$

$$\therefore at_a = \frac{174 + 8.4}{2} \times 11\tfrac{1}{2} = 1050 \text{ ampere-turns.}$$

$k_{14} = 1.56$, corresponding to a pole-face density of

$$\mathfrak{B}''_p = \frac{2,600,000}{11\tfrac{3}{8} \times \pi \times \frac{112}{360} \times 9} = \frac{2,600,000}{100} = 2600 \text{ lines p. sq. in.;}$$

$$\therefore at_r = 1.56 \times \frac{180 \times 92.53}{2} \times \frac{34}{180} = 2460 \text{ ampere-turns.}$$

Hence, magnetizing force left for field frame:

$$at_m = 14,000 - (6350 + 1050 + 2460) = \mathbf{4140} \text{ ampere-turns.}$$

3. *Total Magnetic Flux; Actual Leakage Factor.*—The magnet frame is entirely of cast iron; the dimensions of its magnetic path, according to Fig. 362, are:

$$l''_m = 2 \times (8\tfrac{3}{4} + 5) + 6\tfrac{1}{2} + 3 \times \frac{\pi}{2} = 37.7 \text{ inches.}$$

$$S''_m = \frac{33 \times 7 \times 9 + 4.7 \times 9\tfrac{7}{8} \times 9}{37.7} = 66 \text{ square inches.}$$

Inserting the above values into (209), p. 259, we obtain:

$$m''_m = \frac{4140}{37.7} = 110 \text{ ampere-turns per inch.}$$

According to Table LXXXVIII., p. 336, this specific magnetizing force corresponds to a magnetic density in highly permeable cast iron, of

$$\mathfrak{B}''_m = 50,000 \text{ lines per square inch,}$$

from which, by formula (210), p. 259, follows the total magnetic flux:

$$\Phi' = 66 \times 50{,}000 = \mathbf{3{,}300{,}000} \text{ maxwells.}$$

The actual leakage coefficient, consequently, from (214), p. 262, is:

$$\lambda = \frac{3{,}300{,}000}{2{,}600{,}000} = \mathbf{1.27}.$$

The probable leakage factor computed from the dimensions of the frame has, on page 616, been found $\lambda = 1.22$, which is 4 per cent. below the actual value.

143. Leakage Calculation for a Smooth Ring, One-Material Frame, Double Magnet Type Dynamo:

40 KW "Immisch" Dynamo.[1]
690 Volts. 59 Amps. 480 Revs. per Min.

a. Probable Leakage Factor.—(From Fig. 363).

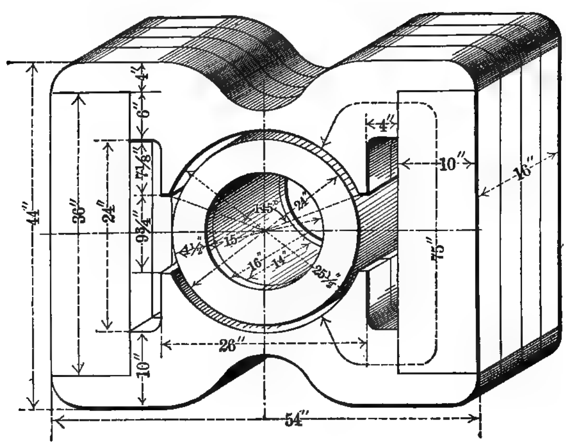


Fig. 363.—40-KW "Immisch" Dynamo.

By (167), p. 226:

$$\mathfrak{T}_1 = \frac{\frac{1}{4}\left(24 \times \pi + 25\tfrac{1}{2} \times \pi \times \frac{145}{180}\right) \times \frac{16 + 14}{2}}{1.3 \times (25\tfrac{1}{2} - 24)} = \frac{520}{1.95} = 267.$$

[1] For data of this machine see Gisbert Kapp's "Transmission of Energy," third edition, p. 272.

By (194), p. 242:

$$\mathfrak{P}_s = 2\left[\frac{(54+16)\times 10 + 4\times 16}{24} + \frac{16\times 7\frac{1}{8}}{9\frac{3}{4}+7\frac{1}{8}} + \frac{2\times 16}{9\frac{3}{4}}\right] = 85.$$

Hence, by formula (157), p. 218, in which for the present type $\mathfrak{P}_1$, $\mathfrak{P}_2$, and $\mathfrak{P}_4$ are zero,

$$\lambda = \frac{267+85}{267} = \frac{352}{267} = \mathbf{1.32.}$$

b. Actual Leakage Factor.—

The armature is wound with 760 turns of No. 9 B. & S. wire, resistance .36 ohm; the field winding consists of 984 series turns (No. 4 S. W. G.) per core, two coils in parallel, joint resistance .25 ohm.

By (9), p. 37:

$$E' = 690 + 59\,(.36 + .25) = 690 + 36 = 726 \text{ volts.}$$

By (138), p. 202:

$$\Phi = \frac{6\times 726\times 10^9}{760\times 480} = 12{,}000{,}000 \text{ maxwells.}$$

By (139), p. 202:

$$\mathcal{H}'' = \frac{12{,}000{,}000}{520} = 23{,}100 \text{ lines per square inch.}$$

By (228), p. 339:

$$at_g = .3133\times 23{,}100\times 1.95 = 14{,}100 \text{ ampere-turns.}$$

By (232), p. 341:

$$S''_{a_1} = 2\times(16-2)\times 4\tfrac{1}{2}\times .865 = 109 \text{ square inches}$$

By (233), p. 341:

$$S''_{a_2} = 2\times(16-2)\times 4\tfrac{1}{2}\times \sqrt{\frac{24\frac{1}{2}}{4\frac{1}{2}} - 1}\times .865$$

$$= 230 \text{ square inches.}$$

By (231), p. 341:

$$m''_a = \frac{290+10.2}{2} = 150 \text{ ampere-turns per inch.}$$

By (236), p. 343:

$$l''_a = 19\frac{1}{2} \times \pi \times \frac{90 + 17\frac{1}{2}}{360} + 4\frac{1}{2} = 22\frac{3}{4} \text{ inches.}$$

By (230), p. 340:

$$at_a = 150 \times 22\frac{3}{4} = 3400 \text{ ampere-turns.}$$

The angle of lead was measured to be about 20°, therefore by (250), p. 352:

$$at_r = 1.40 \times 760 \times \frac{59}{2} \times \frac{20}{180} = 3500 \text{ ampere-turns.}$$

The total magnetizing force of the machine is:

$$AT = 984 \times \frac{59}{2} = 29,000 \text{ ampere-turns.}$$

The frame is all wrought iron, having a uniform cross-section of

$$S_m = 10 \times 16 = 160 \text{ square inches,}$$

and the length of each circuit in the frame is:

$$l''_m = 75 \text{ inches.}$$

Hence we have:

$$75 \times m''_m = 29,000 - (14,100 + 3400 + 3500)$$
$$= 8000 \text{ ampere-turns.}$$

from which:

$$m''_m = \frac{8000}{75} = 106.7 \text{ ampere-turns per inch.}$$

Consulting Table LXXXVIII., p. 336, we find:

$$\mathcal{B}''_m = \frac{\Phi'}{160} = 102,000 \text{ lines per square inch;}$$

or, the total flux:

$$\Phi' = 160 \times 102,000 = 16,400,000 \text{ maxwells.}$$

$$\therefore \lambda = \frac{16,400,000}{12,000,000} = \mathbf{1.36.}$$

The probable leakage factor found, in this case, is about 3 per cent. smaller than the actual one.

144. Leakage Calculation for a Smooth Drum, Combination Frame, Upright Horseshoe Type Dynamo:

200 KW "Edison" Bipolar Railway Generator.[1] 500 Volts. 400 Amps. 450 Revs. per Min.

a. Probable Leakage Factor.—(From Fig. 364).

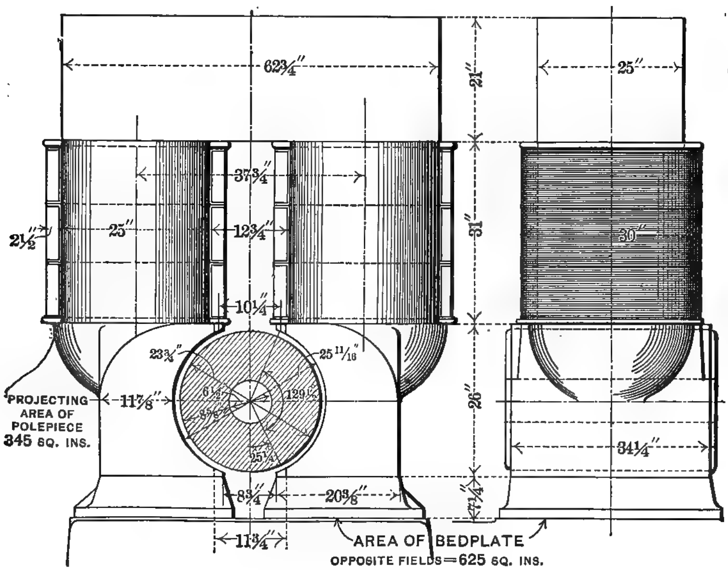


Fig. 364.—200-KW "Edison" Bipolar Railway Generator.

By (167), p. 226:

$$\mathfrak{L}_1 = \frac{\frac{1}{4}\left(23\tfrac{3}{4} \times \pi + 25\tfrac{11}{16} \times \pi \times \frac{129\frac{1}{2}}{180}\right) \times 34\tfrac{1}{4}}{1.30 \times (25\tfrac{11}{16} - 23\tfrac{3}{4})} = \frac{1135}{2.52} = 451.$$

By (178), p. 232:

$$\mathfrak{L}_2 = \frac{25 \times \pi \times 31}{2 \times 12\tfrac{3}{4} + 1.5 \times 25} = 38.7.$$

[1] For description see *Electrical Engineer*, vol. xiii. p. 321 (March 23, 1891); *Electrical World*, vol. xix. p. 220 (March 26, 1892).

By (188), p. 239:

$$\mathfrak{P}_3 = \frac{\frac{1}{2}\left[34\tfrac{1}{4} \times (20\tfrac{3}{8} + \tfrac{1}{2} \times 26) + 625\right]}{2 \times 7\tfrac{1}{4}} = 60.9.$$

By (199), p. 245:

$$\mathfrak{P}_4 = \frac{345}{31} + \frac{34\tfrac{1}{4} \times 26}{2 \times 31 + (26 + 21) \times \frac{\pi}{2}} = 16.5.$$

By (157), p. 218:

$$\lambda = \frac{451 + 38.7 + 60.9 + 16.5}{451} = \frac{567.1}{451} = \mathbf{1.26.}$$

b. Actual Leakage Factor.—

The total E. M. F. generated, by considering the losses in armature and series field windings, is found: $E' = 520$ volts; and there are 228 conductors on the armature periphery; therefore by (138), p. 202:

$$\Phi = \frac{6 \times 520 \times 10^9}{228 \times 450} = 30{,}500{,}000 \text{ maxwells.}$$

$$\therefore \quad \mathfrak{H}'' = \frac{30{,}500{,}000}{1135} = 27{,}000 \text{ lines per square inch.}$$

By (228), p. 339:

$$at_g = .3133 \times 27{,}000 \times 2.52 = 21{,}300 \text{ ampere-turns.}$$

By (232) and (233), p. 341:

$$S''_{a_1} = 2 \times 34\tfrac{1}{4} \times 8\tfrac{5}{8} \times .85 = 502 \text{ square inches.}$$

$$S''_{a_2} = 2 \times 34\tfrac{1}{4} \times 8\tfrac{5}{8} \times \sqrt{\frac{23\tfrac{3}{4}}{8\tfrac{5}{8}} - 1} \times .85 = 665 \text{ sq. ins.}$$

Therefore:

$$\mathfrak{B}''_{a_1} = \frac{30{,}500{,}000}{502} = 60{,}500; \quad \mathfrak{B}''_{a_2} = \frac{30{,}500{,}000}{665} = 45{,}700;$$

and by (231), p. 341:

$$m''_a = \frac{13.2 + 8.6}{2} = 10.9 \text{ ampere-turns per inch.}$$

By (236), p. 343:

$$l''_a = 15\tfrac{1}{8} \times \pi \times \frac{90 + 25\tfrac{1}{4}}{360} + 8\tfrac{5}{8} = 23.9 \text{ inches.}$$

By (230), p. 340:

$$at_a = 10.9 \times 23.9 = 260 \text{ ampere-turns.}$$

By (250), p. 352:

$$at_r = 2.15 \times 114 \times \frac{400 + 3.6}{2} \times \frac{25\frac{1}{4}}{180}$$
$$= 7000 \text{ ampere-turns.}$$

The magnet winding consists of about 8000 shunt turns and of 46 series turns. The shunt-circuit has a resistance of 139 ohms, making the shunt field current at normal load

$$I_{sh} = \frac{500}{139} = 3.6 \text{ amperes;}$$

hence, the total magnetizing force actually exciting this machine at full load:

$$AT = 8000 \times 3.6 + 46 \times 400 = 47,200 \text{ ampere-turns;}$$

and by (207), p. 258:

$$at_m = 47,200 - (21,300 + 260 + 7000) = 18,640 \text{ ampere-turns.}$$

The section of the cores is:

$$S''_m = 25^2 \times \frac{\pi}{4} = 490.9 \text{ square inches;}$$

and that of the yoke:

$$S''_y = 25 \times 21 = 525 \text{ square inches;}$$

the resultant area in wrought iron, therefore, can be taken at about $S''_{w.i.} = 500$ square inches.

The cross-section at centre of polepieces is:

$$34\frac{1}{4} \times 11\frac{7}{8} = 405 \text{ square inches,}$$

and the vertical cross-section is:

$$34\frac{1}{4} \times 26 = 885 \text{ square inches.}$$

Increasing the minimal area by one-third of the difference between the maximum and minimum area, we obtain:

$$S''_{c.i.} = 405 + \frac{885 - 405}{3} = 565 \text{ square inches,}$$

which we will take as the resultant area of the circuit in cast iron.

The lengths of the magnetic circuit are:
in wrought iron, $l''_{w.i.} = 120$ inches; in cast iron, $l''_{c.i.} = 36$ inches.
By (213), p. 261, we consequently have the equation:

$$120 \times m''_{w.i.} + 36 \times m''_{c.i.} = 18{,}640,$$

which is satisfied by

$$\Phi' = 37{,}500{,}000,$$

for, by employing this value of Φ', we obtain:

$$\mathfrak{B}''_{w.i.} = \frac{\Phi'}{S''_{w.i.}} = \frac{37{,}500{,}000}{500} = 75{,}000; \; m''_{w.i.} = 24.7;$$

$$\mathfrak{B}''_{c.i.} = \frac{\Phi'}{S''_{c.i.}} = \frac{37{,}500{,}000}{565} = 66{,}300; \; m''_{c.i.} = 436;$$

therefore, the left member of the above equation becomes:

$$120 \times 24.7 + 36 \times 436 = 2960 + 15{,}700 = 18{,}600,$$

which is practically identical with the actual number of ampere-turns.

Hence, the actual leakage factor:

$$\lambda = \frac{37{,}500{,}000}{30{,}500{,}000} = \mathbf{1.23.}$$

In this instance, the probable value obtained is about 2½ per cent. in excess of the actual value.

145. Leakage Calculation for a Toothed Ring, One-Material Frame, Multipolar Dynamo:

360 KW "Thomson-Houston" Fourpolar Railway Generator.[1]

600 Volts. 600 Amps. 375 Revs. per Min.

a. Probable Leakage Factor.—(From Figs. 365 and 366).
Effective total length of armature conductor:

$$L_e = 90 \times 4 \times \frac{25}{12} \times \frac{2 \times 82^\circ}{180} = 683 \text{ feet.}$$

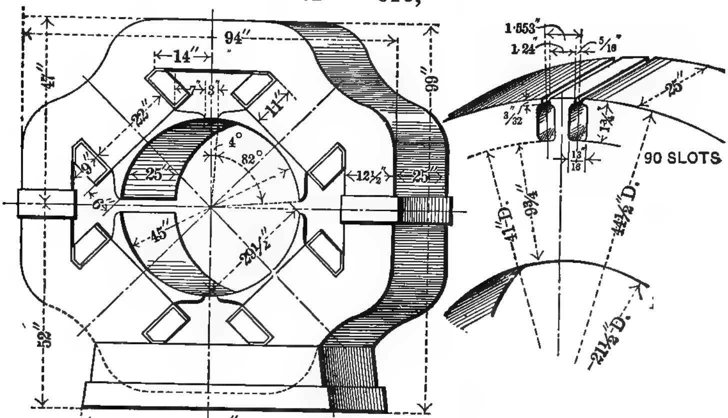
[1] This machine, but bored for a 48-inch armature, is used in the power station of the West-End Railway Company of Boston, Mass.; for description see *Electrical Engineer*, vol. xii. p. 456 (October 21, 1891).

Conductor velocity:

$$v_c = \frac{44\frac{1}{2} - 1\frac{3}{4} - \frac{1}{4}) \times \pi}{12} \times \frac{375}{60} = 70 \text{ feet per second.}$$

The total E. M. F. is

$$E' = 620,$$



Figs. 365 and 366.—360-KW Thomson-Houston Fourpolar Railway Generator.

hence, by (144), p. 205:

$$\mathfrak{H}'' = \frac{2 \times 620 \times 10^8}{72 \times 683 \times 70} = 36{,}000 \text{ lines per square inch.}$$

$$\therefore \; v_c \times \mathfrak{H}'' = 70 \times 36{,}000 = 2{,}520{,}000.$$

Ratio of radial clearance between armature and field to pitch of slots:

$$\frac{.25}{1.553} = .16;$$

therefore, by Table LXVI., p. 225: $k_{12} = 1.4$, and by Table LXVII., p. 230: $k_{12} = 2.2$; average: $k_{12} = 1.8$.

Hence, by (175), p. 230:

$$\mathfrak{P}_1 = \frac{\frac{1}{4}[45 \times \pi + (1.24 + .094) \times 90] \times \frac{2 \times 82^\circ}{180} \times 25}{1.8 \times (45 - 44\frac{1}{2})}$$

$$= \frac{1490}{.9} = 1656.$$

By (181), p. 233:

$$\mathfrak{P}_2 = \frac{25 \times (11 + 14)}{9} + 4 \times \frac{11 \times \frac{22}{2} + 14 \times 12\frac{1}{2}}{9 \times (11 + 12\frac{1}{2}) \times \frac{\pi}{4}}$$

$$= 69.5 + 43 = 112.5.$$

By (197), p. 243:

$$\mathfrak{P}_3 = 4 \times \left(\frac{7 \times 25}{3 + 7 \times \frac{\pi}{2}} + \frac{\frac{1}{2}(22 + 29\frac{1}{2}) \times \frac{1}{2}(1 + 6)}{3 + 13 \times \frac{\pi}{2}} + \frac{2 \times 25}{3}\right)$$

$$= 132.5.$$

$$\therefore \lambda = \frac{1656 + 112.5 + 132.5}{1656} = \frac{1901}{1656} = 1.15.$$

Ratio of width of slot to pitch:

$$\tfrac{13}{16} : 1.553 = .523.$$

for which Table LXV., p. 219, gives a factor of armature leakage of

$$\lambda_1 = 1.05;$$

hence, the total probable leakage coefficient:

$$\lambda' = 1.05 \times 1.15 = \mathbf{1.21.}$$

b. Actual Leakage Factor.—

The machine is compound-wound, having 16,600 shunt ampere-turns and 5500 series ampere-turns on each magnet; the total exciting power per circuit, two coils being magnetically in series, therefore, is:

$$AT = 2 \times (16,600 + 5500) = 44,200 \text{ ampere-turns.}$$

By (228), p. 339:

$$at_g = .3133 \times 36,000 \times .9 = 10,140 \text{ ampere-turns.}$$

By (232), p. 341:

$$S''_{a_1} = 4 \times 9\tfrac{3}{4} \times 25 \times .85 = 828 \text{ square inches}$$

$$\therefore \mathfrak{B}''_{a_1} = \frac{55,000,000}{828} = 66,500 \text{ lines per square inch.}$$

By (233), p. 341:

$$S''_{a_2} = 4 \times \frac{29\frac{1}{2}}{2} \times 25 \times .85 = 1252 \text{ square inches.}$$

$$\therefore \mathfrak{B}''_{a_2} = \frac{55,000,000}{1252} = 44,000 \text{ lines per square inch.}$$

By (231), p. 341:

$$m''_a = \frac{18.6 + 8.4}{2} = 13.5 \text{ ampere-turns per inch.}$$

By (236), p. 343:

$$l''_a = 31\tfrac{1}{4} \times \pi \times \frac{\frac{90}{2} + 4^\circ}{360} + 9\tfrac{3}{4} + 2 \times 1\tfrac{3}{4} = 26\tfrac{1}{2} \text{ inches.}$$

By (230), p. 340:

$$at_a = 13.5 \times 26\tfrac{1}{2} = 360 \text{ ampere-turns.}$$

The shunt current is 16 amperes, and the angle of brush lead, by measurement, about 5°, hence by (250), p. 352:

$$at_r = 2 \times 360 \times \frac{616}{4} \times \frac{5^\circ}{180} = 3100 \text{ ampere-turns.}$$

The magnetizing force left for the magnet frame, consequently, is:

$$at_m = 44,200 - (10,140 + 360 + 3100) = 30,600 \text{ ampere-turns.}$$

The magnet frame is of cast iron; each circuit has a length of $l''_m = 90$ inches; the total cross-section of the cores is:

$$2 \times 22 \times 25 = 1100 \text{ square inches,}$$

and that of the yokes:

$$4 \times 12\tfrac{1}{2} \times 25 = 1250 \text{ square inches.}$$

Taking

$$S''_m = 1125 \text{ square inches}$$

as the resultant sectional area, the value of Φ' is found as follows:

$$90 \times m''_m = 30,600;$$

$$m''_m = \frac{30,600}{90} = 340 \text{ ampere-turns per inch;}$$

$$\mathfrak{B}''_m = \frac{\Phi'}{1125} = 62,500 \text{ lines per square inch;}$$

$$\Phi' = 1125 \times 62,500 = 70,500,000 \text{ maxwells.}$$

The useful flux is:

$$\Phi = \frac{2 \times 6 \times 620 \times 10^9}{360 \times 375} = 55,000,000 \text{ maxwells,}$$

consequently, the actual leakage factor:

$$\lambda' = \frac{70,500,000}{55,000,000} = 1.28.$$

The formula for the probable leakage factor, for this machine, gave a value of $5\frac{1}{2}$ per cent. below this actual figure.

146. Calculation of a Series Motor for Constant Power Work:

Inverted Horseshoe Type. Toothed-Drum Armature. Wrought-Iron Cores and Polepieces, Cast-Iron Yoke.

25 HP. 210 Volts. 850 Revs. per Min.

a. Conversion into Generator of Equal Electrical Activity.—

Assuming a gross efficiency of 93 per cent., and an electrical efficiency of 95 per cent. (see Table XCIX., p. 422), the electrical energy active in the armature of the motor is, by (382), p. 420:

$$P' = \frac{746 \times 25}{.93} = 20,000 \text{ watts.}$$

and the E. M. F. active, by (383), p. 421:

$$E' = 210 \times .95 = 200 \text{ volts;}$$

hence, by (384), p. 421, the current capacity:

$$I' = \frac{20,000}{200} \times 100 \text{ amperes,}$$

which, in the present case of a series motor, is also the current intensity to be supplied to the motor terminals.

Intake of motor, by (381), p. 420:

$$P_2 = \frac{20{,}000}{.95} = \mathbf{21{,}000} \text{ watts.}$$

b. Calculation of Armature.—

According to § 146, *a*, the armature has to be designed to give a total E. M. F. of 200 volts and a total current of 100 amperes, at a speed of 850 revolutions. For the reason advanced on p. 63, a toothed armature with its projections highly saturated at full load is chosen. In order to obtain high efficiencies at small loads, the armature, as explained in § 116, must overpower the field, and therefore a low conductor velocity and a small field density must be taken:

$\beta_1 = .75$; $e = 62.5 \times 10^{-8}$ volt; $v_c = 40$ feet per second;

$\mathcal{H}'' = 20{,}000$ lines per square inch.

By (26), p. 55:

$$L_a = \frac{200 \times 10^8}{62.5 \times 40 \times 20{,}000} = \mathbf{400} \text{ feet.}$$

By (27), p. 57:

$$\delta_a^2 = 300 \times 100 = \mathbf{30{,}000} \text{ circular mils.}$$

2 No. 8 B. & S. (.128″ + .016″) have $2 \times 16{,}510 = 33{,}020$ circular mils area.

By (30), p. 58:

$$d'_a = 230 \times \frac{40}{850} = 10\tfrac{13}{16} \text{ inches.}$$

Approximate size of slot, by Table XV., p. 70:

$$\tfrac{7}{8}'' \times \tfrac{19}{64}''.$$

12 No. 8 B. & S. wires, arranged in 6 layers (see Fig. 368) with .020″ slot-lining give a slot,

$$\tfrac{15}{16}'' \times \tfrac{21}{64}''.$$

Making the pitch ½ inch, the number of slots is obtained, Fig. 367:

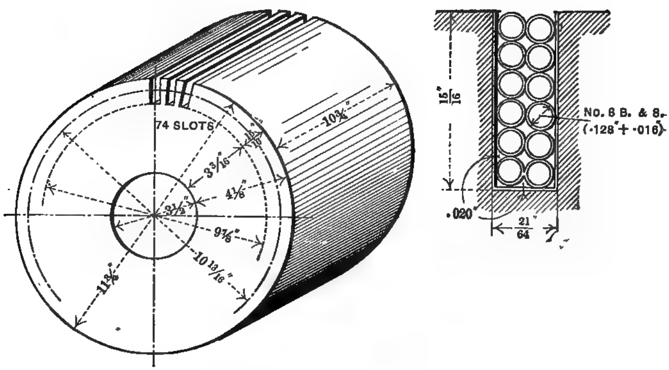
$$n'_c = \frac{11\tfrac{3}{4} \times \pi}{\tfrac{1}{2}} = 74.$$

Hence, by (40), p. 76:

$$l_a = \frac{400 \times 12}{74 \times 6} = \mathbf{10\tfrac{3}{4}} \text{ inches;}$$

and by (138), p. 202:

$$\Phi = \frac{6 \times 200 \times 10^9}{74 \times 6 \times 850} = \mathbf{3{,}180{,}000} \text{ maxwells;}$$



Figs. 367 and 368.—Dimensions of Armature Core, 25-HP Inverted Horseshoe Type Series Motor.

making the maximum density in the teeth at full load:

$$\mathfrak{B}''_t = \frac{3{,}180{,}000}{\left(\frac{9\frac{7}{8} \times \pi}{74} - \frac{21}{64}\right) \times 74 \times 10\tfrac{3}{4} \times .90 \times \frac{.75}{2}}$$

$$= \mathbf{130{,}000} \text{ lines per square inch.}$$

The shape-ratio of the armature core is:

$$l_a : d_a = \frac{10\frac{3}{4}}{9\frac{7}{8}} = 1.1,$$

therefore by Table XXIV., p. 96:

$$L_t = 2.90 \times 400 = 1160 \text{ feet;}$$

whence:

$$wt_a = .00000303 \times 33{,}020 \times 1160 = \mathbf{116} \text{ lbs.}$$

and

$$r_a = \frac{1}{4 \times 2} \times 1160 \times .000626 = \mathbf{.092} \text{ ohm, at } 15.5^\circ \text{ C.}$$

c. Energy Losses in Armature, and Temperature Increase.—Shaft diameter, by (123), p. 185:

$$d_c = 1.3 \times \sqrt[4]{\frac{20,800}{850}} = 3 \text{ inches;}$$

internal diameter of discs:

$3\frac{1}{2}$ inches.

$S''_{a_1} = (9\frac{7}{8} - 3\frac{1}{2}) \times 10\frac{3}{4} \times .90 = 61.7$ square inches.

$S''_{a_2} = 11\frac{3}{4} \times 10\frac{3}{4} \times .90 = 113.5$ square inches.

$\mathfrak{B}''_{a_1} = \frac{3,180,000}{61.7} = 51,500$; $\mathfrak{B}''_{a_2} = \frac{3,180,000}{113.5} = 28,000$;

$m''_a = \frac{10 + 5.1}{2} = \mathbf{7.5}$ ampere-turns per inch.

Average density:

$\mathfrak{B}''_a = \mathbf{40,000}$ lines per square inch.

By (72), p. 112:

$$M = \frac{\left(\frac{1}{2} \times (11\frac{3}{4} + 3\frac{1}{2}) \times \pi \times 4\frac{1}{8} - 74 \times \frac{15}{16} \times \frac{21}{64}\right) \times 10\frac{3}{4} \times .9}{1728}$$

$= .427$ cubic inch.

$N_1 = \frac{850}{60} = 14.2$ cycles per second.

By (68), p. 109:

$$P_a = 1.2 \times 104^2 \times .092 = 1194 \text{ watts.}$$

By (73), p. 112:

$$P_h = 11.55 \times 14.2 \times .427 = 70 \text{ watts.}$$

By (76), p. 120:

$$P_e = .046 \times 14.2^2 \times .427 = 4 \text{ watts.}$$

By (65), p. 107:

$$P_A = 1194 + 70 + 4 = \mathbf{1268} \text{ watts.}$$

By (78), p. 125:

$S_A = 11\frac{3}{4} \times \pi \times [10\frac{3}{4} + 1.8 \times (.5 \times 11\frac{3}{4} + 2 \times \frac{15}{16})]$

$= \mathbf{913}$ square inches.

Ratio of pole area to radiating surface:

$$\frac{11\frac{7}{8} \times \pi \times 10\frac{3}{4} \times .75}{913} = .33;$$

for this ratio, and for $v_c = 40$ feet per second, Table XXXVI., p. 127, gives:

$$\theta'_a = 44° \text{ C.},$$

hence $\theta_a = 44 \times \frac{1268}{913} = \mathbf{61° \text{ C.}}$

$\therefore \quad r'_a = .092 \times (1 + .004 \times 61) = \mathbf{.115}$ ohm, at $76.5°$ C.

d. Dimensioning of Magnet Frame.

In order to secure a small excitation, the density in the wrought iron is taken:

$$\mathfrak{B}''_{\text{w.i.}} = 75{,}000 \text{ lines per square inch;}$$

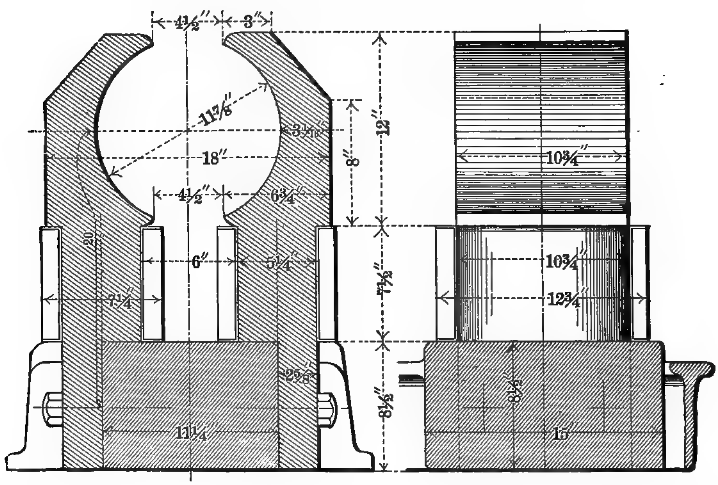


Fig. 369.—Dimensions of Magnet Frame, 25 HP Inverted Horseshoe Type Series Motor.

and that in the cast iron:

$$\mathfrak{B}''_{\text{c.i.}} = 30{,}000 \text{ lines per square inch.}$$

$$\Phi' = 1.20 \times 3{,}180{,}000 = 3{,}820{,}000 \text{ maxwells.}$$

$$S''_{\text{w.i.}} = \frac{3{,}820{,}000}{75{,}000} = \mathbf{51} \text{ square inches.}$$

$$S_{\text{c.i.}} = \frac{3,820,000}{30,000} = \mathbf{127} \text{ square inches.}$$

Cross-section of cores, rectangle, $5\frac{1}{2}'' \times 5\frac{1}{4}''$, between two semi-circles of $5\frac{1}{4}''$ diameter; (Figs. 369 and 370):

$$5\tfrac{1}{2} \times 5\tfrac{1}{4} + 5\tfrac{1}{4}^2 \times \frac{\pi}{4} = \mathbf{50.5} \text{ square inches.}$$

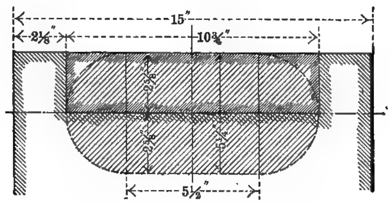


Fig. 370.—Joint of Magnet Core and Yoke, 25 HP Inverted Horseshoe Type Series Motor.

Length of cores, by Table LXXXIII., p. 321:

$$l_m = \mathbf{7\tfrac{1}{2}} \text{ inches.}$$

Cross-section of yoke: $\mathbf{15'' \times 8\frac{1}{2}''}$ (= 127.5 square inches).
Core projection, rectangular: $10\frac{3}{4} \times 2\frac{5}{8} \times 8\frac{1}{2}$.
Area of contact of same with yoke, Fig. 370:

$$(10\tfrac{3}{4} + 2 \times 2\tfrac{5}{8}) \times 8\tfrac{1}{2} + \frac{50.5}{2} = \mathbf{160} \text{ square inches.}$$

Polepieces:

$$d_p = 11\tfrac{3}{4} + 2 \times \tfrac{1}{8} = \mathbf{12} \text{ inches.}$$

$$l'_p = 12 \times \sin 22\tfrac{1}{2}^\circ = \mathbf{4\tfrac{1}{2}} \text{ inches.}$$

e. Calculation of Magnetic Leakage.

Width of tooth:

$$b_t = \frac{11\frac{3}{4} \times \pi}{74} - \tfrac{21}{64} = .5 - .328 = .172 \text{ inch.}$$

Ratio of radial clearance to pitch:

$$\frac{\frac{1}{8}}{.5} = .25.$$

Product of field density and conductor velocity:

$$\mathcal{H}'' \times v_c = 20,000 \times 40 = 800,000.$$

By (169), p. 227:

$$\mathfrak{P}' = \frac{\frac{1}{4}\,(12 \times \pi \times .75 + 1.5 \times .172 \times 74 \times .88) \times 10\frac{3}{4}}{1.7 \times (12 - 11\frac{3}{4})}$$

$$= \frac{120}{.45} = 257.$$

By (170), p. 228:

$$\mathfrak{P}'' = \frac{(9\frac{7}{8} \times \pi - 74 \times \frac{21}{64})}{\frac{15}{16}} \times 10\frac{3}{4} \times .90 \times .75 \times 30 = 1370.$$

By (171), p. 228:

$$\mathfrak{P}''' = 74 \times .75 \times \frac{\frac{21}{64} \times 10\frac{3}{4}}{\frac{15}{16}} = 209$$

By (168), p. 227:

$$\mathfrak{P}_1 = \frac{257 \times (1370 + 209)}{257 + 1370 + 209} = \mathbf{221.}$$

By (179) p. 232:

$$\mathfrak{P}_2 = \frac{5\frac{1}{2} \times 7\frac{1}{2}}{2 \times 6} + \frac{5\frac{1}{4} \times \pi \times 7\frac{1}{2}}{2 \times 6 + 1.5 \times 5\frac{1}{4}} = 3.5 + 6.3 = \mathbf{9.8.}$$

By (192), p. 241:

$$\mathfrak{P}_3 = \frac{3 \times 10\frac{3}{4}}{4\frac{1}{2} + 3 \times \frac{\pi}{2}} + \frac{2 \times 12 \times 3\frac{1}{16}}{4\frac{1}{2} + 6\frac{3}{4} \times \frac{\pi}{2}} = 3.5 + 4.8 = \mathbf{8.3.}$$

By (202), p. 246:

$$\mathfrak{P}_4 = \frac{10\frac{3}{4} \times 6\frac{3}{4} - 50.5}{7\frac{1}{2}} + \frac{10\frac{3}{4} \times 8}{7\frac{1}{2} + (8 + 8\frac{1}{2}) \times \frac{\pi}{2}} = 3 + 4.3 = \mathbf{7.3}$$

$$\lambda = \frac{221 + 9.8 + 8.3 + 7.3}{221} = \frac{246.4}{221} = 1.12.$$

$$\lambda' = 1.05 \times 1.12 = \mathbf{1.18.}$$

f. Calculation of Magnetizing Forces.

at_g = .3133 × 20,000 × .45 = **2820** ampere-turns.
at_a = 7.5 × 12 = **90** ampere-turns.

$at_{\text{w.i.}} = 24.7 \times 40 =$ **990** ampere-turns.
$at_{\text{c.i.}} = 50 \times 11\frac{1}{4} =$ **590** ampere-turns.

$$at_{\text{r}} = 1.25 \times \frac{74 \times 6 \times 100}{2} \times \frac{.73 \times 22\frac{1}{2}}{180} = \mathbf{2530} \text{ ampere-turns.}$$

$$AT = 2820 + 90 + 990 + 590 + 2530 = \mathbf{7000} \text{ ampere-turns.}$$

g. Calculation of Magnet Winding.

The total magnetizing force being kept exceedingly low, a very small winding depth will be sufficient to accommodate the winding. Taking $h_{\text{m}} = 1$ inch, formula (291), p. 374, will give the mean length of one turn:

$$l_{\text{T}} = 2 \times 5\tfrac{1}{2} + (5\tfrac{1}{4} + 1) \times \pi = 31 \text{ inches.}$$

$$N_{\text{se}} = \frac{7000}{52} = 135 \text{ turns, total, or } \mathbf{68} \text{ turns per core.}$$

$$L_{\text{se}} = \frac{68 \times 31}{12} = \mathbf{176} \text{ feet per core.}$$

$$S_{\text{M}} = 2 \times 7\tfrac{1}{2} \times \left[5\tfrac{1}{2} \times \left(\frac{5\frac{1}{4}}{2} + 1\right) \times \pi\right] = 253 \text{ sq. in. p. magnet.}$$

For a rise of 20° C., we find:

$$r_{\text{se}} = \frac{20}{75} \times \frac{253}{52^2} \times \frac{1}{1 + .004 \times 20} = \mathbf{.0231} \text{ ohm per core.}$$

$$\lambda_{\text{se}} = \frac{176}{.0231} = \mathbf{7620} \text{ feet per ohm.}$$

The nearest gauge wire is No. **2** B. W. G. (.284″ + .020″), with a specific length of 7813 feet per ohm. The number of turns of this wire filling one layer on the cores is:

$$\frac{7\frac{1}{2} - \frac{1}{2}}{.284 + .020} = \frac{7}{.304} = \mathbf{23};$$

therefore, the number of layers required:

$$\frac{68}{23} = \mathbf{3}.$$

Actual winding depth:

$$h'_{\text{m}} = 3 \times .304 = \mathbf{.912} \text{ inch.}$$

Actual excitation:

$$AT = 2 \times 23 \times 3 \times 52 = \mathbf{7176} \text{ ampere-turns.}$$

Joint series resistance, warm:

$$r'_{se} = \frac{.0231}{2} \times (1 + .004 \times 20) = \mathbf{.0125} \text{ ohm, at } 35.5° \text{ C.}$$

Total weight of wire, bare:

$$wt_{se} = 2 \times 23 \times 3 \times \frac{31}{12} \times .244 = \mathbf{87} \text{ lbs.}$$

h. Speed Calculations.—

E. M. F. lost in armature and series winding:

$$100 \times (.115 + .0125) = 12.75 \text{ volts.}$$

Actual E. M. F. active in armature:

$$E' = 210 - 12.75 = \mathbf{198.25} \text{ volts.}$$

Torque, by (93), p. 138:

$$\tau = \frac{11.74}{10^{10}} \times 100 \times 444 \times 3,180,000 = 166 \text{ foot-lbs.}$$

Specific generating power:

$$e'' = \frac{198.25 \times 60}{850} = 14 \text{ volts at 1 revolution per second;}$$

hence, by (389), p. 426, the speed at any supply voltage, E:

$$N_2 = 60 \times \left(\frac{E}{14} - 8.52 \times \frac{(.115 + .0125) \times 166}{14^2}\right)$$
$$= \mathbf{4.3\ E - 53.}$$

For $E = \mathbf{210}$ volts: $N_2 = 903 - 53 = \mathbf{850}$ revs. per min.

" $E = \mathbf{200}$ volts: $N_2 = 860 - 53 = \mathbf{807}$ revs. per min., etc.

i. Calculation of Efficiencies.—

Electrical efficiency, at normal load:

$$\eta_e = \frac{200 \times 100 - (.115 + .0125) \times 100^2}{200 \times 100} = .94, \text{ or } \mathbf{94\%}.$$

Commercial efficiency at normal load:

$$\eta_c = \frac{200 \times 100 - (100^2 \times .1275 + 74 + 1500)}{200 \times 100}$$

$$= \frac{20,000 - (1275 + 1574)}{20,000} = \frac{17,151}{20,000} = \mathbf{.86.}$$

Commercial efficiency, at $\frac{3}{4}$ load (the energy loss in armature and series-field windings varying practically as the square of the load, and hysteresis and friction losses being independent of the load):

$$\eta_c = \frac{\frac{3}{4} \times 20{,}000 - [(\frac{3}{4})^2 \times 1275 + 1574]}{\frac{3}{4} \times 20{,}000} = \frac{12{,}709}{15{,}000} = \mathbf{.85.}$$

Commercial efficiency, at $\frac{1}{2}$ load:

$$\eta_c = \frac{\frac{1}{2} \times 20{,}000 - [(\frac{1}{2})^2 \times 1275 + 1574]}{\frac{1}{2} \times 20{,}000} = \frac{8107}{10{,}000} = \mathbf{.81.}$$

Commercial efficiency at $\frac{1}{4}$ load:

$$\eta_c = \frac{\frac{1}{4} \times 20{,}000 - [(\frac{1}{4})^2 \times 1275 + 1574]}{\frac{1}{4} \times 20{,}000} = \frac{3346}{5000} = \mathbf{.67.}$$

Commercial efficiency, at 50 per cent. overload:

$$\eta_c = \frac{1\frac{1}{2} \times 20{,}000 - [(1\frac{1}{2})^2 \times 1275 + 1574]}{1\frac{1}{2} \times 20{,}000} = \frac{25{,}556}{30{,}000} = \mathbf{.85.}$$

The latter is lower than the efficiency at normal load.

147. Calculation of a Shunt Motor for Intermittent Work:

Bipolar Iron Clad Type. Smooth Ring Armature. Cast-Iron Frame.

15 HP. 125 Volts. 1400 Revs. per Min.

a. Conversion into Generator of Equal Electrical Activity.—

Assuming an efficiency of 89%, the electrical activity is:

$$P' = \frac{746 \times 15}{.89} = \mathbf{12{,}600} \text{ watts.}$$

From Table VIII., p. 56:

$$E' = 125 - .06 \times 125 = \mathbf{117.5} \text{ volts.}$$

Current in armature, at full load, by (384), p. 421:

$$I' = \frac{12,600}{117.5} = \mathbf{107} \text{ amperes.}$$

Intake, by (381), p. 420:

$$P_2 = \frac{12,600}{.90} = \mathbf{14,000} \text{ watts.}$$

b. Calculation of Armature.—

In this case we want a weak armature of few ampere-turns and a strong field with large exciting power. Hence the conductor velocity and the field density must both be taken very high:

$\beta_1 = .80$; $e = 65 \times 10^{-8}$ volt p. ft.; $v_c = 92.5$ feet p. second; $\mathcal{H}'' = 26,000$ lines per square inch.

$$L_a = \frac{117.5 \times 10^8}{65 \times 92.5 \times 26,000} = \mathbf{76} \text{ feet.}$$

$$\delta_a^2 = 250 \times 107 = \mathbf{26,800} \text{ circular mils.}$$

2 No. **11** B. W. G. (.120″ + .016″) have a sectional area of $\delta^2_a = 2 \times 14,400 = \mathbf{28,800}$ circular mils.

$$d'_a = 230 \times \frac{92.5}{1400} = 15.3 \text{ inches.}$$

By Table IX., p. 59:

$$d_a = 15.3 \times .98 = \mathbf{15} \text{ inches.}$$

Allowing $7\frac{1}{2}$ inches for 50 division strips of .15″ width, we have:

$$N_w = \frac{15 \times \pi - 7\frac{1}{2}}{.120 + .016} = \frac{39.6}{.136} = \mathbf{288} \text{ wires, or } \mathbf{144} \text{ conductors per layer.}$$

$$l_a = \frac{76 \times 12}{144} = \mathbf{6\tfrac{3}{8}} \text{ inches.}$$

$$\Phi = \frac{6 \times 117.5 \times 10^9}{144 \times 1400} = \mathbf{3,500,000} \text{ maxwells.}$$

$$b_a = \frac{3,500,000}{2 \times 107,500 \times 6\frac{3}{8} \times .85} = \mathbf{3} \text{ inches.}$$

$$b'_a = 3 \times \sqrt{\frac{15}{3} - 1} = \mathbf{6} \text{ inches.}$$

$$\mathfrak{B}''_{a_1} = \frac{3,500,000}{2 \times 6\frac{3}{8} \times 3 \times .85} = 107,300 \text{ lines per square inch;}$$

$$\mathfrak{B}''_{a_2} = \frac{3,500,000}{2 \times 6\frac{3}{8} \times 6 \times .85} = 53,700 \text{ lines per square inch;}$$

$$m''_a = \frac{186 + 10.7}{2} = 98.4 \text{ ampere-turns per inch.}$$

Average density: $\mathfrak{B}''_a = \mathbf{101,000}$ lines per square inch.

$$L_t = \frac{2 \times (6\frac{3}{8} + 3) + \frac{1}{4} \times \pi}{6\frac{3}{8}} \times 76 = 233 \text{ feet.}$$

$$wt_a = 233 \times 2 \times .0436 = \mathbf{20\tfrac{1}{2}} \text{ lbs.}$$

$$r_a = \frac{1}{4 \times 2} \times 233 \times .00072 = \mathbf{.021} \text{ ohm, at } 15.5^\circ \text{ C.}$$

c. Energy Losses in Armature, and Temperature Increase.—

$$M = \frac{12 \times \pi \times 6\frac{3}{8} \times 3 \times .85}{1728} = .355 \text{ cubic foot.}$$

$$N_1 = \frac{1400}{60} = 23.33 \text{ cycles per second.}$$

$$P_a = 1.2 \times 107^2 \times .021 = 290 \text{ watts.}$$

$$P_h = 50.8 \times 23.33 \times .355 = 363 \text{ watts.}$$

$$P_e = .295 \times 23.33^2 \times .355 = 57 \text{ watts.}$$

$$P_A = 290 + 363 + 57 = \mathbf{710} \text{ watts.}$$

$$S_A = 2 \times 12 \times \pi \times (6\tfrac{3}{8} + 3 + 4 \times \tfrac{1}{4}) = \mathbf{780} \text{ sq. inches.}$$

$$\theta_a = 42 \times \frac{710}{780} = \mathbf{38^\circ} \text{ C.}$$

$$r'_a = .021 \times (1 + .004 \times 38) = \mathbf{.024} \text{ ohm, at } 53.5^\circ \text{ C.}$$

d. Dimensioning of Magnet Frame.—

$$\Phi' = 1.15 \times 3,500,000 = \mathbf{4,025,000} \text{ maxwells.}$$

$$S''_m = \frac{4,025,000}{42,500} = \mathbf{95} \text{ square inches.}$$

Breadth of cores:

$$\frac{95}{6\frac{3}{8}} = \mathbf{15} \text{ inches.}$$

Breadth of polepieces:

$$15\tfrac{3}{4} \times \sin 72^\circ = \mathbf{15} \text{ inches.}$$

These two dimensions being equal, no separate polepieces are required, and the frame may be cast in one piece, as shown in Fig. 371.

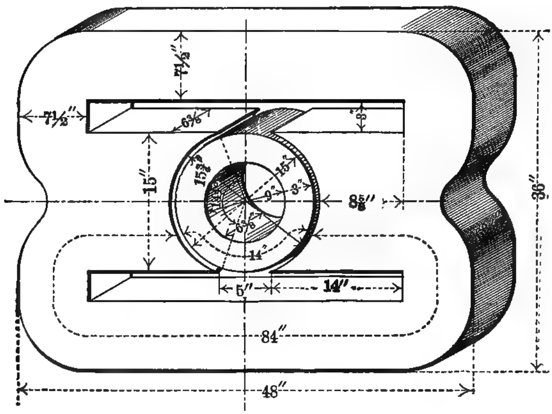


Fig. 371.—Dimensions of Armature Core and Field-Magnet Frame, 15-HP Bipolar Iron-Clad Type Shunt Motor.

e. Calculation of Magnetizing Forces.—

$at_g = .3133 \times 26{,}008 \times 1.2 \times \tfrac{3}{4} = \mathbf{7320}$ ampere-turns.

$at_a = 98.4 \times 14 = \mathbf{1380}$ ampere-turns.

$at_m = 101 \times 83 = \mathbf{8300}$ ampere-turns.

$$at_r = 1.80 \times \frac{144 \times 107}{2} \times \frac{18^\circ}{180} = \mathbf{1400} \text{ ampere-turns.}$$

$AT = 7320 + 1380 + 8300 + 1400 = \mathbf{18{,}400}$ ampere-turns.

f. Calculation of Magnet Winding.—

Since the motor is not intended for continuous work, a high increase, $\theta_m = 40^\circ$ C., is permitted. Regulating resistance, at full load, $r_x = 23$ per cent.

Height of winding space, estimated: $2\tfrac{1}{2}$ inches.

$l_T' = 2\,(6\tfrac{3}{8} + 15) + 2\tfrac{1}{2} \times \pi = \mathbf{50}$ inches.

$$\lambda_{sh} = \frac{18,400}{125} \times \frac{50}{12} \times 1.23 \times (1 + .004 \times 40)$$
$$= \mathbf{874} \text{ feet per ohm,}$$

corresponding to No. **13** B. W. G. (.095″ + .010″).

$$S_M = 2 \times (6\tfrac{3}{8} + 15 + 2\tfrac{1}{2}\,\pi) \times 2 \times 8\tfrac{1}{2} = \mathbf{1000} \text{ sq. inches.}$$

$$P'_{sh} = \frac{40}{75} \times 1000 \times 1.23 = \mathbf{655} \text{ watts.}$$

$$N_{sh} = \frac{18,400 \times 125}{655} = \mathbf{3500} \text{ turns, total, or } \mathbf{1750} \text{ per core.}$$

Number of turns per layer:

$$\frac{8\tfrac{1}{2}}{.095 + .010} = \mathbf{80};$$

Number of layers required:

$$\frac{1750}{80} = \mathbf{22}.$$

Depth of magnet winding:

$$h'_m = 22 \times (.095 + .010) = \mathbf{2.32} \text{ inches.}$$

$$L_{sh} = 2 \times 80 \times 22 \times \frac{50}{12} = \mathbf{14,650} \text{ feet.}$$

$$r_{sh} = \frac{14,650}{874} = \mathbf{16\ 75} \text{ ohms, shunt resistance, at } 15.5^\circ \text{ C.}$$

$$r'_{sh} = 16.75 \times (1 + .004 \times 40)$$
$$= \mathbf{19.45} \text{ ohms, shunt resistance, at } 55.5^\circ \text{ C.}$$

$$r''_{sh} = 19.45 \times 1.23$$
$$= \mathbf{23.9} \text{ ohms, res. of entire shunt-circuit, at full load.}$$

$$I_{sh} = \frac{125}{23.9} = \mathbf{5.23} \text{ amperes, shunt current, at normal load.}$$

Total actual magnetizing force:

$$AT = 2 \times 80 \times 22 \times 5.23 = \mathbf{18,400} \text{ ampere-turns.}$$

Total weight, bare:

$$wt_{sh} = \frac{16.75}{.0419} = \mathbf{400} \text{ lbs., or } \mathbf{200} \text{ lbs. per core.}$$

g. Speed Calculations.—

E. M. F. consumed by armature winding:

$$107 \times .024 = \mathbf{2.6} \text{ volts.}$$

E. M. F. active in armature:

$$E' = 125 - 2.6 = \mathbf{122.4} \text{ volts.}$$

Torque:

$$\tau = \frac{11.74}{10^{10}} \times 107 \times 144 \times 3,500,000 = \mathbf{63.3} \text{ foot-pounds.}$$

Specific generating power:

$$e'' = \frac{122.4 \times 60}{1400} = \mathbf{5.25} \text{ volts.}$$

Speed, at any voltage, E:

$$N_2 = 60 \times \left(\frac{E}{5.25} - 8.52 \times \frac{.024 \times 63.3}{5.25^2}\right) = \mathbf{11.42\,E - 28.}$$

For $E = \mathbf{125}$: $N_2 = 1428 - 28 = \mathbf{1400}$ revs. per minute.
" $E = \mathbf{110}$: $N_2 = 1256 - 28 = \mathbf{1228}$ revs. per minute.
" $E = \mathbf{100}$: $N_2 = 1142 - 28 = \mathbf{1114}$ revs. per minute.

h. Calculation of Current for Various Loads.—

Current for full load, by (392), p. 428:

$$I' = \frac{125 - \sqrt{125^2 - 4 \times .024 \times (746 \times 15 + 2000)}}{2 \times .024}$$
$$= \mathbf{107} \text{ amperes.}$$

Current for $\frac{1}{4}$ load:

$$I' = \frac{125 - \sqrt{125^2 - 4 \times .024 \times (746 \times 15 \times \frac{1}{4} + 2000)}}{2 \times .024}$$
$$= \mathbf{40} \text{ amperes.}$$

Current for $\frac{1}{2}$ load:

$$I' = \frac{125 - \sqrt{125^2 - 4 \times .024 \times (746 \times 15 \times \frac{1}{2} + 2000)}}{2 \times .024}$$
$$= \mathbf{63} \text{ amperes.}$$

Current for $\frac{3}{4}$ load:

$$I' = \frac{125 - \sqrt{125^2 - 4 \times .024 \times (746 \times 15 \times \frac{3}{4} + 2000)}}{2 \times .024}$$

$= \mathbf{86}$ amperes.

Current for 25 per cent. overload:

$$I' = \frac{125 - \sqrt{125^2 - 4 \times .024 \times (746 \times 15 \times 1\frac{1}{4} + 2000)}}{2 \times .024}$$

$= \mathbf{126}$ amperes.

Current for 50 per cent. overload:

$$I' = \frac{125 - \sqrt{125^2 - 4 \times .024 \times (746 \times 15 \times 1\frac{1}{2} + 2000)}}{2 \times .024}$$

$= \mathbf{159}$ amperes.

Current for maximum commercial efficiency, by (393), p. 429:

$$I' = \sqrt{\frac{535 + 2000}{.024} + \left(\frac{535}{125}\right)^2} - \frac{535}{125} = \mathbf{320} \text{ amperes,}$$

from which follows that the maximum commercial efficiency is obtained at about five times the normal load.

Current for maximum electrical efficiency, by (394), p. 429:

$$I' = \sqrt{\frac{535}{.024} + \left(\frac{535}{125}\right)^2} - \frac{535}{125} = \mathbf{145} \text{ amperes,}$$

which corresponds to about $1\frac{1}{2}$ times the normal load.

i. Calculation of Efficiencies.—

Electrical efficiency, at normal load:

$$\eta_e = \frac{122.4 \times 107 - 107^2 \times .024 - 5.23^2 \times 19.45}{122.4 \times 107}$$

$$= \frac{13,100 - 275 - 535}{13,100} = \frac{12,290}{13,100} = .94, \text{ or } \mathbf{94\%}.$$

Commercial efficiency, at normal load:

$$\eta_c = \frac{122.4 \times 107 - (107^2 \times .024 + 535 + 352 + 2000)}{13,100}$$

$$= \frac{10,213}{13,100} = .78 \text{ or } \mathbf{78\%}.$$

Commercial efficiency at $\frac{3}{4}$ load:

$$\eta_c = \frac{122.4 \times 86 - (86^2 \times .024 + 2887)}{122.4 \times 86} = \frac{7455}{10,520} = \mathbf{.71.}$$

Commercial efficiency at $\frac{1}{2}$ load:

$$\eta_c = \frac{122.4 \times 63 - (63^2 \times .024 + 2887)}{122.4 \times 63} = \frac{4738}{7720} = \mathbf{.61.}$$

Commercial efficiency at $\frac{1}{4}$ load:

$$\eta_c = \frac{122.4 \times 40 - (40^2 \times .024 + 2887)}{122.4 \times 40} = \frac{1975}{4900} = \mathbf{.40.}$$

Commercial efficiency at 25 per cent. overload:

$$\eta_c = \frac{122.4 \times 126 - (126^2 \times .024 + 2887)}{122.4 \times 126} = \frac{12,153}{15,420} = \mathbf{.79.}$$

Commercial efficiency at 50 per cent. overload:

$$\eta_c = \frac{122.4 \times 159 - (159^2 \times .024 + 2887)}{122.4 \times 159} = \frac{16,006}{19,500} = \mathbf{.82.}$$

In this case the efficiencies at overload are higher than the normal load efficiency.

148. Calculation of a Compound Motor for Constant Speed at Varying Load:

Radial Outerpole Type. 4 Poles. Toothed Ring Armature. Cast-Steel Frame. 75 HP. 440 Volts. 500 Revs. per Min.

a. Conversion into Generator of Equal Electrical Activity.—

$P' =$ **60,000** watts (by Table XCIX., p. 422).

$E' = 440 - .045 \times 440 =$ **420** volts.

$$I' = \frac{60,000}{420} = \mathbf{143} \text{ amperes.}$$

b. Calculation of Armature.—

$\beta_1 = .70$; $e = 55 \times 10^{-8}$ volt p. ft.; $v_c = 65$ feet p. sec.; $\mathcal{H}'' = 30,000$ lines per square inch.

$$L_a = \frac{2 \times 420 \times 10^8}{55 \times 65 \times 30,000} = \mathbf{785} \text{ feet.}$$

$$\delta_a^2 = 400 \times 143 = \mathbf{57,200} \text{ circular mils.}$$

4 No. 11 B. W. G. wires (.120″ + .016″), have an actual area of:

$$4 \times 14{,}400 = \mathbf{57{,}600} \text{ circular mils.}$$

$$d'_a = 230 \times \frac{65}{500} = \mathbf{29\tfrac{7}{8}} \text{ inches.}$$

For this diameter, Table XV., p. 70, gives a slot of $1\frac{5}{8} \times \frac{7}{16}$ inch; actual slot for 36 No. 11 B. W. G. wires, see Fig. 372, is $\mathbf{1\frac{11}{16}}$ inch deep and $\mathbf{\frac{15}{32}}$ inch wide.

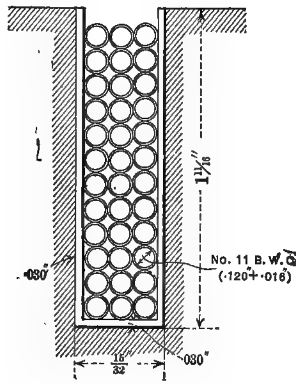


Fig. 372.—Dimensions of Armature-Slot, 75 HP Fourpolar Compound Motor.

Number of slots:

$$n'_c = \frac{(29\frac{7}{8} + 1\frac{11}{16}) \times \pi}{\frac{7}{8}} = \mathbf{112.}$$

$$l_a = \frac{785 \times 12}{112 \times \frac{36}{4}} = \mathbf{9\tfrac{1}{2}} \text{ inches.}$$

$$\Phi = \frac{6 \times 2 \times 420 \times 10^9}{112 \times 9 \times 500} = 10{,}000{,}000 \text{ maxwells.}$$

$$b_a = \frac{10{,}000{,}000}{4 \times 105{,}000 \times 9\frac{1}{2} \times .875} = \mathbf{2\tfrac{7}{8}} \text{ inches.}$$

$$\mathcal{B}''_{a_1} = 105{,}000 \text{ lines; } \mathcal{B}''_{a_2} = 40{,}500 \text{ lines;}$$

$$m''_a = \frac{137 + 7.6}{2} = \mathbf{72.3} \text{ ampere-turns per inch.}$$

Average density:

$$\mathfrak{B}''_a = \mathbf{96{,}000} \text{ lines per square inch.}$$

$$L_t = \frac{2 \times (9\tfrac{1}{2} + 2\tfrac{7}{8}) + 1\tfrac{11}{16} \times \pi}{12} \times 112 \times 9 = 2540 \text{ feet.}$$

$$wt_a = 2540 \times 4 \times .0436 = \mathbf{442} \text{ lbs. bare wire.}$$

$$r_a = \frac{1}{4 \times 4} \times 2540 \times .000717 = \mathbf{.114} \text{ ohm at } 15.5^\circ \text{ C.}$$

c. Energy Losses in Armature, and Temperature Increase.—

$$M = \frac{(27 \times \pi \times 4\tfrac{9}{16} - 1\tfrac{11}{16} \times \tfrac{15}{32} \times 112) \times 9\tfrac{1}{2} \times .875}{1728}$$

$$= 1.44 \text{ cubic foot.}$$

$$N_1 = \frac{500}{60} \times 2 = 16.67 \text{ cycles per second.}$$

$$P_a = 1.2 \times 143^2 \times .114 = \mathbf{2800} \text{ watts.}$$

$$P_h = 46.85 \times 16.67 \times 1.44 = \mathbf{1120} \text{ watts.}$$

$$P_e = .0665 \times 16.67^2 \times 1.44 = \mathbf{30} \text{ watts.}$$

$$P_A = 2800 + 1120 + 30 = \mathbf{3950} \text{ watts.}$$

$$S_A = 27 \times \pi \times 2 \times (9\tfrac{1}{2} + 2\tfrac{7}{8} + 1\tfrac{11}{16} \times \pi = \mathbf{3000} \text{ square inches.}$$

$$\theta_a = 41 \times \frac{3950}{3000} = \mathbf{54^\circ} \text{ C.}$$

$$r'_a = .114 \times (1 + .004 \times 54^\circ) = \mathbf{.139} \text{ ohm, at } 69\tfrac{1}{2}^\circ \text{ C.}$$

d. Dimensioning of Magnet Frame.—(Fig. 373.)

$$\Phi' = 1.20 \times 10{,}000{,}000 = \mathbf{12{,}000{,}000} \text{ maxwells.}$$

Width of frame (equal to length of armature core): **9½** inches.

Breadth of cores:

$$\frac{12{,}000{,}000}{2 \times 80{,}000 \times 9\tfrac{1}{2}} = \mathbf{8} \text{ inches.}$$

Thickness of yoke:

$$\frac{12{,}000{,}000}{4 \times 80{,}000 \times 9\tfrac{1}{2}} = \mathbf{4} \text{ inches.}$$

Length of cores:

$$l_m = \mathbf{7\tfrac{1}{2}} \text{ inches.}$$

Breadth of polepieces:

$$h_p = 32\tfrac{1}{16} \times \sin 31\tfrac{1}{2}^\circ = \mathbf{16\tfrac{1}{2}} \text{ inches.}$$

Distance between pole-corners:

$$l'_p = 32\tfrac{1}{16} \times \sin 13\tfrac{1}{2}^\circ = 7\tfrac{1}{2} \text{ inches.}$$

e. Calculation of Magnetizing Forces.—

The E. M. F. at no load being,

$$E_0 = 440 \text{ volts},$$

and that at full load being

$$E' = 440 - 143 \times .139 \times 1.25 = 415 \text{ volts},$$

the shunt winding is to be calculated to supply the total magnetizing force necessary to produce 440 volts, and the series

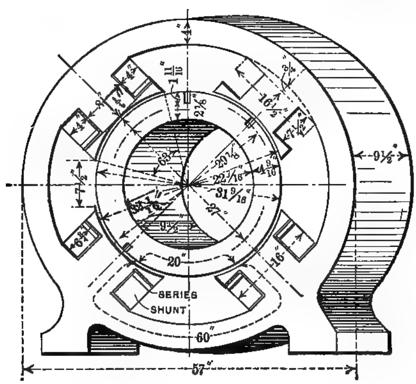


Fig. 373.—Dimensions of Magnet Frame, 75 HP Fourpolar Compound Motor.

winding, in order to regulate for constant speed at all loads, must provide the difference between the magnetizing forces required for 440 and 415 volts, respectively, and must be connected so as to act in opposition to the shunt winding. (*Differential winding.*)

Magnetizing Force Required at No Load:

$$\Phi_0 = \frac{6 \times 2 \times 440 \times 10^9}{112 \times 9 \times 500} = 10,500,000 \text{ maxwells.}$$

$$\mathcal{H}''_0 = \frac{2 \times 440 \times 10^8}{72 \times \dfrac{112 \times 9 \times 9\frac{1}{2}}{12} \times .80 \times 65} = 29,500 \text{ lines p. sq. in}$$

$$\mathcal{H}''_0 \times v_c = 29,500 \times 65 = 1,920,000.$$

Ratio of clearance to pitch:

$$\tfrac{1}{4} \div \frac{31\tfrac{9}{16} \times \pi}{112} = .28.$$

From Table LXVII., p. 230: $k_{12} = 1.90$;

$$\therefore at_{g_0} = .3133 \times 29{,}500 \times \tfrac{1}{2} \times 1.90 = 9000 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{a_{1_0}} = \frac{10{,}500{,}000}{4 \times 9\tfrac{1}{2} \times 2\tfrac{7}{8} \times .875} = 110{,}000 \text{ lines per sq. inch;}$$

$$\mathfrak{B}''_{a_{2_0}} = \frac{10{,}500{,}000}{4 \times 9\tfrac{1}{2} \times 7\tfrac{1}{2} \times .875} = 42{,}500 \text{ lines per sq. inch;}$$

$$\therefore at_{a_0} = \frac{290 + 8}{2} \times 20 = 3000 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{m_0} = \frac{1.2 \times 10{,}500{,}000}{2 \times 8 \times 9\tfrac{1}{2}} = 83{,}000 \text{ lines per square inch;}$$

$$\therefore at_{m_0} = 36.1 \times 60 = 2200 \text{ ampere-turns.}$$

$$\therefore AT_0 = 9000 \times 3000 + 2200 = \mathbf{14{,}200} \text{ ampere-turns.}$$

Magnetizing Force required at Full Load.—

$$\Phi = \frac{6 \times 2 \times 415 \times 10^9}{112 \times 9 \times 500} = \mathbf{9{,}900{,}000} \text{ maxwells.}$$

$$\mathfrak{H}'' = 29{,}500 \times \frac{415}{440} = 27{,}800 \text{ lines per square inch;}$$

$$\therefore at_g = .3133 \times 27{,}800 \times \tfrac{1}{2} \times 1.90 = 8200 \text{ ampere-turns.}$$

$$\mathfrak{B}''_{a_1} = 103{,}700 \text{ lines; } \mathfrak{B}''_{a_2} = 40{,}000 \text{ lines;}$$

$$\therefore at_a = \frac{122.5 + 7.5}{2} \times 20 = 1300 \text{ ampere-turns.}$$

$$\mathfrak{B}''_m = 78{,}300 \text{ lines per square inch;}$$

$$\therefore at_m = 29.3 \times 60 = 1750 \text{ ampere-turns.}$$

$$at_r = 1.25 \times 112 \times 9 \times \frac{143}{4} \times \frac{.40 \times 13\tfrac{1}{2}^{\circ}}{180} = 1350 \text{ amp.-turns.}$$

$$\therefore AT = 8200 + 1300 + 1750 + 1350 = \mathbf{12{,}600} \text{ amp.-turns.}$$

Magnetizing Force for Series Differential Winding.—

$$AT_{se} = AT - AT_0 = 12{,}600 - 14{,}200 = -\mathbf{1600} \text{ amp.-turns.}$$

As seen from the above, the armature reaction, by increasing the excitation needed for full load, in a motor reduces the difference between full load and no load magnetomotive force; and by properly adjusting the magnetizing forces required for the various portions of the magnetic circuit, the difference between the ampere-turns required to overcome the reluctances of the circuit at no load and full load, respectively, can, indeed, be brought within the amount of the armature reactive ampere-turns, so that no series winding at all is needed for regulation, the armature-reaction (which may have to be made extra large for this purpose, either by widening the polepieces or in giving the brushes a greater backward lead) taking its place.

In the present machine, this can be achieved by increasing the radial depth of the armature core from $2\frac{7}{8}''$ to $3\frac{1}{2}''$, whereby the average specific magnetizing force is reduced to $m''_{a_0} = 29.5$ ampere-turns per inch at no load and to $m''_a = 23.5$ ampere-turns per inch at full load, making the corresponding magnetizing forces $at_{a_0} = 590$ ampere-turns and $at_a = 470$ ampere-turns, respectively. Substituting these figures for those in the above calculation, the total exciting power at no load is found $AT_0 = 11,790$ ampere-turns, and at full load $AT = 11,770$ ampere-turns. The remaining small difference of 20 ampere-turns is negligible, and we have then a self-regulating shunt-motor of practically constant speed for all variations of load.

f. Calculation of Magnet Winding.

Series Winding.—

$$N_{se} = \frac{1600}{143} = \mathbf{12} \text{ turns per magnetic circuit,}$$

or **6** turns per core.

Allowing 1000 circular mils per ampere, and taking **2** cables of **7** wires each, the size of each wire is:

$$\delta^2_{a_1} = \frac{1000 \times 143}{2 \times 7} = 10,200 \text{ circular mils,}$$

or No. **10** B. & S. (.102″).

Assuming $h_m = 4$ inches, twelve cables of $3 \times (.102'' + .008'') = .33$ inch diameter will just fill the winding space, and but one layer, axially, is therefore required.

$$l_T = 2 \times (8 + 9\tfrac{1}{2}) + 4 \times \pi = 47\tfrac{1}{2} \text{ inches,}$$

$$wt_{se} = 12 \times 14 \times \frac{47\tfrac{1}{2}}{12} \times .0315 = \mathbf{21} \text{ lbs. per pair of cores,}$$

or **42** lbs. total.

$$r_{se} = 12 \times \frac{47\tfrac{1}{2}}{12} \times \frac{.001}{14} = \mathbf{.0034} \text{ ohm per pair of magnets, at } 15.5° \text{ C.}$$

Shunt Winding.—

For $\theta_m = 25°$ C., and $r_x = 45\%$.

Connecting all four shunt coils in series, the potential across a pair of coils is 220 volts, and the size of the wire required:

$$\lambda_{sh} = \frac{14,200}{220} \times \frac{47\tfrac{1}{2}}{12} \times 1.45 \times (1 + .004 \times 25°)$$
$$= \mathbf{408} \text{ feet per ohm,}$$

which is the specific length of No. **16** B. W. G. wire $(.065'' + .007'')$.

Allowing $\frac{1}{4}''$ of the length of the core for width of bobbin flanges, the radiating surface of one pair of shunt coils is:

$$S_M = 2 \times (8 + 9\tfrac{1}{2} + 4 \times \pi) \times 2 \times (7\tfrac{1}{2} - \tfrac{1}{4})$$
$$= \mathbf{870} \text{ square inches.}$$

$$P_{sh} = \frac{25}{75} \times 870 - 143^2 \times .0034 \times (1 + .004 \times .25)$$
$$= 290 - 77 = \mathbf{213} \text{ watts.}$$

$$P'_{sh} = 213 \times 1.45 = \mathbf{309} \text{ watts.}$$

$$N_{sh} = \frac{14,200 \times 220}{309} = \mathbf{10,100} \text{ turns.}$$

Allowing $\frac{1}{2}$ inch for the series winding and its insulation, the length available for the shunt winding is $6\frac{3}{4}$ inches, which holds:

$$\frac{6\tfrac{3}{4}}{.072} = 94 \text{ No. 16 B. W. G. wires;}$$

hence, the height actually occupied by the shunt winding:

$$h'_m = \frac{10,100}{2 \times 94} \times .072 = 54 \times .072 = \mathbf{3.89} \text{ inches.}$$

$$wt_{sh} = 2 \times 94 \times 54 \times \frac{47\frac{1}{2}}{12} \times .0128 = \mathbf{512} \text{ lbs. per pair of magnets, or } \mathbf{1024} \text{ lbs., total.}$$

$$r_{sh} = \frac{4 \times 94 \times 54 \times 47\frac{1}{2}}{12 \times 409} = \mathbf{196} \text{ ohms, total, at } 15.5^\circ \text{ C.}$$

$$r'_{sh} = 196 \times (1 + .004 \times 25^\circ) = \mathbf{215.5} \text{ ohms, at } 40.5^\circ \text{ C.}$$

$$r''_{sh} = 215.5 \times 1.45 = \mathbf{312} \text{ ohms, entire shunt-circuit, at full load.}$$

$$I_{sh} = \frac{440}{312} = \mathbf{1.41} \text{ ampere, shunt current, full load.}$$

Actual magnetizing force at full load:

$$AT = 2 \times 94 \times 54 \times 1.41 - 12 \times 143 = \mathbf{14,300} - \mathbf{1720} = \mathbf{12,580} \text{ ampere-turns.}$$

g. Speed Calculations.—

Actual counter E. M. F. of motor at full load:

$$E' = 440 - 143 \times (.139 + .004) = 440 - 20\tfrac{1}{2} = \mathbf{419\tfrac{1}{2}} \text{ volts.}$$

Useful flux at full load:

$$\Phi = 10,000,000 \text{ maxwells.}$$

Useful flux at no load:

$$\Phi_0 = 10,500,000 \text{ maxwells.}$$

Torque, at full load:

$$\tau = \frac{11.74}{10^{10}} \times \frac{143 \times 1008}{2} \times 10,000,000 = \mathbf{847} \text{ foot-lbs.}$$

Torque, at no load (energy for overcoming frictions estimated at $P_0 = 5000$ watts):

$$\tau_0 = \frac{11.74}{10^{10}} \times \frac{5000}{440} \times \frac{1008}{2} \times 10,500,000 = \mathbf{71} \text{ foot-lbs.}$$

Specific generating power, at full load:

$$e'' = 10,000,000 \times \frac{1008}{2} \times 10^{-8} = 50.4 \text{ volts,}$$

at no load:

$$e''_0 = 10{,}500{,}000 \times \frac{1008}{2} \times 10^{-8} = 52.9 \text{ volts.}$$

Speed, at full load:

$$N_2 = 60 \times \left(\frac{419\frac{1}{2}}{50.4} - 8.52 \times \frac{(.139 + .004) \times 847}{50.4^2}\right)$$

$$= 60 \times (8.34 - .0405) = \mathbf{498} \text{ revolutions per minute.}$$

Speed, at no load:

$$N_2 = 60 \times \left(\frac{440}{52.9} - 8.52 \times \frac{(.139 + .004) \times 71}{52.9^2}\right)$$

$$= 60 \times (8.32 - .0031) = \mathbf{500} \text{ revolutions per minute.}$$

The difference in speed at full and no load being only 2 revolutions per minute, the condition of constant speed is fulfilled.

149. Calculation of a Unipolar Dynamo:

Cylinder Single Type. Cast-Steel Frame. Cast-Iron Armature.

300 KW. 10 Volts. 30,000 Amps. 1000 Revs. per Min.

a. Diameter of Armature, Dimensioning of Frame, and Current Output.—

By (423), p. 447:

$$d_a = 400 \times \sqrt{\frac{10}{1000}} = \mathbf{40} \text{ inches.}$$

The minimum diameter for the given voltage, by (426), p. 448, would be:

$$d_a = 3.45 \times 10 = 34\tfrac{1}{2} \text{ inches,}$$

which would correspond to a maximum speed of

$$N = .33 \times \frac{200^2}{10} = 1333 \text{ revolutions per minute.}$$

The dimensions of the machine, by § 118, are (see Fig. 374): Length of field, by (409), p. 444:

$$l_p = .3 \times 40 = \mathbf{12} \text{ inches.}$$

Radial thickness of armature, by (412), p. 445:

$$b_a = .2 \times \sqrt{40} = \mathbf{1\tfrac{1}{4}} \text{ inch.}$$

Radial distance of poles, by (413), p. 445:

$$b_p = .25 \times \sqrt{40} = \mathbf{1\tfrac{5}{8}} \text{ inch.}$$

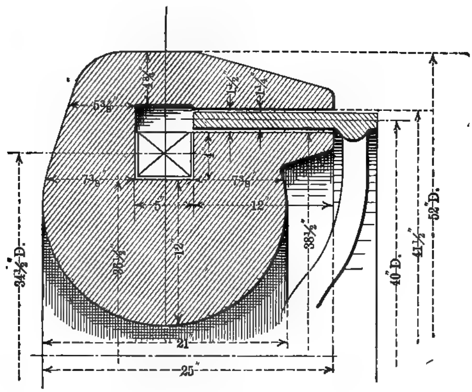


Fig. 374.—Dimensions of 300 KW Unipolar Cylinder Dynamo. 10 Volts. 30,000 Amps. 1000 Revs.

Height of winding space, by (411), p. 444:

$$h_m = .1 \times 40 = \mathbf{4} \text{ inches.}$$

Length of winding space, by (410), p. 444

$$l_m = .125 \times 40 = \mathbf{5} \text{ inches.}$$

Thickness of yoke part of frame, opposite inner surface of exterior shell, by (414), p. 445:

$$b_y = .14 \times 40 - .042 \times \sqrt{40} = \mathbf{5\tfrac{3}{8}} \text{ inches.}$$

Thickness of frame opposite bottom of winding space, by (415), p. 445:

$$b_m = .175 \times 40 + .055 \times \sqrt{40} = \mathbf{7\tfrac{3}{8}} \text{ inches.}$$

Radial thickness of outer shell, by (416), p. 446:

$$h_y = .125 \times 40 - .03 \times \sqrt{40} = \mathbf{4\tfrac{3}{4}} \text{ inches.}$$

Radial thickness of inner shell, by (417), p. 446:

$$h_c = .26 \times 40 + .23 \times \sqrt{40} = \mathbf{12} \text{ inches.}$$

Total length of frame, by (418), p. 446:

$$l_F = .625 \times 40 = \mathbf{25} \text{ inches.}$$

Length of magnetic circuit in frame, by (419), p. 446:

$$l''_m = 1.2 \times 40 = \mathbf{48} \text{ inches.}$$

Current capacity, by (428), p. 448:

$$I' = 125 \times \sqrt{40^3} = \mathbf{30{,}600} \text{ amperes.}$$

b. Calculation of Magnetizing Forces.—

By (435), p. 450:

$$at_g = 750 \times \sqrt{40} = 4740 \text{ ampere-turns.}$$

By (436), p. 450:

$$at_a = 17.6 \times \sqrt{40} = 110 \text{ ampere-turns.}$$

By (437), p. 450:

$$at_m = 53 \times 40 = 2120 \text{ ampere-turns.}$$

Total magnetizing force:

$$AT = 4740 + 110 + 2120 = \mathbf{6970} \text{ ampere-turns.}$$

c. Calculation of Magnet Winding.—

By (438), p. 451:

$$l_T = 2.83 \times 40 - .785 \times \sqrt{40} = \mathbf{108} \text{ inches.}$$

By (439), p. 451:

$$S_M = .39 \times 40^2 - .1 \times \sqrt{40^3} = \mathbf{600} \text{ square inches.}$$

By (440), p. 451:

$$wt_m = .0074 \times 40^3 - .002 \times \sqrt{40^5} = \mathbf{453} \text{ lbs.}$$

By (329), p. 390:

$$\theta_m = \frac{\left[31.3 \times \left(6.970 \times \frac{108}{12}\right)^2 \times \frac{75}{600}\right]}{453 - .004 \times \left[31.3 \times (6.970 \times 9)^2 \times \frac{75}{600}\right]}$$

$$= \frac{15{,}400}{453 - 61.6} = \mathbf{39\tfrac{1}{2}}{}^\circ \text{ C.}$$

Size of wire, for 20 per cent. extra resistance:

$$\lambda_{sh} = \frac{6970}{10} \times 9 \times 1.20 \times (1 + .004 \times 39\tfrac{1}{2})$$

$$= \mathbf{8718} \text{ feet per ohm, or No. } \mathbf{1} \text{ B. W. G. } (.300'' + .020'').$$

Number of wires per layer:

$$\frac{5 - 2 \times .1}{.320} = \mathbf{15}.$$

Number of layers:

$$\frac{4 - .1}{.320} = \mathbf{12}.$$

Total length:

$$L_{sh} = 15 \times 12 \times 9 = \mathbf{1620} \text{ feet.}$$

Resistance:

$$r_{sh} = 1620 \times .0001147 = \mathbf{.186} \text{ ohm, at } 15.5^\circ \text{ C.}$$

$$r'_{sh} = .186 \times (1 + .004 \times 39\tfrac{1}{2}^\circ) = \mathbf{.215} \text{ ohm, at } 55^\circ \text{ C.}$$

$$r''_{sh} = .215 \times 1.20 = \mathbf{.254} \text{ ohm, entire shunt-circuit.}$$

$$I_{sh} = \frac{10}{.254} = \mathbf{39.4} \text{ amperes.}$$

Actual excitation:

$$AT = 15 \times 12 \times 39.4 = \mathbf{7080} \text{ ampere-turns.}$$

d. Weight-Efficiency.

The weight of this machine, complete, will be in the neighborhood of 10,000 lbs., thus making its weight-efficiency about **30** watts per pound.

150. Calculation of a Dynamotor:

Bipolar Double Horseshoe Type. Cast-Steel Frame. Toothed Ring Armature. 5½ KW. 1450 Revs. per Min. Primary: 500 Volts, 11 Amps. Secondary: 110 Volts, 44 Amps.

a. Ratio of Armature Turns; Current Output of Secondary Winding.

Electromotive forces active in armature:

$$E'_1 = 500 - .064 \times 500 = \mathbf{468} \text{ volts,}$$

$$E'_2 = 110 + .064 \times 110 = \mathbf{117} \text{ volts,}$$

Ratio of number of armature turns:

$$\frac{N_{a_1}}{N_{a_2}} = \frac{E'_1}{E'_2} = \frac{468}{117} = \mathbf{4}.$$

Electrical activity in primary winding:

$$E'_1 \times I'_1 = 468 \times 11 = \mathbf{5150} \text{ watts.}$$

Current intensity of secondary winding:

$$I'_2 = \frac{5150}{117} = \mathbf{44} \text{ amperes.}$$

b. Calculation of Armature.—

$\beta_1 = .75$; $e = 62.5 \times 10^{-8}$ volt p. ft.; $v_c = 67\frac{1}{2}$ feet p. sec.; $\mathcal{H}'' = 25{,}000$ lines per square inch.

The relation of the two windings being fixed by the ratio of the E. M. Fs. active in the same, it is only necessary to calculate one of them. For the primary winding we have:

$$L_{a_1} = \frac{468 \times 10^8}{62.5 \times 67\frac{1}{2} \times 25{,}000} = \mathbf{444} \text{ feet.}$$

$$\delta_{a_1}{}^2 = 382 \times 11 = \mathbf{4225} \text{ circular mils,}$$

or No. **16** B. W. G. ($.065'' + .015'' = .080''$).

Arranging **24** of these wires in **8** layers of **3** each, the depth of the slot is obtained:

$h_a = 8 \times .080 + .012 + .035 = .687''$, or $\frac{11}{16}$ inch; and its width:

$$b_s = 3 \times .080 + 2 \times .012 = .264'', \text{ or } \tfrac{17}{64} \text{ inch}$$

$$d'_a = 230 \times \frac{67\frac{1}{2}}{1450} = \mathbf{10\tfrac{11}{16}} \text{ inches;}$$

$$d''_a = 10\tfrac{11}{16} + \tfrac{11}{16} = \mathbf{11\tfrac{3}{8}} \text{ inches.}$$

$$n'_c = \frac{11\frac{3}{8} \times \pi}{2 \times \frac{17}{64}} = \mathbf{68} \text{ slots;}$$

One-half of the winding space being occupied by each winding, 34 slots of 24 wires constitute one winding. The secondary

winding requiring four times the area, 4 No. 16 wires in multiple form one secondary conductor. Making the primary commutator of 68, and the secondary of 34 divisions, the primary, or motor winding consists of **68** coils of **12** turns of **1** No. **16** B. W. G.; and the secondary, or generator winding, of **34** coils of **6** turns of **4** No. **16** B. W. G. wires. Hence the length of the armature core:

$$l_a = \frac{444 \times 12}{34 \times 24} = \mathbf{6\tfrac{1}{2}} \text{ inches.}$$

$$\Phi = \frac{6 \times 444 \times 10^9}{(34 \times 24) \times 1450} = \mathbf{2{,}240{,}000} \text{ maxwells.}$$

$$b_a = \frac{2{,}240{,}000}{2 \times 85{,}000 \times 6\frac{1}{2} \times .90} = \mathbf{2\tfrac{1}{4}} \text{ inches.}$$

The length, weight, and resistance of the primary winding are, respectively:

$$L_{t_1} = \frac{2 \times (6\frac{1}{2} + 2\frac{1}{4}) + \frac{11}{16} + \pi}{12} \times 34 \times 24 = \mathbf{1355} \text{ feet.}$$

$$wt_{a_1} = .00000303 \times 4225 \times 1355 = \mathbf{17\tfrac{1}{2}} \text{ lbs.}$$

$$r_{a_1} = \frac{1}{4} \times 1355 \times .00245 = \mathbf{.83} \text{ ohm, at } 15.5^\circ \text{ C.}$$

The length of the secondary winding is one-quarter that of the primary, the weight is precisely the same, and the resistance is $\frac{1}{4^2}$ times that of the primary, or

$$r_{a_2} = \frac{.83}{16} = \mathbf{.052} \text{ ohm, at } 15.5^\circ \text{ C.}$$

c. Dimensioning of Magnet Frame.—(Fig. 375.)

$$\Phi' = 1.34 \times 2{,}240{,}000 = \mathbf{3{,}000{,}000} \text{ maxwells.}$$

$$S_m = \frac{3{,}000{,}000}{2 \times 85{,}000} = \mathbf{17.7} \text{ square inches.}$$

Cylindrical cores being selected, their diameter is:

$$d_m = \sqrt{17.7 \times \frac{4}{\pi}} = \mathbf{4\tfrac{3}{4}} \text{ inches.}$$

Length of cores:

$$l_m = 6\tfrac{1}{2} \text{ inches.}$$

Width of yoke:

$$6\tfrac{1}{2} \text{ inches.}$$

Thickness of yoke:

$$\frac{17.7}{6\frac{1}{2}} = 2\tfrac{3}{4} \text{ inches.}$$

Bore of field:

$$d_p = 11\tfrac{3}{8} + 2 \times \tfrac{3}{16} = 11\tfrac{3}{4} \text{ inches.}$$

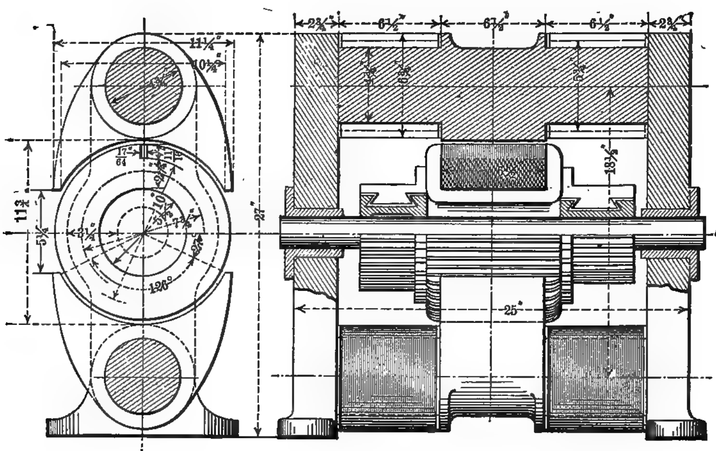


Fig. 375.—Dimensions of Armature Core and Magnet Frame, $5\frac{1}{2}$ KW. Double Horseshoe Type Motor-Generator.

Chord of polar embrace:

$$h_p = 11\tfrac{3}{4} \times \sin 63° = 10\tfrac{1}{4} \text{ inches.}$$

Total width of polepieces:

$$h'_p = 10\tfrac{1}{4} + 2 \times \tfrac{1}{2} = 11\tfrac{1}{4} \text{ inches.}$$

Distance between pole-corners:

$$l'_p = 11\tfrac{3}{4} \times \sin 27° = 5\tfrac{1}{4} \text{ inches.}$$

d. Calculation of Magnetizing Forces.—

$$L_e = 36 \times 24 \times \frac{6\frac{1}{2}}{12} \times .88 = 412 \text{ feet.}$$

By (144), p. 205:

$$\mathfrak{H}'' = \frac{468 \times 10^8}{72 \times 412 \times 67\frac{1}{2}} = 23{,}400 \text{ lines per square inch.}$$

$\therefore\ at_g = .3133 \times 23{,}400 \times \frac{3}{8} = \mathbf{2750}$ ampere-turns.

Since in a dynamotor there is practically no armature-reaction, the factor of field-deflection is here $k_{12} = 1$.

By (237), p. 343:

$$l''_a = 7\tfrac{3}{4} \times \pi \times \frac{90 + 27}{360} + 2\tfrac{1}{4} + 2 \times \tfrac{11}{16} = 11\tfrac{3}{4} \text{ inches.}$$

$\mathfrak{B}''_{a_1} = 85{,}000$ lines; $\mathfrak{B}''_{a_2} = 42{,}500$ lines;

$$m''_a = \frac{39.7 + 8}{2} = 23.9 \text{ ampere-turns per inch.}$$

$\therefore\ at_a = 23.9 \times 11\frac{3}{4} = \mathbf{280}$ ampere-turns.

$l''_m = 45$ inches, length of circuit in frame.

$\mathfrak{B}''_m = 85{,}000$; $m''_m = 44$ ampere-turns per inch (cast steel).

$\therefore\ at_m = 44 \times 45 = \mathbf{2000}$ ampere-turns.

$AT = 2750 + 280 + 2000 = \mathbf{5030}$ ampere-turns.

e. Calculation of Magnet Winding.—
Given: $\theta_m = 25°$ C.; $r_x = 45\%$.

Assuming as the height of the winding space

$h_m = \mathbf{1}$ inch, we have:

$l_T = (4\frac{3}{4} + 1) \times \pi = \mathbf{18}$ inches;

$S_M = 6\frac{3}{4} \times \pi \times 2 \times (6\frac{1}{2} - \frac{1}{2}) = \mathbf{255}$ square inches.

Connecting all four cores in series across the primary terminals, each circuit of two cores will take one-half the voltage, or 250 volts, hence:

$$\lambda_{sh} = \frac{5030}{250} \times \frac{18}{12} \times 1.45 \times (1 + .004 \times 25°)$$
$$= \mathbf{48.2} \text{ feet per ohm,}$$

which corresponds to No. **23** B. & S. (.023″ + .005″).

$$P'_{sh} = \frac{25}{70} \times 255 \times 1.45 = 123 \text{ watts.}$$

$$N_{sh} = \frac{5030 \times 250}{123} = \mathbf{10{,}200} \text{ turns, per magnetic circuit, or}$$
$$\mathbf{5100} \text{ turns per core.}$$

Each layer can hold

$$\frac{6\frac{1}{2} - \frac{1}{2}}{.028} = \mathbf{214} \text{ wires.}$$

Number of layers required:

$$\frac{5030}{214} = \mathbf{24.}$$

Actual depth of winding:

$$h'_m = 24 \times .028 = \mathbf{.67} \text{ inch.}$$

$$L_{sh} = 4 \times 214 \times 24 \times \frac{18}{12} = \mathbf{30{,}800} \text{ feet, total, 4 cores.}$$

$$r_{sh} = 30{,}800 \times .02 = \mathbf{616} \text{ ohms, at } 15.5^\circ \text{ C.}$$
$$r'_{sh} = 616 \times (1 + .004 \times 25) = \mathbf{678} \text{ ohms, at } 40.5^\circ \text{ C.}$$
$$r''_{sh} = 678 \times 1.45 = \mathbf{983} \text{ ohms, entire shunt-circuit.}$$

$$I_{sh} = \frac{500}{983} = \mathbf{.51} \text{ ampere.}$$

Actual excitation, full load:

$$AT = 2 \times 214 \times 24 \times .51 = \mathbf{5230} \text{ ampere-turns.}$$

Weight of magnet winding:

$$wt_{sh} = 30{,}800 \times .00155 = \mathbf{48} \text{ pounds, bare;}$$
$$wt'_{sh} = 1.08 \times 48 = \mathbf{52} \text{ pounds, covered; or } \mathbf{13} \text{ pounds of No. } \mathbf{23} \text{ B. \& S. wire per core.}$$

APPENDIX I.

TABLES OF DIMENSIONS OF MODERN DYNAMOS.

TABLES OF DIMENSIONS OF MODERN DYNAMOS.

In order to guide the student in the selection of values for the various variable quantities which he has to assume in the course of a dynamo calculation, tables of dimensions of modern machines are here given, covering all the usual cases of dynamo design. In Tables CVII. to CXII. the dimensions of some of the machines of the Crocker-Wheeler, General Electric, and Westinghouse Companies are compiled, and in Tables CXIII. and CXIV. the armature dimensions of all the machines contained in Tables CVII. to CXII. are tabulated according to size and speed, so as to be readily employed for reference.

In the case of an example calling for the design of a machine which is to run at a speed not sufficiently near any of the speeds given for that output in Tables CXIII. or CXIV., a standard of comparison may be obtained by interpolating between the next higher and lower speeds found for the same output and type of armature. For instance, if a 200-KW ring-type dynamo is to be calculated for a speed of 300 revs. per min., the result of formula (30), page 58, should be compared with the diameter obtained by interpolating between the dimensions given in Table CXIII. for the 200-KW high-speed ring armature and the 200-KW medium-speed ring armature, thus:

Diameter, 200-KW Ring Armature, 450 Revs. per min., $32\frac{1}{2}$ in.
" 200- " " " 175 " " " 50 "
" 200- " " " 300 " " " by interpolation:

$$32\tfrac{1}{2} + \left[\frac{450 - 300}{450 - 175} \times (50 - 32\tfrac{1}{2})\right] = 32\tfrac{1}{2} + 9\tfrac{1}{2} = 42 \text{ inches.}$$

The letters at the head of the columns refer to the dimensions designated by the same letters in the corresponding figure. All dimensions are given in inches.

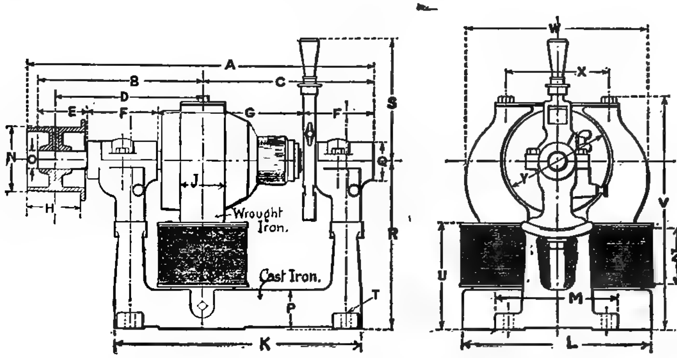


Fig. 376.—*Crocker-Wheeler* Bipolar Motor.

TABLE CVII.—DIMENSIONS OF CROCKER-WHEELER BIPOLAR MEDIUM-SPEED RING-ARMATURE MOTORS. (See Fig. 376.)

HP.	KW.	R. p. m.	Weight.	A	B	C	D	E	F	G	H	J	K	L
$\frac{1}{12}$	.1	1800	19	$7\frac{11}{16}$	$3\frac{13}{16}$	$3\frac{11}{16}$	3	$1\frac{5}{8}$	$1\frac{1}{8}$	$3\frac{5}{8}$	1	2	$5\frac{3}{4}$	$5\frac{1}{8}$
$\frac{1}{8}$	.15	1600	27	$9\frac{7}{8}$	$4\frac{11}{16}$	$4\frac{27}{32}$	$4\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{7}{8}$	$4\frac{1}{2}$	1	$1\frac{5}{8}$	$7\frac{1}{2}$	$6\frac{1}{2}$
$\frac{1}{4}$	.25	1400	70	$14\frac{15}{16}$	$6\frac{5}{8}$	$7\frac{11}{16}$	$6\frac{1}{8}$	$1\frac{1}{2}$	$3\frac{1}{8}$	$6\frac{5}{8}$	$2\frac{1}{4}$	$1\frac{7}{8}$	$11\frac{3}{8}$	$8\frac{5}{8}$
$\frac{1}{2}$	.5	1200	100	$17\frac{3}{4}$	$8\frac{3}{4}$	$9\frac{1}{16}$	$7\frac{3}{16}$	$2\frac{5}{8}$	$3\frac{3}{4}$	$7\frac{11}{16}$	3	$2\frac{1}{8}$	$13\frac{1}{16}$	10
1	1	1000	205	$19\frac{1}{4}$	$9\frac{1}{4}$	$9\frac{3}{4}$	$7\frac{3}{4}$	$2\frac{11}{16}$	$3\frac{7}{8}$	$8\frac{9}{16}$	$3\frac{1}{2}$	$2\frac{3}{16}$	$14\frac{7}{16}$	$13\frac{1}{4}$
2	1.5	975	288	$22\frac{1}{2}$	$10\frac{5}{16}$	$11\frac{3}{4}$	$8\frac{3}{4}$	$2\frac{3}{4}$	$4\frac{7}{16}$	$10\frac{7}{16}$	4	$3\frac{5}{16}$	$17\frac{1}{4}$	$14\frac{1}{4}$
3	2.25	950	410	$26\frac{13}{16}$	$11\frac{13}{16}$	$14\frac{3}{8}$	$10\frac{5}{8}$	$3\frac{1}{8}$	5	$13\frac{1}{16}$	$4\frac{1}{4}$	$3\frac{9}{16}$	$21\frac{7}{16}$	$16\frac{1}{4}$
5	4	925	510	$28\frac{3}{16}$	$12\frac{31}{32}$	$14\frac{13}{16}$	$10\frac{7}{8}$	$3\frac{25}{32}$	5	14	5	$4\frac{13}{16}$	$22\frac{3}{8}$	$19\frac{1}{4}$
$7\frac{1}{2}$	6.5	875	760	33	$15\frac{1}{16}$	$16\frac{15}{16}$	$13\frac{1}{16}$	4	$5\frac{3}{4}$	$16\frac{1}{2}$	6	$5\frac{5}{8}$	$26\frac{1}{4}$	$21\frac{1}{4}$
10	8.5	850	920	$36\frac{1}{4}$	17	$18\frac{1}{2}$	$14\frac{1}{4}$	5	$6\frac{1}{2}$	$17\frac{1}{2}$	7	$6\frac{1}{4}$	$28\frac{1}{2}$	$23\frac{3}{16}$

HP.	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
$\frac{1}{12}$	$3\frac{1}{2}$	$1\frac{1}{2}$	$\frac{3}{8}$	1	$\frac{9}{16}$	$5\frac{11}{16}$	..	$\frac{1}{4}$	$3\frac{15}{16}$	$7\frac{7}{8}$	$5\frac{1}{8}$	3	3	$2\frac{3}{4}$
$\frac{1}{8}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$\frac{3}{8}$	$1\frac{7}{16}$	$1\frac{3}{16}$	$6\frac{1}{8}$	..	$\frac{5}{16}$	$4\frac{1}{16}$	$8\frac{1}{2}$	$5\frac{7}{16}$	$3\frac{3}{8}$	$3\frac{19}{32}$	$2\frac{1}{2}$
$\frac{1}{4}$	$4\frac{9}{16}$	3	$\frac{9}{16}$	$1\frac{7}{8}$	$1\frac{1}{2}$	$7\frac{3}{4}$	..	$\frac{3}{8}$	$4\frac{15}{16}$	$10\frac{11}{16}$	$8\frac{1}{4}$	$4\frac{5}{8}$	$4\frac{3}{4}$	$2\frac{7}{8}$
$\frac{1}{2}$	$6\frac{5}{8}$	3	$\frac{11}{16}$	$2\frac{1}{8}$	2	9	$7\frac{13}{32}$	$\frac{1}{2}$	$5\frac{3}{4}$	$12\frac{9}{16}$	$9\frac{1}{8}$	$5\frac{1}{2}$	$5\frac{5}{8}$	$3\frac{1}{2}$
1	$8\frac{3}{4}$	4	$\frac{7}{8}$	$2\frac{13}{16}$	$2\frac{1}{4}$	$10\frac{5}{8}$	$7\frac{13}{32}$	$\frac{1}{2}$	$6\frac{3}{4}$	15	$12\frac{7}{16}$	$7\frac{1}{4}$	7	$3\frac{5}{8}$
2	9	5	1	$3\frac{3}{16}$	$2\frac{3}{8}$	$11\frac{1}{16}$	$7\frac{3}{4}$	$\frac{1}{2}$	$6\frac{7}{8}$	$15\frac{3}{4}$	$13\frac{3}{4}$	8	$7\frac{3}{4}$	$3\frac{1}{2}$
3	$11\frac{1}{2}$	6	1	$2\frac{9}{16}$	$2\frac{5}{8}$	$11\frac{13}{16}$	$9\frac{5}{8}$	$\frac{5}{8}$	$7\frac{3}{8}$	$18\frac{3}{4}$	$13\frac{1}{2}$	$7\frac{1}{4}$	$8\frac{1}{4}$	$4\frac{3}{8}$
5	12	7	$1\frac{1}{8}$	$3\frac{3}{8}$	$2\frac{5}{8}$	$13\frac{3}{8}$	$9\frac{5}{8}$	$\frac{5}{8}$	$8\frac{1}{2}$	$21\frac{3}{8}$	$15\frac{1}{4}$	$7\frac{1}{2}$	$9\frac{3}{16}$	$4\frac{3}{8}$
$7\frac{1}{2}$	$14\frac{1}{4}$	8	$1\frac{1}{4}$	4	$3\frac{1}{8}$	$14\frac{5}{8}$	$10\frac{3}{8}$	$\frac{5}{8}$	$9\frac{1}{4}$	$23\frac{1}{2}$	$17\frac{7}{8}$	$7\frac{1}{2}$	$10\frac{1}{8}$	$4\frac{3}{4}$
10	$15\frac{3}{4}$	9	$1\frac{1}{2}$	$4\frac{1}{4}$	$3\frac{1}{2}$	$16\frac{1}{4}$	$11\frac{1}{2}$	$\frac{5}{8}$	$10\frac{3}{16}$	$25\frac{3}{4}$	$19\frac{3}{16}$	$8\frac{3}{4}$	11	$5\frac{1}{2}$

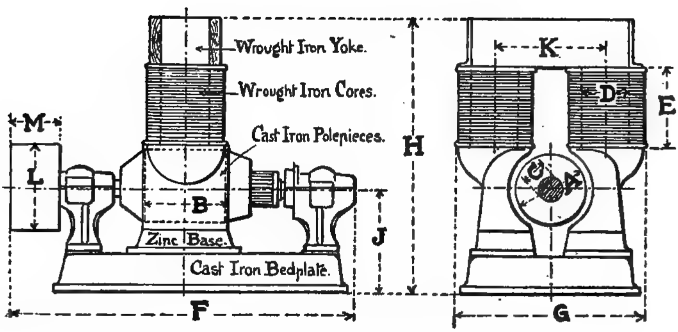


Fig. 377.—*Edison* Bipolar Dynamo.

TABLE CVIII.—DIMENSIONS OF EDISON BIPOLAR HIGH-SPEED DRUM-ARMATURE DYNAMOS AND MOTORS. (See Fig. 377.)

Generator.		Motor.		Weight.	Armature.			Magnets.		Frame.					Pulley.	
KW.	R.p.m.	H.P.	R.p.m.		A	B	C	D	E	F	G	H	J	K	L	M
.5	2400	.5	2100	90	$2\frac{5}{16}$	4	$\frac{11}{16}$	$2\frac{5}{8}$	$4\frac{1}{2}$	$16\frac{5}{8}$	$9\frac{1}{2}$	$12\frac{1}{2}$	$4\frac{5}{16}$	$4\frac{7}{8}$	$3\frac{1}{2}$	2
.75	2400	.75	2100	120	$2\frac{5}{8}$	5	$\frac{7}{8}$	$2\frac{3}{4}$	$5\frac{1}{2}$	$21\frac{1}{4}$	$9\frac{3}{4}$	$13\frac{3}{4}$	$4\frac{5}{16}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$2\frac{1}{2}$
1.5	2100	1.5	1600	270	$3\frac{25}{32}$	$6\frac{1}{2}$	$\frac{15}{16}$	$3\frac{3}{4}$	$6\frac{3}{4}$	$24\frac{1}{4}$	$13\frac{1}{2}$	19	7	$7\frac{7}{16}$	4	3
3	1000	3	1450	550	$4\frac{1}{2}$	8	$1\frac{1}{8}$	5	$7\frac{5}{8}$	33	17	23	$8\frac{1}{2}$	$9\frac{1}{4}$	$7\frac{1}{2}$	3
6	1800	6	1350	830	$5\frac{11}{16}$	$9\frac{1}{4}$	$1\frac{3}{8}$	$5\frac{3}{4}$	$8\frac{3}{4}$	$38\frac{3}{4}$	20	$26\frac{7}{8}$	$10\frac{1}{16}$	11	8	4
8.5	1700	9	1325	1090	$5\frac{7}{8}$	10	$1\frac{5}{8}$	$6\frac{5}{16}$	$10\frac{5}{8}$	$44\frac{1}{8}$	22	$30\frac{1}{2}$	$11\frac{1}{8}$	12	$8\frac{1}{2}$	5
12	1600	14	1275	1470	$6\frac{1}{4}$	12	$1\frac{7}{8}$	$7\frac{3}{8}$	$12\frac{1}{4}$	$48\frac{3}{8}$	$23\frac{1}{2}$	34	$12\frac{1}{8}$	$12\frac{1}{2}$	9	6
15	1500	17.5	1150	2130	$6\frac{3}{4}$	$13\frac{1}{2}$	$2\frac{1}{4}$	$8\frac{1}{4}$	$13\frac{5}{8}$	57	$25\frac{3}{4}$	38	$13\frac{5}{8}$	$13\frac{7}{8}$	$11\frac{1}{4}$	8
20	1400	24	1100	2880	$7\frac{3}{4}$	15	$2\frac{1}{2}$	$9\frac{1}{4}$	$15\frac{1}{8}$	$65\frac{3}{4}$	$28\frac{3}{8}$	$41\frac{3}{4}$	$14\frac{5}{8}$	$15\frac{1}{8}$	12	9
25	1300	30	950	3570	$8\frac{3}{4}$	$16\frac{1}{2}$	3	$10\frac{1}{8}$	$16\frac{1}{4}$	$68\frac{1}{4}$	$30\frac{1}{4}$	$45\frac{1}{8}$	$15\frac{5}{8}$	$16\frac{1}{4}$	13	10
30	1200	36	880	4340	$9\frac{3}{16}$	18	$3\frac{1}{4}$	$11\frac{1}{8}$	$16\frac{7}{8}$	$73\frac{1}{2}$	$31\frac{3}{8}$	$47\frac{5}{8}$	$16\frac{1}{2}$	$16\frac{3}{8}$	14	11
45	1000	60	750	6800	$11\frac{1}{16}$	$20\frac{1}{2}$	$3\frac{3}{4}$	$12\frac{3}{4}$	$20\frac{1}{8}$	$83\frac{1}{2}$	36	$57\frac{1}{2}$	$19\frac{1}{2}$	$19\frac{3}{4}$	17	12
60	700	70	525	9790	$12\frac{7}{8}$	$24\frac{1}{2}$	4	$15\frac{1}{8}$	$23\frac{7}{8}$	$92\frac{1}{2}$	$40\frac{3}{4}$	$65\frac{7}{8}$	$21\frac{3}{4}$	$22\frac{1}{2}$	24	13
100	650	120	550	16200	$16\frac{3}{8}$	25	$4\frac{3}{4}$	$17\frac{1}{2}$	24	105	$47\frac{1}{2}$	$73\frac{1}{2}$	$24\frac{3}{4}$	$26\frac{1}{2}$	26	16
150	450	180	450	31790	$23\frac{3}{4}$	$26\frac{1}{2}$	6	$21\frac{1}{2}$	31	$119\frac{1}{8}$	$59\frac{1}{4}$	$92\frac{1}{2}$	$31\frac{3}{4}$	33	44	21
200	450	240	450	39000	$23\frac{3}{4}$	$34\frac{1}{4}$	$7\frac{3}{4}$	23	31	$135\frac{1}{2}$	$67\frac{3}{4}$	94	$31\frac{3}{4}$	$37\frac{3}{4}$	14	24

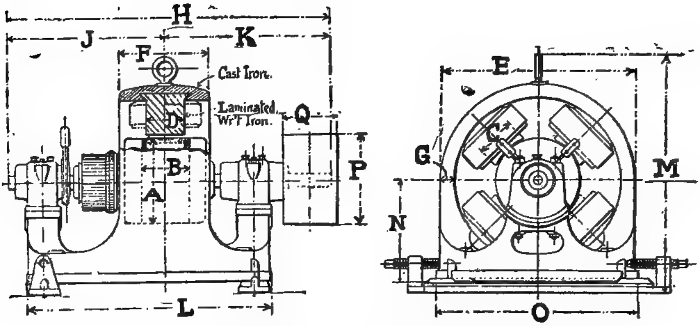


Fig. 378.—*Westinghouse* Four-Pole Dynamo.

TABLE CIX.—DIMENSIONS OF WESTINGHOUSE FOUR-POLE MEDIUM AND HIGH-SPEED DRUM-ARMATURE DYNAMOS AND MOTORS. (See Fig. 378.)

Machine No.	Medium Speed.			High Speed.			Weight.	Armature.			Magnets.	
	KW.	HP.	R. p. m.	KW.	HP.	R. p. m.		A	B	Nbr. of Slots.	C	D
1	.56	¾	1300	.75	1	1900	190	5¾	2⅝	31	2½	2½
2	1.5	2	1200	1.875	2½	1700	290	6½	3¼	31	3⅛	2⅞
3	2.62	3½	1050	3.75	5	1600	550	7¾	4⅝	47	4½	3½
4	3.75	5	950	5.62	7½	1350	740	8½	5¼	41	5	3⅞
5	5.62	7½	850	7.5	10	1250	940	9½	5½	47	5¼	4½
6	7.5	10	750	11.25	15	1150	1190	10½	6	47	5¾	5
7	11.25	15	650	15	20	1050	1550	11⅝	6¾	47	6⅜	5¼
8	15	20	600	22.5	30	975	2460	13	8	47 & 71	7½	6¼
9	....	..	....	30	40	950	2950	13⅞	8½	61	8¼	6¾
10	22.5	30	575	37.5	50	900	3410	15	8½	41 & 61	8	7¼
11	30	40	550	45	60	850	3900	15¾	8½	47 & 59	8	8
12	37.5	50	550	56.25	75	800	4300	16½	9	71 & 55	8½	8½

Machine No.	Yoke.			General Dimensions.							Pulley.	
	E	F	G	H	J	K	L	M	N	O	P	Q
1	13¾	5½	¾	22	11	11	16½	17¾	7⅛	14⅜	3½	2½
2	14¾	6½	1	26¼	12½	13¾	19⅞	18¾	7⅝	15½	5	3
3	18	8½	1⅜	32	15¾	16¼	21⅛	22⅞	9⅜	19	6	4
4	19½	9½	1¾	36	16¾	19¼	26⅝	24⅜	10⅛	20¼	8	5
5	21½	10	1¾	38	18⅝	19⅜	28¼	26⅝	11⅛	22⅜	8	5
6	24	10½	2	40¾	19⅛	21⅝	30⅛	30	12½	25¼	9	6
7	26½	12	2¼	44½	20⅞	23⅝	32⅞	32½	13¾	27⅞	14	7
8	30¼	13½	2⅞	50½	25⅛	25⅜	33⅜	36⅜	15⅝	31¾	14	8
9	33	14	3¼	54½	25⅞	28⅝	39⅞	40	17	34½	15	11
10	34½	14½	3⅜	56½	26⅝	29⅞	42	41½	17¾	36⅛	15	11
11	35⅞	14⅞	3$\frac{7}{16}$	65¾	34⅞	30⅞	47½	43⅝	18½	37½	20	11
12	37	15¼	3½	66	33¾	32¼	47⅞	44	19	38¾	20	11

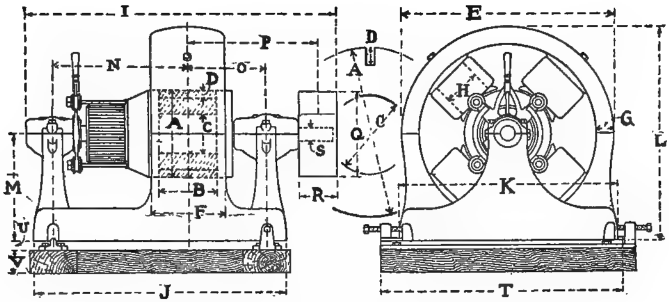


Fig. 379.—*General Electric* Four-Pole Generator.

TABLE CX.—DIMENSIONS OF GENERAL ELECTRIC FOUR-POLE MODERATE AND HIGH-SPEED RING-ARMATURE GENERATORS.
(See Fig. 379.)

No.	Medium Speed. KW.	Medium Speed. R. p. m.	High Speed. KW.	High Speed. R p. m.	Weight. Lbs.	Armature. A	Armature. B	Armature. C	Armature. D	Armature. No. of Slots.	Magnet Frame. E	Magnet Frame. F	Magnet Frame. G	Magnet Frame. H
1	6.5	950	9	1450	970	$9\frac{1}{4}$	6	$4\frac{5}{8}$	.834	46	$23\frac{7}{8}$	9	$1\frac{13}{16}$	$3\frac{31}{32}$
2	13.5	850	17.5	1175	1810	13	$6\frac{1}{2}$	$6\frac{1}{2}$	.804	52	31	12	$2\frac{3}{16}$	$5\frac{7}{32}$
3	20	700	30	1050	3200	16	7	$7\frac{1}{8}$	.948	66	37	15	3	$6\frac{3}{4}$
4	30	675	45	975	4780	18	$8\frac{3}{4}$	$9\frac{1}{4}$	.944	116	44	$16\frac{1}{2}$	$3\frac{3}{8}$	$7\frac{5}{16}$
5	50	600	65	875	6930	$20\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{5}{16}$	1.076	98	50	18	$3\frac{3}{4}$	$8\frac{1}{16}$
6	75	550	85	750	8560	22	$9\frac{1}{4}$	10	1.208	96	54	19	$4\frac{1}{2}$	$9\frac{1}{8}$

No.	General Dimensions. I	J	K	L	M	N	O	P	Pulley. Q	R	S	Rails. T	U	Base. V
1	38	$31\frac{3}{4}$	26	24	12	$16\frac{1}{2}$	$10\frac{3}{4}$	$16\frac{1}{8}$	11	$4\frac{1}{2}$	$1\frac{3}{8}$	$34\frac{3}{4}$	$1\frac{3}{8}$	3
2	$49\frac{1}{2}$	39.	33	31	$15\frac{1}{2}$	$19\frac{7}{8}$	$13\frac{1}{8}$	$21\frac{3}{8}$	$12\frac{1}{4}$	$8\frac{1}{2}$	2	$43\frac{1}{2}$	$1\frac{1}{2}$	3
3	$60\frac{1}{2}$	$45\frac{1}{2}$	40	37	$18\frac{1}{2}$	$22\frac{3}{4}$	$15\frac{3}{4}$	$26\frac{3}{4}$	15	$10\frac{1}{2}$	$2\frac{1}{4}$	52	$1\frac{5}{8}$	4
4	$67\frac{3}{4}$	$52\frac{3}{4}$	$46\frac{1}{2}$	$44\frac{1}{2}$	$22\frac{1}{2}$	$26\frac{3}{8}$	$18\frac{3}{8}$	$29\frac{7}{8}$	$20\frac{1}{2}$	11	$2\frac{3}{4}$	55	$1\frac{5}{8}$	4
5	$78\frac{7}{8}$	$57\frac{5}{8}$	53	50	25	$29\frac{3}{8}$	$18\frac{7}{8}$	$33\frac{1}{8}$	23	$18\frac{1}{2}$	$3\frac{1}{4}$	$62\frac{3}{4}$	$1\frac{3}{4}$	6
6	92	$63\frac{1}{2}$	56	54	27	$31\frac{1}{4}$	$19\frac{3}{4}$	39	25	$24\frac{1}{2}$	$3\frac{7}{8}$	$65\frac{1}{2}$	2	6

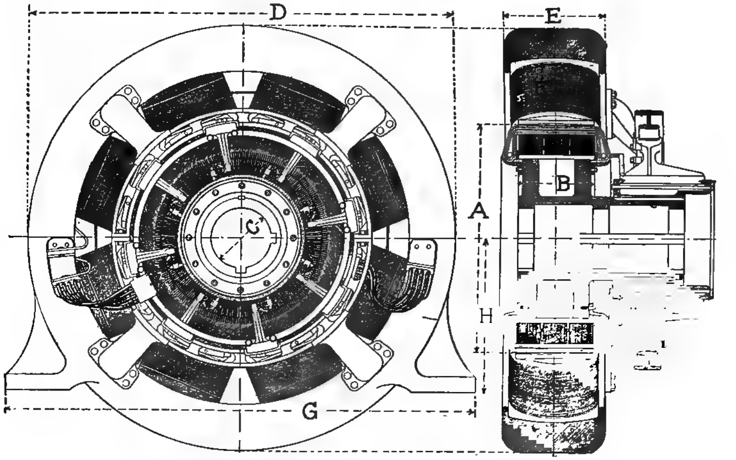


Fig. 380.—*Crocker-Wheeler* Multipolar Generator.

TABLE CXI.—DIMENSIONS OF CROCKER-WHEELER MULTIPOLAR LOW, MEDIUM, AND HIGH-SPEED SURFACE-WOUND RING-ARMATURE DYNAMOS. (See Fig. 380.)

Low Speed.		Medium Speed.		High Speed.		Nbr. of Poles.	Armature.				Field Frame.				
KW.	R. p. m.	KW.	R. p. m.	KW.	R.p.m.		A	B	C	No. of Slots.	D	E	F	G	H
		2.5	1200	3.5	1600	4	$7\frac{3}{16}$	$3\frac{1}{8}$	$1\frac{1}{2}$	40	$17\frac{3}{8}$	$7\frac{1}{4}$	$2\frac{7}{8}$	$17\frac{1}{2}$	$6\frac{3}{4}$
		4.5	1150	5.75	1400	4	$8\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{2}$	44	$19\frac{5}{8}$	8	$3\frac{3}{16}$	$19\frac{3}{4}$	$7\frac{3}{4}$
		6.5	1050	8.25	1300	4	$9\frac{1}{4}$		$1\frac{3}{4}$	44	$21\frac{5}{8}$	$8\frac{1}{2}$	$3\frac{9}{16}$	$21\frac{3}{4}$	$8\frac{3}{4}$
		9	950	11.5	1300	4	11	5	$1\frac{3}{4}$	87	$25\frac{1}{2}$	$9\frac{3}{4}$	$4\frac{1}{8}$	$25\frac{1}{2}$	$9\frac{3}{4}$
		13	900	17.5	1300	4	$11\frac{5}{8}$	$6\frac{5}{8}$	2	73	28	$11\frac{1}{2}$	$4\frac{3}{16}$	28	$10\frac{3}{4}$
2	500					6	12	$3\frac{1}{4}$	$1\frac{3}{4}$	62	$21\frac{1}{4}$	$6\frac{3}{4}$	$3\frac{1}{8}$	$21\frac{1}{4}$	$8\frac{1}{2}$
		18	875	23	1150	4	13	7	2	99	$30\frac{1}{8}$	$12\frac{1}{8}$	4	$30\frac{1}{8}$	$11\frac{1}{4}$
		22.5	850	27	1125	4	14	$7\frac{1}{2}$	$2\frac{1}{2}$	78	$32\frac{1}{4}$	$13\frac{1}{2}$	$4\frac{5}{16}$	$32\frac{1}{4}$	$12\frac{1}{8}$
4	425					6	15	$3\frac{3}{4}$	2	50	$25\frac{1}{4}$	$6\frac{3}{4}$	$3\frac{1}{8}$	$25\frac{1}{4}$	$9\frac{1}{2}$
		30	775	40	1050	4	15	8	$2\frac{3}{4}$	95	36	$14\frac{1}{2}$	$5\frac{3}{16}$	36	$13\frac{5}{8}$
		45	700	55	930	4	$16\frac{1}{2}$	9	3	70	$40\frac{1}{2}$	$15\frac{1}{2}$	$6\frac{1}{4}$	$40\frac{1}{2}$	15
7	400					8	19	$4\frac{1}{4}$	3	80	$30\frac{1}{4}$	8	$3\frac{5}{8}$	32	12
		65	625	80	750	4	19	$10\frac{1}{2}$	$3\frac{1}{2}$	96	$48\frac{1}{2}$	$16\frac{3}{4}$	$7\frac{3}{4}$	$48\frac{1}{2}$	18
9	350					8	21	5	3	73	$33\frac{1}{4}$	$8\frac{1}{2}$	$3\frac{7}{8}$	$33\frac{1}{4}$	13
15	325					8	$22\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{4}$	65	36	11	$3\frac{3}{4}$	$40\frac{1}{2}$	$14\frac{3}{4}$
20	300					6	$20\frac{1}{4}$	$6\frac{3}{4}$	$3\frac{1}{2}$	76	$39\frac{1}{4}$	$13\frac{1}{2}$	$5\frac{3}{16}$	$39\frac{1}{2}$	15
30	300					6	$22\frac{1}{4}$	$8\frac{1}{4}$	4	109	$44\frac{1}{4}$	$13\frac{3}{4}$	$5\frac{5}{8}$	$44\frac{1}{2}$	16
50	275	90	560	110	720	6	24	$9\frac{5}{8}$	$4\frac{1}{4}$	141	$48\frac{1}{2}$	$15\frac{7}{8}$	$6\frac{1}{4}$	$48\frac{1}{2}$	18
60	275					6	28	11	$4\frac{1}{2}$	126	$54\frac{1}{2}$	$17\frac{1}{4}$	$6\frac{1}{2}$	$54\frac{1}{2}$	20
75	250	130	500	150	550	6	$32\frac{1}{2}$	$12\frac{1}{2}$	6	111	$64\frac{1}{4}$	18	$7\frac{7}{8}$	68	24
		200	450			6	$32\frac{1}{2}$	$13\frac{1}{2}$	8	105	$68\frac{1}{4}$	$18\frac{3}{4}$	$9\frac{3}{16}$	$68\frac{1}{2}$	25
100	250					8	37	$10\frac{7}{8}$	$6\frac{1}{2}$	156	67	17	$7\frac{3}{4}$	68	24
125	250					8	45	13	7	208	$76\frac{3}{4}$	18	$7\frac{7}{8}$	80	26
150	130	200	175	250	225	8	50	$16\frac{1}{2}$	12	108	90	$20\frac{1}{4}$	$10\frac{3}{8}$	99	$29\frac{1}{2}$
200	100	250	125	300	150	8	57	19	15	168	107	$25\frac{1}{2}$	14	128	32
		300	100			12	77	$14\frac{1}{2}$		198	$118\frac{3}{4}$	23	12		
350	88	400	100	500	125	12	87	$15\frac{3}{4}$		160	135	$25\frac{1}{2}$	$13\frac{3}{8}$		
				600	100	12	95	$17\frac{3}{4}$		152	151	$28\frac{1}{2}$	$15\frac{3}{8}$		

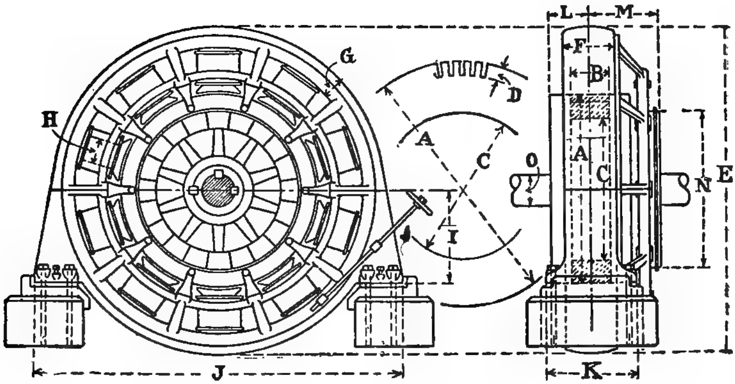


Fig. 381.—*General Electric* Multipolar Generator.

TABLE CXII.—DIMENSIONS OF GENERAL ELECTRIC MULTIPOLAR LOW-SPEED RING-ARMATURE GENERATORS. (See Fig. 381.)

No.	KW.	R. p. m.	No. of Poles.	Armature.					Approximate Weight.	
				A	B	C	D	No. of Slots.	Armature and Commutator.	Generator Complete.
1	150	200	6	46	$12\frac{3}{8}$	$26\frac{5}{16}$	$1\frac{5}{16}$	130	6400	29000
2	300	150	8	68	14	45	$1\frac{27}{32}$	272	17000	55000
3	400	120	8	72	$18\frac{1}{4}$	48	$1\frac{3}{4}$	240	22000	79000
4	500	120	10	84	$17\frac{1}{4}$	62	$1\frac{3}{4}$	280	25000	81000
5	800	80	14	120	$19\frac{1}{2}$	98	$1\frac{3}{4}$	364	50000	135000
6	1600	75	22	164	19	144	$1\frac{3}{4}$	440	74000	180000

No.	Magnet Frame.				General Dimensions.						
	E	F	G	H	I	J	K	L	M	N	O
1	99	25	$8\frac{11}{16}$	$13\frac{3}{4}$	23	114	35	$12\frac{7}{8}$	$19\frac{3}{4}$	32	9
2	125	27	10	$15\frac{1}{4}$	32	141	41	$16\frac{1}{8}$	$29\frac{3}{8}$	49	$11\frac{1}{2}$
3	135	$32\frac{1}{2}$	11	18	41	150	48	$18\frac{11}{16}$	$35\frac{7}{16}$	49	15
4	$144\frac{1}{2}$	30	$10\frac{3}{4}$	$16\frac{3}{4}$	44	154	45	$17\frac{3}{4}$	31	60	16
5	187	32	$11\frac{3}{4}$	18	53	201	48	$18\frac{5}{16}$	$34\frac{5}{16}$	84	19
6	230	24	$12\frac{5}{8}$	$16\frac{3}{4}$	63	245	48	$18\frac{3}{8}$	$37\frac{1}{2}$	120	24

TABLE CXIII.—RING-ARMATURE DIMENSIONS.

High Speed.				Medium Speed.				Low Speed.			
KW.	R. p. m.	Diam.	Length.	KW.	R. p. m.	Diam.	Length.	KW.	R. p. m.	Diam.	Length.
				.1	1800	3	2				
				.15	1600	$3\frac{19}{32}$	$1\frac{5}{8}$				
				.25	1400	$4\frac{3}{4}$	$1\frac{7}{8}$				
				.5	1200	$5\frac{5}{8}$	$2\frac{1}{8}$				
				1	1000	7	$2\frac{3}{16}$				
				1.5	975	$7\frac{3}{4}$	$3\frac{5}{16}$				
				2.25	950	$8\frac{1}{4}$	$3\frac{9}{16}$	2	500	12	$3\frac{1}{4}$
				2.5	1200	$7\frac{3}{16}$	$3\frac{1}{8}$				
3.5	1600	$7\frac{3}{16}$	$3\frac{1}{8}$	4	925	$9\frac{3}{16}$	$4\frac{13}{16}$	4	425	15	$3\frac{3}{4}$
				4.5	1150	$8\frac{1}{2}$	$3\frac{1}{2}$				
5.75	1400	$8\frac{1}{2}$	$3\frac{1}{2}$	6.5	1050	$9\frac{1}{4}$					
				6.5	950	$9\frac{1}{4}$	6				
				6.5	875	$10\frac{1}{8}$	$5\frac{5}{8}$	7	400	19	$4\frac{1}{4}$
8.25	1300	$9\frac{1}{4}$		8.5	850	11	$6\frac{1}{4}$				
9	1450	$9\frac{1}{4}$	6	9	950	11	5	9	350	21	5
11.5	1300	11	5	13	900	$11\frac{5}{8}$	$6\frac{5}{8}$				
17.5	1300	$11\frac{5}{8}$	$6\frac{5}{8}$	13.5	850	13	$6\frac{1}{2}$	15	325	$22\frac{1}{2}$	$6\frac{1}{2}$
17.5	1175	13	$6\frac{1}{2}$	18	875	13	7				
23	1150	13	7	20	700	16	7	20	300	$20\frac{1}{4}$	$6\frac{3}{4}$
27	1125	14	$7\frac{1}{2}$	22.5	850	14	$7\frac{1}{2}$				
30	1050	16	7	30	775	15	8	30	300	$22\frac{1}{4}$	$8\frac{1}{4}$
40	1050	15	8	30	675	18	$8\frac{3}{4}$				
45	975	18	$8\frac{3}{4}$	45	700	$16\frac{1}{2}$	9	50	275	24	$9\frac{5}{8}$
55	930	$16\frac{1}{2}$	9	50	600	$20\frac{1}{4}$	$8\frac{3}{4}$	60	275	28	11
65	875	$20\frac{1}{4}$	$8\frac{3}{4}$	65	625	19	$10\frac{1}{2}$	75	250	$32\frac{1}{2}$	$12\frac{1}{2}$
80	750	19	$10\frac{1}{2}$	75	550	22	$9\frac{1}{4}$	100	250	37	$10\frac{7}{8}$
85	750	22	$9\frac{1}{4}$	90	560	24	$9\frac{5}{8}$	125	250	45	13
110	720	24	$9\frac{5}{8}$	130	500	$32\frac{1}{2}$	$12\frac{1}{2}$	150	200	46	$12\frac{3}{8}$
150	550	$32\frac{1}{2}$	$12\frac{1}{2}$					150	130	50	$16\frac{1}{2}$
200	450	$32\frac{1}{2}$	$13\frac{1}{2}$					200	175	50	$16\frac{1}{2}$
				250	225	50	$16\frac{1}{2}$	200	100	57	19
								250	125	57	19
								300	150	57	19
								300	150	68	14
								300	100	77	$14\frac{1}{2}$
								350	88	87	$15\frac{3}{4}$
								400	120	72	$18\frac{1}{4}$
								400	100	87	$15\frac{3}{4}$
								500	125	87	$15\frac{3}{4}$
								500	120	84	$17\frac{1}{4}$
								600	100	95	$17\frac{3}{4}$
								800	80	120	$19\frac{1}{2}$
								1600	75	164	19

It will be noted in Tables CXIII. and CXIV., where machines of different manufacturers are placed side by side, that the armature dimensions of dynamos for similar output and speed in some instances vary greatly from one another, which goes to show that a wide range is given to the judgment of the designer in assuming the variable quantities, such as peripheral speed, shape-ratio, etc. For instance, in Table CXIII. two 300-KW 150-revolution armatures are given, the first having a diameter of 57 inches and a length of 19 inches, while the

TABLE CXIV.—DRUM-ARMATURE DIMENSIONS.

High Speed.				Medium Speed.				Low Speed.			
KW.	R. p. m.	Diam.	Length.	KW.	R. p. m.	Diam.	Length.	KW.	R. p. m.	Diam.	Length.
.5	2400	$2\frac{5}{16}$	4					.56	1300	$5\frac{3}{4}$	$2\frac{5}{8}$
.75	2400	$2\frac{5}{8}$	5	.75	1900	$5\frac{3}{4}$	$2\frac{5}{8}$	1.5	1200	$6\frac{1}{2}$	$3\frac{1}{4}$
1.5	2100	$3\frac{25}{32}$	$6\frac{1}{2}$	1.875	1700	$6\frac{1}{2}$	$3\frac{1}{4}$	2.62	1050	$7\frac{3}{4}$	$4\frac{5}{8}$
3	1900	$4\frac{1}{2}$	8	3.75	1600	$7\frac{3}{4}$	$4\frac{5}{8}$	3.75	950	$8\frac{1}{2}$	$5\frac{1}{4}$
6	1800	$5\frac{11}{16}$	$9\frac{1}{4}$	5.62	1350	$8\frac{1}{2}$	$5\frac{1}{4}$	5.62	850	$9\frac{1}{2}$	$5\frac{1}{2}$
8.5	1700	$5\frac{7}{8}$	10	7.5	1250	$9\frac{1}{2}$	$5\frac{1}{2}$	7.5	750	$10\frac{1}{2}$	6
12	1600	$6\frac{1}{4}$	12	11.25	1150	$10\frac{1}{2}$	6	11.25	650	$11\frac{5}{8}$	$6\frac{3}{4}$
15	1500	$6\frac{3}{4}$	$13\frac{1}{2}$	15	1050	$11\frac{5}{8}$	$6\frac{3}{4}$	15	600	13	8
20	1400	$7\frac{3}{4}$	15	22.5	975	13	8	22.5	575	15	$8\frac{1}{2}$
25	1300	$8\frac{3}{4}$	$16\frac{1}{2}$	30	950	$13\frac{1}{8}$	$8\frac{1}{2}$	30	550	$15\frac{3}{4}$	$8\frac{1}{2}$
30	1200	$9\frac{3}{16}$	18	37.5	900	15	$8\frac{1}{2}$	37.5	550	$16\frac{1}{2}$	9
45	1000	$11\frac{1}{16}$	$20\frac{1}{2}$	45	850	$15\frac{3}{4}$	$8\frac{1}{2}$				
60	700	$12\frac{7}{8}$	$24\frac{1}{2}$	56.25	800	$16\frac{1}{2}$	9				
100	650	$16\frac{3}{8}$	25								
150	450	$23\frac{3}{4}$	$26\frac{1}{2}$								
200	450	$23\frac{3}{4}$	$34\frac{1}{4}$								

second has a diameter of 68 inches and a length of 14 inches. The peripheral velocity in the first machine is

$$v_c = \frac{57 \times \pi}{12} \times \frac{150}{60} = 37.3 \text{ feet per sec.,}$$

while in the second it is

$$v_c = \frac{68 \times \pi}{12} \times \frac{150}{60} = 44.6 \text{ feet per sec.}$$

The length is $\frac{19}{57}$, or $\frac{1}{3}$, of the diameter in the first armature, and $\frac{14}{68}$, or about $\frac{1}{5}$, of the diameter in the second armature.

APPENDIX II.

WIRE TABLES AND WINDING DATA.

WIRE TABLES AND WINDING DATA.

THE tables here compiled will be found useful in connection with dynamo calculation.

Table CXV., on pages 676 and 677, gives the resistance per foot and per pound, the weight per ohm, and the length per ohm of pure copper wire at 20° C. (68° F.), 60° C. (140° F.), and 100° C. (212° F.). The figures are substantially those adopted by the American Institute of Electrical Engineers upon the recommendation of the Committee on Standards, in 1893. The table is based on Matthiesson's standard of resistivity for soft copper, which is 1.5939 microhms per cu. cm. (1 cm. length, 1 sq. cm. cross-section), corresponding to 10.32 ohms per mil-foot (1 ft. length, 1 mil diameter, or 1 circular mil area), at 0° C. (32° F.). Specific gravity of copper wire, 8.90. The temperature coefficient of copper is taken as $(1 + .00388\, t)$, in which t is the elevation of temperature in degrees C. The data are given for all sizes of the American, or Browne & Sharpe (*B. & S.*) gauge as well as of the Birmingham wire gauge (*B. W. G.*). The wires are arranged according to size, so that the nearest standard gauge wire corresponding to any given resistance, weight, or length, can be obtained by referring to but one table.

Table CXVI., page 678, gives the winding data for B. & S. and B. W. G. wires, when insulated for use as armature wires. In the first five columns the gauge numbers, diameters, sectional areas, and resistances of the sizes commonly employed are repeated for convenience; the sixth column, headed "Diameter of Insulated Wire," gives the diameter of the respective wire when insulated with a double cotton covering (*D. C. C.*) which is the usual insulation on armature wires. The figures given in this column are in each case the maximum diameter of three samples of insulated wires furnished by different manufacturers. The next two columns, Nos. 7 and 8,

Size of Wire.				Cool at 20° C. (68° Fahr.)					
Gauge Number.		Diameter.	Area.	Resistance.		Weight.		Length.	
B & S.	B.W.G.	Inches.	Cir. Mils.	Ohms p. lb.	Ohms p. ft.	lbs. p. ohm.	lbs. p. ft.	ft. p. ohm.	feet p. lb.
0000		.460	211,600	.0000764	.0000488	13,140	.6412	20,495	1.560
	0000	.454	206,116	.0000802	.0000501	12,470	.6246	19,970	1.601
	000	.425	180,625	.0001045	.0000571	9,570	.5472	17,500	1.827
000		.4096	167,772	.0001215	.0000615	8,260	.5084	16,250	1.967
	00	.380	144,400	.0001634	.0000715	6,120	.4376	13,990	2.285
00		.365	133,225	.000193	.0000776	5,190	.4033	12,890	2.480
	0	.340	115,600	.000255	.0000893	3,920	.3503	11,200	2.885
0		.325	105,625	.000307	.0000980	3,260	.3199	10,230	3.126
	1	.300	90,000	.000421	.0001147	2,377	.2727	8,718	3.667
1		.289	83,521	.000488	.0001234	2,055	.2536	8,106	3.943
	2	.284	80,656	.000524	.0001280	1,909	.2444	7,812	4.092
	3	.259	67,081	.000757	.000154	1,320	.2033	6,497	4.919
2		.258	66,564	.000776	.000156	1,290	.2011	6,428	4.973
	4	.238	56,644	.001062	.000182	940	.1717	5,487	5.824
3		.229	52,441	.001235	.000196	813	.1595	5,098	6.270
	5	.220	48,400	.001455	.000213	688	.1467	4.688	6.817
4		.204	41,616	.001960	.000247	510	.1265	4,043	7.905
	6	.203	41,209	.00201	.000251	498	.1249	3,992	8.006
5		.182	33,124	.00312	.000312	320	.1003	3,205	9.970
	7	.180	32,400	.00325	.000319	308	.0982	3,138	10.135
	8	.165	27,225	.00460	.000379	218	.0825	2,637	12.12
6		.162	26,244	.00496	.000393	202	.0795	2,542	12.57
	9	.148	21,904	.00710	.000471	141	.0664	2,122	15.06
7		.1443	20,822	.00789	.000496	127	.0631	2,017	15.85
	10	.134	17,956	.01060	.000575	95	.0544	1,739	18.38
8		.1285	16,512	.01255	.000625	80	.0500	1,600	19.98
	11	.120	14,400	.0164	.000717	61.0	.0436	1,395	22.91
9		.1144	13,087	.0200	.000730	50 3	.0397	1,268	25.21
	12	.109	11,881	.0242	.000869	41 4	.0360	1,151	27.78
10		.1019	10,384	.0316	.000994	31.6	.0315	1,006	31.78
	13	.095	9,025	.0418	.001144	23.9	.0274	874	36.56
11		.0907	8,226	.0505	.001255	19.9	.0249	797	40.11
	14	.083	6,889	.0718	.00150	13.9	.0209	667	47.89
12		.0808	6,529	.0802	.00158	12.5	.0198	633	50.53
13	15	.072	5,184	.1275	.00200	7.9	.0157	502	63.65
	16	.065	4,225	.191	.00244	5.24	.0128	400	78.13
14		.0641	4,109	.203	.00251	4.95	.0125	398	80.32
	17	.058	3,364	.301	.00307	3.32	.0102	326	98.04
15		.0571	3,260	.323	.00317	3.12	.00988	316	101.2
16		.0508	2,581	.513	.00400	1.96	.00782	250	127.8
	18	.049	2,401	.591	.00430	1.69	.00728	233	137.4
17		.0453	2,052	.815	.00505	1.24	.00622	199	160.8
	19	.042	1,764	1.095	.00585	.91	.00535	171	187.0
18		.0403	1,624	1.296	.00636	.77	.00492	157	203.0
19		.0359	1,289	2.05	.00801	.49	.00391	125	256
	20	.035	1,225	2.27	.00843	.44	.00371	119	269
20	21	.032	1,024	3.25	.01010	.31	.00310	99.2	322
21		.0285	812	5.17	.0127	.19	.00246	78.7	406
	22	.028	784	5.54	.0132	.18	.00238	75.9	421
22		.0253	640	8.32	.0161	.12	.00194	62.0	516
	23	.025	625	8.73	.0165	.115	.00189	60.5	528
23		.0226	511	13.06	.0202	.077	.00155	49.5	646
	24	.022	484	14.54	.0213	.069	.00147	46.9	682
24	25	.020	400	21.30	.0258	.047	.00121	38.8	825
25	26	.018	324	32.45	.0319	.0308	.000982	31.4	1,019
26	27	.016	256	52.00	.0403	.0192	.000776	24 8	1,289
27	28	.014	196	88.7	.0527	.0113	.000594	19.0	1,684
	29	.013	169	119.3	.0611	.0084	.000512	16.4	1,952
28		.0126	159	135.1	.0650	.0074	.000481	15.4	2,078
	30	.012	144	164.3	.0717	.0061	.000436	14.00	2,291
29		.0113	128	209.0	.0808	.0048	.000387	12.40	2,584
30	31	.010	100	341.0	.1032	.00293	.000303	9.69	3,300
31	32	.009	81	520	.1275	.00193	.000246	7.85	4,073
32	33	.008	64	832	.1613	.00120	.000194	6.20	5,155
33	34	.007	49	1419	.2111	.000705	.000149	4.74	6,734
34		.0063	40	2164	.260	.000462	.000120	3.85	8,313
35		.0056	31	3465	.329	.000289	.000095	3.04	10,530
36	35	.005	25	5453	.413	.0001 3	.0000758	2.42	13,200

Size of Wire.		Warm at 60° C. (140° Fahr.)				Hot at 100° C. (212° Fahr.)			
Gauge Number.		Resistance.		Weight.	Length.	Resistance.		Weight.	Length.
B. & S.	B.W.G.	Ohms p. lb.	Ohms p. ft.	lbs. p. ohm.	ft. p. ohm.	ohms p. lb.	ohms p. ft.	lbs. p. ohm.	ft. p. ohm.
0000		.0000871	.0000558	11,480	17,920	.0000980	.0000629	10,200	15,910
	0000	.0000917	.0000573	10,900	17,460	.0001033	.0000645	9,681	15,500
	000	.0001195	.0000654	8,368	15,300	.0001346	.0000736	7,429	13,580
000		.0001385	.0000704	7,220	14,200	.0001560	.0000793	6,410	12,610
	00	.000187	.0000818	5,350	12.230	.0002105	.0000921	4,750	10,860
00		.000220	.0000887	4,539	11,250	.0002481	.0001000	4,031	10.004
	0	.000292	.0001021	3,427	9,797	.0003287	.0001151	3,042	8,688
0		.000350	.0001118	2,857	8,945	.0003942	.0001259	2,537	7,913
	1	.000481	.0001312	2,078	7,622	.000542	.0001478	1,845	6,766
1		.000557	.0001411	1,797	7,087	.000627	.000159	1,595	6,289
	2	.000599	.0001464	1,669	6,831	.000675	.000165	1,482	6,064
	3	.000866	.0001761	1,155	5,679	.000975	.000198	1,025	5.043
2		.000885	.0001780	1,129	5,618	.000997	.000200	1,003	4,990
	4	.001215	.0002084	823	4,798	.001368	.000235	731	4,261
3		.001408	.0002244	710	4,456	.001586	.000253	631	3,956
	5	.001664	.000244	610	4,098	.001874	.000275	534	3,639
4		.002237	.000283	447	3,536	.002520	.000319	397	3,139
	6	.00230	.000287	436	3,490	.002586	.000323	387	3,099
5		.00356	.000357	281	2,802	.004010	.000402	249	2,488
	7	.00371	.000365	269	2,743	.004180	.000411	239	2,435
	8	.00526	.000434	190	2,306	.00592	.000489	169	2,047
6		.00566	.000450	177	2,222	.00638	.000507	159	1,974
	9	.00812	.000539	123	1,855	.00915	.000607	109	1,647
7		.00899	.000567	111	1,763	.01013	.000639	98.7	1,565
	10	.0121	.000658	83	1,521	.01362	.000741	73.4	1,350
8		.0143	.000715	70	1,399	.01611	.000805	62.1	1,242
	11	.0188	.000820	53.2	1,220	.02117	.000923	47.2	1,083
9		.0228	.000902	44.0	1,101	.02563	.001016	39.0	984
	12	.0276	.000994	36.2	1,006	.03111	.001119	32.1	894
10		.0362	.001137	27.7	880	.04074	.001281	24.6	781
	13	.0479	.001309	20.9	764	.05391	.001474	18.6	678
11		.0576	.001436	17.4	696	.06487	.001617	15.4	618
	14	.0821	.001713	12.2	584	.0925	.00193	10.80	518
12		.0914	.00181	10.9	553	.1030	.00204	9.71	491
13	15	.1450	.00228	6.9	439	.1634	.00257	6.12	390
	16	.218	.00280	4.58	358	.246	.00315	4.07	318
14		.231	.00288	4.33	348	.260	.00324	3.85	309
	17	.344	.00351	2 91	285	.388	.00395	2.68	253
15		.367	.00362	2.73	276	.413	.00408	2.42	245
16		.585	.00458	1.71	219	.659	.00515	1.52	194
	18	.676	.00492	1.48	203	.761	.00554	1.31	181
17		.926	.00575	1.080	174	1.043	.00648	.959	154
	19	1.253	.00669	.790	149	1.411	.00754	.709	133
18		1.478	.00727	.676	138	1.665	.00819	.601	122
19		2.35	.00916	.426	109 0	2.64	.01031	.379	97
	20	2.60	.00964	.385	104.0	2.93	.01086	.342	92
20	21	3.72	.01153	.269	86.7	4.19	.0130	.239	77
21		5.91	.01454	.169	68.8	6.66	.0164	.150	61
	22	6.34	.01506	.158	66.4	7.14	.0170	.140	59
22		9.52	.01845	.105	54.2	10.72	.0208	.093	48
	23	9.98	.0189	.100	52.9	11.24	.0213	.0890	47
23		14.94	.0231	.067	43.2	16.83	.0260	.0594	38.4
	24	16.63	.0244	.060	40.9	18.73	.0275	.0534	36.4
24	25	24.4	.0295	.041	33.9	27.44	.0333	.0364	30.1
25	26	37.1	.0365	.0269	27.4	41.81	.0411	.0239	24.4
26	27	59.5	.0461	.0168	21.7	67.00	.0519	.0149	19.3
27	28	101.4	.0603	.0099	16.6	114	.0679	.00876	14.7
	29	136.5	.0699	.0073	14.3	154	.0787	.00651	12.7
28		154.5	.0744	.0065	13.4	174	.0838	.00574	11.9
	30	188	.0820	.00530	12.20	212	.0923	.00472	10.80
29		239	.0925	.00420	10.80	269	.1042	.00372	9.60
30	31	390	.1181	.00257	8.47	439	.1330	.00228	7.52
31	32	594	.1458	.001680	6.86	669	.1643	.00149	6.09
32	33	952	.1845	.001050	5.42	1,072	.2078	.000933	4.81
33	34	1623	.2415	.000616	4.14	1,828	.2719	.000547	3.68
34		2475	.2975	.000404	3.36	2,788	.335	.000359	2.98
35		3964	.3766	.000252	2.66	4,464	.424	.000224	2.36
36	35	6237	.4723	.000160	2.12	7,026	.532	.000142	1.88

TABLE CXVI.—Data of Armature Wire. (D.C.C.)

Size of Wire. Gauge Number. B. & S.	Size of Wire. Gauge Number. B.W.G.	Size of Wire. Diameter. Inches.	Size of Wire. Area. Cir. Mils.	Resistance at 20° C. (68° Fahr.) Ohms per Foot.	Diameter of Insulated Wire. (D.C.C.) Inches.	Number of D.C.C. Wires per Inch.	Number of D.C.C. Wires per Square Inch.	Weight of D.C.C. Wire. Lbs. p. Foot.	Resistance at 20° C. (68° F.) per Cubic Inch of D.C.C. Wire. Ohms.
	1	.300	90,000	.0001147	.320	3.1	9.8	.279	.000094
1		.289	83,521	.0001234	.309	3.25	10.5	.260	.000107
	2	.284	80,656	.0001280	.304	3.3	10.8	.250	.000116
	3	.259	67,081	.000154	.279	3.6	12.9	.208	.000167
2		.258	66,564	.000156	.278	3.6	13.0	.206	.000169
	4	.238	56,644	.000182	.258	3.9	15.1	.176	.000230
3		.229	52,441	.000196	.249	4.0	16.2	.164	.000267
	5	.220	48,400	.000213	.240	4.2	17.4	.150	.000310
4		.204	41,616	.000247	.224	4.45	20.0	.130	.000415
	6	.203	41,209	.000251	.223	4.5	20.2	.1285	.000423
5		.182	33,124	.000312	.200	5.0	25.0	.1030	.000651
	7	.180	32,400	.000319	.198	5.1	25.5	.1010	.000680
	8	.165	27,225	.000379	.183	5.5	30.0	.0852	.00095
6		.162	26,244	.000393	.180	5.6	31.0	.0821	.00106
	9	.148	21,904	.000471	.164	6.1	37.2	.0685	.00147
7		.1443	20,822	.000496	.160	6.3	39.0	.0652	.00162
	10	.134	17,956	.000575	.150	6.7	44.6	.0563	.00215
8		.1285	16,512	.000625	.145	6.9	47.5	.0518	.00249
	11	.120	14,400	.000717	.136	7.4	54.2	.0454	.00325
9		.1144	13,087	.000789	.130	7.7	58.3	.0415	.00442
	12	.109	11,881	.000869	.125	8.0	64.0	.0376	.00465
10		.1019	10,384	.000994	.117	8.5	73.0	.0331	.00607
	13	.095	9,025	.001144	.110	9.1	82.8	.0289	.00792
11		.0907	8,226	.001255	.106	9.4	89.0	.0264	.00934
	14	.083	6,889	.00150	.095	10.5	110.5	.0220	.01383
12		.0808	6,529	.00158	.093	10.8	115.6	.0208	.01526
13	15	.072	5,184	.00199	.084	12.0	145.2	.0166	.02410
	16	.065	4,225	.00244	.077	13.0	169.0	.0136	.0345
14		.0641	4,109	.00251	.075	13.3	177.8	.0133	.0373
	17	.058	3,364	.00307	.068	14.7	216.0	.0108	.0553
15		.0571	3,260	.00317	.067	14.9	222.8	.01050	.0590
16		.0508	2,581	.00400	.061	16.4	268.7	.00835	.0896
	18	.049	2,401	.00430	.059	17.0	289.0	.00780	.1038
17		.0453	2,052	.00503	.055	18.2	330.6	.00672	.1392
	19	.042	1,764	.00585	.052	19.3	371.0	.00580	.1815
18		.0403	1,624	.00636	.050	20.0	400.0	.00533	.2124
19		.0359	1,289	.00801	.046	21.7	473.5	.00426	.3172
	20	.035	1,225	.00843	.045	22.2	495.0	.00407	.3480
20	21	.032	1,024	.01010	.042	23.8	566.9	.00343	.4790
21		.0285	812	.0127	.039	25.7	660.0	.00276	.7375
	22	.028	784	.0132	.038	26.3	692.5	.00266	.7620
22		.0253	640	.0161	.035	28.6	876.3	.00221	1.0963
	23	.025	625	.0165	.035	28.6	876.3	.00215	1.210
23		.0226	511	.0202	.033	30.3	918.3	.00179	1.555
	24	.022	484	.0213	.032	31.3	980.0	.00170	1.746
24	25	.020	400	.0258	.030	33.3	1111.1	.00142	2.373
25	26	.018	324	.0319	.028	35.7	1276.4	.00117	3.436
26	27	.016	256	.0403	.026	38.5	1479.3	.00094	5.022

Size of Wire.				Diameter of Insulated Wire. (S.C.C.) Inches.	S.C.C. Wire in Coils.			Weight.		Percentage of Solid Copper in Any Volume of Winding.	Resistance at 20° C. (68° F.) per Cubic Inch of Winding. Ohms.
Gauge Number.		Diameter.	Area.		Nbr. of Turns p. Inch.	Nbr. of Layers p. Inch.	Nbr. of Turns p.Sq.In.	Pounds p. Foot S.C.C.	Pounds p. Cu. Inch of Winding.		
B. & S.	B.W.G.	Inches.	Cir. Mils.								
	1	.300	90,000	.320*	3.1*	3.5*	10.8*	.279*	.251*	76.8 *	.0001036*
1		.289	83,521	.309*	3.25*	3.65*	11.8*	.260*	.256*	77.9 *	.0001217*
	2	.284	80,656	.304*	3.3*	3.7*	12.2*	.250*	.254*	77.6 *	.0001305*
	3	.259	67,081	.279*	3.6*	4.05*	14.5*	.208*	.247*	76.7 *	.0001865*
2		.258	66,564	.278*	3.6*	4.1*	14.7*	.206*	.253*	76.9 *	.0001905*
	4	.238	56,644	.258*	3.9*	4.4*	17.1*	.176*	.251*	76.4 *	.0002600*
3		.229	52,441	.249*	4.0*	4.5*	18*	.164*	.246*	74.7 *	.000296*
	5	.220	48,400	.240*	4.2*	4.7*	19.8*	.150*	.248*	75.6 *	.000353*
4		.204	41,616	.216	4.6	5.15	23.7	.129	.255	78.0	.000492
	6	.203	41,209	.215	4.65	5.2	24.2	.128	.258	78.6	.00051
5		.182	33,124	.194	5.15	5.8	29.9	.103	.257	78.2	.00078
	7	.180	32,400	.192	5.2	5.85	30.4	.100	.254	77.7	.00081
	8	.165	27,225	.177	5.65	6.35	35.9	.0845	.253	77.2	.00113
6		.162	26,244	.172	5.8	6.5	37.8	.081	.257	78.3	.00124
	9	.148	21,904	.158	6.3	7.1	43.7	.068	.248	75 5	.00172
7		.1443	20,822	.154	6.5	7.3	47.5	.0645	.255	78.1	.00196
	10	.134	17,956	.144	6.95	7.8	54.2	.0556	.251	76.8	.00261
8		.1285	16,512	.139	7.2	8.1	58.3	.0512	.249	76.0	.00308
	11	.120	14,400	.130	7.7	8.65	66.5	.0447	.248	75.4	.00350
9		.1144	13,087	.124	8.0	9.0	72	.0407	.244	72.6	.00475
	12	.109	11,881	.119	8.4	9.45	79.5	.0369	.245	74.5	.00577
10		.1019	10,384	.112	8.9	10	89	.0324	.240	73.0	.00738
	13	.095	9,025	.105	9.5	10.7	102	.0282	.239	72.8	.00975
11		.0907	8,226	.101	9.9	11.1	110	.0257	.235	71.3	.01154
	14	.083	6,889	.090	11.1	12.5	139	.0214	.248	75.7	.0173
12		.0808	6,529	.088	11.3	12.7	144	.0203	.243	74.3	.0190
13	15	.072	5,184	.079	12.6	14.2	179	.0162	.241	73.1	.0298
	16	.065	4,225	.072	13.9	15.6	217	.0132	.239	72.4	.0443
14		.0641	4,109	.071	14.0	15.7	220	.0129	.237	71.6	.0462
	17	.058	3,364	.065	15.4	17.3	266	.0106	.235	70.7	.0683
15		.0571	3,260	.064	15.6	17.5	273	.01024	.233	70.3	.072
16		.0508	2,581	.058	17.2	19.3	332	.00815	.226	67.5	.111
	18	.049	2,401	.056	17.8	20	356	.00760	.225	67.5	.128
17		.0453	2,052	.051	19.6	22	431	.00645	.232	69.8	.181
	19	.042	1,764	.047	21.2	23.8	504	.00554	.233	70.3	.216
18		.0403	1,624	.045	22.2	25	555	.00510	.236	71.1	.295
19		.0359	1,289	.041	24.4	26.8	654	.00408	.223	66.5	.437
	20	.035	1,225	.040	25	28.1	703	.00387	.227	68.0	.495
20	21	.032	1,024	.037	27	30.4	821	.00325	.223	66.3	.690
21		.0285	812	.034	29.4	33	970	.00261	.211	62.1	1.030
	22	.028	784	.033	30.3	34	1,030	.00252	.216	63.8	1.134
22		.0253	640	.030	33.0	37	1,220	.00206	.209	61.5	1.643
	23	.025	625	.030	33.3	37.5	1,250	.00202	.211	61.6	1.725
23		.0226	511	.028	35.7	40	1,430	.00168	.200	57.7	2.415
	24	.022	484	.027	37	41.6	1,540	.00159	.204	59.0	2.750
24	25	.020	400	.025	40	45	1,800	.00132	.198	56.6	3.88
25	26	.018	324	.023	43.5	49	2,130	.001075	.191	54.5	5.67
26	27	.016	256	.021	47.6	53.5	2,550	.000856	.182	51.5	8.60
27	28	.014	196	.019	52.6	59	3,100	.000660	.170	48.0	13.62
	29	.013	169	.018	55.5	62.5	3,470	.000570	.165	46.3	17.70
28		.0126	159	.018	56	63	3,530	.000538	.158	44 2	19.20
	30	.012	144	.017	58.8	66.2	3,900	.000488	.159	44.2	24.4
29		.0113	128	.016	62.5	70.3	4,400	.000435	.160	44.3	29.6
30	31	.010	100	.015	66.7	75	5,000	.000342	.1425	39.5	43.1

* Double Cotton Insulation.

headed "Number of D. C. C. Wires per Inch" and "Number of D. C. C. Wires per Square Inch," respectively, are convenient for computing the number of conductors that can be placed on a given armature. By multiplying the available

TABLE CXVIII.—LIMITING CURRENTS FOR COPPER WIRES.

SIZE OF WIRE.		Usual Limit given by Fire Underwriters. Amperes.	Current producing a Rise of 40° C. (72° Fahr.) Amperes.	Current producing Smoking Point of Insulation. Amperes.
B. & S.	B.W.G.			
0000		210	360	630
	0000	206	352	616
	000	186	316	552
000		177	298	519
	00	159	263	464
00		150	246	438
	0	135	220	396
0		127	206	372
	1	113	182	335
1		107	172	319
	2	104	167	311
2	3	90	144	273
	4	80	127	248
3		76	120	237
	5	72	114	220
4	6	65	104	207
5		54	90	178
	7	51	87	170
	8	47	80	158
6		46	78	155
	9	41	70	138
7		40	68	134
	10	35	61	130
8		33	58	127
	11	30	54	114
9		28	52	107
	12	26	49	97
10		24	46	87
	13	22	42	80
11		20	40	75
	14	18	35	67
12		17	33	65
13	15	14	30	56
	16	13	27	50
14		12	25	48

circumference of the armature core by the number of wires per inch (column 7), the approximate number of wires per layer of the armature is obtained; and when the sectional area of the winding space is multiplied by the number of wires per

TABLE CXIX.—Carrying Capacity of Copper Wires.

Size of Wire.		Capacity, in Amperes, for Current-Density of:					
B. & S.	B.W.G.	1000 Amps. per sq. in., or 1275 Cir. Mils per Amp.	2000 Amps. per sq. in., or 640 Cir. Mils per Amp.	3000 Amps. per sq. in., or 425 Cir. Mils per Amp.	4000 Amps. per sq. in., or 320 Cir. Mils per Amp.	5000 Amps. per sq. in., or 255 Cir. Mils per Amp.	6000 Amps. per sq. in., or 212 Cir. Mils per Amp.
0000		166	332	498	664	840	996
	0000	162	324	486	648	810	972
	000	142	283	425	567	708	850
000		132	263	395	527	658	790
	00	113	227	340	453	567	680
00		105	209	314	419	523	628
	0	92	183	275	367	458	550
0		83	166	249	332	415	498
	1	71	141	212	283	353	424
1		66	131	197	263	328	394
	2	63	127	190	253	317	380
	3	53	105.5	158	211	263	316
2		52	104.5	157	209	262	314
	4	45	89	133.5	178	223	267
3		41	83	123.5	165	206	247
	5	38	76	114	152	190	228
4		33	65	98	131	163	196
	6	32	64.5	97	129	161	194
5		26	52	78	104	130	156
	7	25.5	51	71.5	102	127.5	153
	8	21.3	43	64	83	107	128
6		20.7	41	62	83	103	124
	9	17.2	34	51.5	69	86	103
7		16.3	33	49	65	82	98
	10	14.1	28	42	56.5	71	85
8		13.0	26	39	52	65	78
	11	11.3	23	34	45	57	68
9		10.3	20	31	41	51.5	62
	12	9.35	19	28	37.4	47	56
10		8.2	16	24.5	32.6	41	49
	13	7.1	14	21.3	28.4	35.5	43
11		6.5	13	19.4	25.9	32.4	39
	14	5.4	10.8	16.3	21.7	27.1	32.5
12		5 1	10.3	15.4	20.6	25.7	30.8
13	15	4.1	8.2	12.3	16.3	20.4	24.5
	16	3.3	6.6	10.0	13.2	16.6	19.9
14		3.2	6.5	9.7	12.9	16.2	19.4
	17	2.65	5 3	8.0	10.6	13.3	15.9
15		2.57	5 25	7.7	10.3	12.8	15.4
16		2.03	4.1	6.1	8.1	10.3	12.2
	18	1.88	3.8	5.7	7.3	9 4	11.3
17		1.61	3.2	4.85	6.4	8.1	9.7
	19	1.39	2.8	4.15	5.6	7.0	8.3
18		1.28	2.56	3.83	5.1	6.4	7.7
19		1.01	2.02	3 04	4.0	5.1	6.1
	20	.96	1.82	2.89	3.8	4.8	5.8
20	21	.81	1.61	2.42	3.2	4.0	4.8
21		.64	1.28	1.92	2.56	3.2	3 83
	22	.62	1.24	1.85	2.48	3.1	3 70
22		.50	1.00	1.51	2 01	2.5	3 02
	23	.49	.98	1.48	1.96	2.45	2.95
23		.40	.80	1.21	1.61	2.0	2.41
	24	.38	.76	1.14	1.42	1.9	2.28
24	25	.32	.62	.95	1.24	1.58	1.89
25	26	.26	.51	.77	1.02	1.28	1.53
26	27	.20	.40	.61	.81	1.01	1.21
27	28	.15	.30	.46	.61	.75	.92
	29	.13	.26	.40	.52	.65	.80
28		.125	.25	.375	.50	.625	.75
	30	.11	.22	.34	.44	.55	.68
29		.10	.20	.30	.40	.50	.60
30	31	.08	.16	.235	.32	.40	.47

square inch (column 8), the approximate total number of wires on the armature is found. The ninth column, headed "Weight of D. C. C. Wire, Pounds per Foot," serves to compute the weight of the armature winding, and the last column (No. 10), headed "Resistance at 20° C. per Cubic Inch of D. C. C. Wire," can be used to find the approximate resistance of the armature wire, and from this the armature resistance, if only the size of the wire and the dimensions of the winding space are given.

In Table CXVII., page 679, data similar to Table CXVI. are given for wires having a single cotton covering (*S. C. C.*)

TABLE CXX.—Carrying Capacity of Circular Copper Rods.

Diameter of Rod, in Inches.	Area of Rod,		Current, in Amperes, Causing a Temperature Rise of			
	in Square Inches.	in Circular Mils.	10° C. (18° F.)	20° C. (36° F.)	30° C. (54° F.)	40° C. (72° F.)
½	.196	250,000	300	400	450	550
¾	.442	562,500	500	650	800	950
1	.785	1,000,000	750	950	1,200	1,400
1¼	1.227	1,562,500	1,000	1,350	1,650	1,900
1½	1.767	2,250,000	1,300	1,800	2,200	2,550
1¾	2.405	3,062,500	1,600	2,300	2,800	3,200
2	3.142	4,000,000	2,000	2,800	3,450	3,950
2¼	3.976	5,062,500	2,350	3,300	4,100	4,700
2½	4.909	6,250,000	2,750	3,900	4,800	5,550
2¾	5.940	7,562,500	3,250	4,500	5,500	6,500
3	7.069	9,000,000	3,600	5,600	6,300	7,200
3¼	8.296	10,562,500	4,100	5,800	7,000	8,000
3½	9.621	12,250,000	4,600	6,500	7,800	9,000
3¾	11.045	14,062,500	5,000	7,200	8,600	10,000
4	12.566	16,000,000	5,200	7,700	9,200	11,000

as is used for magnet winding. The fifth column, headed "Diameter of Insulated Wire, S. C. C.," contains the maximum outside diameter of a number of samples of single covered wire from different manufacturers. The sixth column, marked "Number of Turns per Inch," gives the number of wires which can be laid side by side in the length of one inch; this column serves to find the number of wires per layer on a magnet of given length. The figures in the seventh column represent the number of layers of the given size of wire that can be wound per inch height of winding space. In the table of

armature winding data the corresponding figures are omitted, because in an armature the wires of different layers are usually placed in line, vertically, so that the number of layers per inch is identical with the number of turns per inch. In winding a magnet, however, the wires of each following layer are placed into the hollows left between the wires of the preceding layer; more layers can therefore be wound per inch height on a magnet than on an armature, and the figures in the column headed "Number of Wires per Layer" are thus greater than the corresponding figures in the preceding column. While the number of wires per square inch for an armature is the square of the number of wires per inch, the number of turns per square inch for a magnet (given in column 8) is the product of the number of turns per inch by the number of layers per inch. The two succeeding columns, Nos. 9 and 10, headed respectively "Pounds per Foot S. C. C." and "Pounds per Cubic Inch of Winding," serve to determine the weight of the magnet winding when the total length of wire, or the total space occupied by the winding, is known. From column 11 the percentage of solid copper in any volume wound with S. C. C. magnet wire can be taken. It will be noted that this percentage is greatest for the larger diameters given and diminishes with the size of wire, which is due to the fact that on a small wire the insulation occupies a relatively much greater space than on a large wire. Column 12, finally, gives the resistance of a length of each wire which will just fill a cubic inch of the winding space. Knowing the total contents of the latter, a simple multiplication will give the total resistance of the magnet winding at 20° C.

Table CXVIII., page 680, gives (1) the maximum current allowed by the Fire Underwriters, in various sizes of wires for use in buildings, (2) the currents which produce a rise of 40° C. (72° F.) in the temperature of the wire, and (3) the currents which cause the wires to heat up to the smoking point of cotton insulation (about 180° C. or 356° F.). The Underwriters' limits are only given for comparison; they do not apply to dynamo windings, and therefore do not directly interest the dynamo designer. The figures contained in the two remaining columns are intended as a check on the wire calculation, the former limit corresponding to the usual rise of tempera-

Size of Single Wire.		Nearest Gauge of Wire to Obtain Area Equivalent to Single Wire, if Number of Wires which Make Up One Conductor, is:						
B. & S.	B.W.G.	2	3	4	5	6	7	8
0000		0 B. & S.	3 B.W.G.	3 B. & S.	4 B. & S.	5 B. & S.	7 B.W.G.	6 B. & S.
	0000	0 B. & S.	3 B.W.G.	3 B. & S.	6 B.W.G.	5 B. & S.	7 B.W.G.	6 B. & S.
	000	1 B. & S.	2 B. & S.	5 B.W.G.	6 B.W.G.	7 B.W.G.	6 B. & S.	9 B.W.G.
000		1 B. & S.	4 B.W.G.	4 B. & S.	5 B. & S.	8 B.W.G.	6 B. & S.	7 B. & S.
	00	2 B.W.G.	5 B.W.G.	6 B.W.G.	7 B.W.G.	6 B. & S.	7 B. & S.	10 B.W.G.
00		2 B. & S.	5 B.W.G.	5 B. & S.	8 B.W.G.	9 B.W.G.	10 B.W.G.	8 B. & S.
	0	4 B.W.G.	6 B.W.G.	8 B.W.G.	9 B.W.G.	7 B. & S.	8 B. & S.	11 B.W.G.
0		3 B. & S.	5 B. & S.	6 B. & S.	9 B.W.G.	10 B.W.G.	11 B.W.G.	9 B. & S.
	1	5 B.W.G.	7 B.W.G.	9 B.W.G.	10 B.W.G.	11 B.W.G.	9 B. & S.	12 B.W.G.
1		4 B. & S.	8 B.W.G.	7 B. & S.	8 B. & S.	11 B.W.G.	12 B.W.G.	10 B. & S.
	2	6 B.W.G.	6 B. & S.	7 B. & S	8 B. & S.	9 B. & S.	12 B.W.G.	10 B. & S.
	3	5 B. & S.	9 B.W.G.	8 B. & S.	9 B. & S.	12 B.W.G.	10 B. & S.	11 B. & S.
2		5 B. & S.	9 B.W.G.	8 B. & S.	9 B. & S.	12 B.W.G.	10 B. & S.	11 B. & S.
	4	8 B.W.G.	10 B.W.G.	11 B.W.G.	12 B.W.G.	13 B.W.G.	11 B. & S.	14 B. & S.
3		6 B. & S.	10 B.W.G.	9 B. & S.	10 B. & S.	13 B.W.G.	11 B. & S.	12 B. & S.
	5	6 B. & S.	8 B. & S.	12 B.W.G.	10 B. & S.	11 B. & S.	14 B.W.G.	12 B. & S.
4		7 B. & S.	11 B.W.G.	10 B. & S.	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S.
	6	7 B. & S.	11 B.W.G.	10 B. & S.	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S.
5		8 B. & S.	12 B.W.G.	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S.	14 B. & S.
	7	8 B. & S.	10 B. & S.	11 B. & S.	12 B. & S.	13 B. & S.	16 B.W.G.	14 B. & S.
	8	11 B.W.G.	13 B.W.G.	14 B.W.G.	13 B. & S	16 B.W.G.	14 B. & S.	17 B.W.G.
6		9 B. & S.	13 B.W.G.	12 B. & S.	13 B. & S.	16 B.W.G.	14 B. & S.	15 B. & S.
	9	12 B.W.G.	11 B. & S.	13 B. & S.	16 B.W.G.	14 B. & S.	15 B. & S.	16 B. & S.
7		10 B. & S.	14 B.W.G.	13 B. & S.	14 B. & S.	17 B.W.G.	15 B. & S.	16 B. & S.
	10	13 B.W.G.	12 B. & S.	16 B.W.G.	17 B.W.G.	15 B. & S.	16 B. & S	18 B.W.G.
8		11 B. & S.	13 B. & S.	14 B. & S.	17 B.W.G.	16 B. & S.	18 B.W.G.	17 B. & S.
	11	14 B.W.G.	13 B. & S.	17 B.W.G.	15 B. & S.	16 B. & S.	17 B. & S.	19 B.W.G.
9		12 B. & S.	16 B.W.G.	15 B. & S.	16 B. & S.	18 B.W.G.	17 B. & S.	18 B. & S.
	12	12 B. & S.	14 B. & S.	15 B. & S.	18 B.W.G.	17 B. & S.	18 B. & S.	18 B. & S.
10		13 B. & S.	17 B.W.G.	16 B. & S.	17 B. & S.	19 B.W.G.	18 B. & S.	19 B. & S.
	13	16 B.W.G.	15 B. & S.	18 B.W.G.	19 B.W.G.	18 B. & S.	19 B. & S.	20 B.W.G.
11		14 B. & S.	16 B. & S.	17 B. & S.	18 B. & S.	19 B. & S.	20 B.W.G.	20 B. & S.
	14	17 B.W.G.	18 B.W.G.	19 B.W.G.	18 B. & S.	20 B.W.G.	20 B. & S.	20 B. & S.
12		15 B. & S	17 B. & S.	18 B. & S.	19 B. & S.	20 B. & S.	20 B. & S.	21 B. & S.
13	15	16 B & S.	19 B.W.G.	19 B. & S	20 B. & S.	21 B. & S.	22 B.W.G.	22 B. & S.
	16	17 B. & S.	18 B. & S.	20 B. & S.	21 B. & S.	22 B.W.G.	23 B.W.G.	23 B. & S.
14		17 B. & S.	19 B. & S.	20 B. & S.	21 B. & S.	22 B. & S.	23 B.W.G.	23 B. & S.
	17	18 B. & S.	20 B.W.G.	21 B. & S.	22 B. & S.	23 B.W.G.	24 B.W.G.	24 B. & S.
15		18 B. & S.	20 B. & S.	21 B. & S.	22 B. & S.	23 B.W.G.	24 B.W.G.	24 B. & S.
16		19 B. & S	21 B. & S.	22 B. & S.	23 B. & S.	24 B.W.G.	24 B. & S.	25 B. & S.
	18	20 B.W.G.	21 B. & S.	23 B.W.G.	24 B.W.G.	24 B. & S.	25 B. & S.	25 B. & S.
17		20 B. & S.	22 B.W.G.	23 B. & S	24 B. & S.	25 B. & S.	25 B. & S.	26 B. & S.
	19	20 B. & S	23 B.W.G.	24 B.W.G.	24 B. & S.	25 B. & S.	26 B. & S.	26 B. & S.
18		21 B. & S.	23 B. & S.	24 B. & S.	25 B. & S.	26 B. & S.	26 B. & S.	27 B. & S.
19		22 B. & S.	24 B. & S.	25 B. & S.	26 B. & S.	27 B. & S.	27 B. & S.	28 B. & S.
	20	23 B.W.G.	24 B. & S.	25 B. & S.	26 B. & S.	27 B. & S.	29 B.W.G.	28 B. & S.
20	21	23 B. & S.	25 B. & S.	26 B. & S	27 B. & S.	29 B.W.G.	30 B.W.G.	29 B. & S.

Size of Single Wire. B. & S.	Size of Single Wire. B.W.G.	Nearest Gauge of Wire to Obtain Area Equivalent to Single Wire, if Number of Wires which Make Up One Conductor, is: 9	10	12	14	16	20	24
0000		9 B.W.G.	7 B. & S.	10 B.W.G.	11 B.W.G.	9 B. & S.	10 B. & S.	13 B.W.G.
	0000	9 B.W.G.	7 B. & S.	10 B.W.G.	11 B.W.G.	9 B. & S.	10 B. & S.	13 B.W.G.
	000	7 B. & S.	10 B.W.G.	8 B. & S.	9 B. & S.	12 B.W.G.	13 B.W.G.	11 B. & S.
000		10 B.W.G.	8 B. & S.	11 B.W.G.	12 B.W.G.	10 B. & S.	11 B. & S.	14 B.W.G.
	00	8 B. & S.	11 B.W.G.	12 B.W.G.	10 B. & S	13 B.W.G.	14 B.W.G.	12 B. & S.
00		11 B.W.G.	9 B. & S.	12 B.W.G.	13 B.W.G.	11 B. & S.	14 B.W.G.	15 B.W.G.
	0	9 B. & S.	12 B.W.G.	10 B. & S.	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S.
0		12 B.W.G	10 B. & S.	13 B.W.G.	11 B. & S.	14 B.W.G.	13 B. & S.	16 B.W.G.
	1	10 B. & S.	13 B.W.G.	11 B. & S.	14 B.W.G.	12 B. & S.	16 B.W.G.	14 B. & S.
1		13 B.W.G.	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S.	14 B. & S	17 B.W.G.
	2	13 B.W.G.	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S	14 B. & S.	17 B W.G.
	3	11 B. & S.	14 B.W.G.	12 B. & S.	13 B. & S	14 B. & S.	17 B.W.G.	15 B. & S.
2		11 B. & S	14 B.W.G.	12 B. & S.	13 B. & S.	14 B & S.	17 B.W.G.	15 B. & S.
	4	12 B. & S	12 B. & S.	13 B. & S.	14 B. & S.	17 B.W.G.	15 B. & S.	18 B.W.G.
3		12 B & S	13 B. & S	16 B.W G.	14 B. & S.	15 B. & S.	16 B. & S.	17 B. & S.
	5	13 B. & S	13 B. & S	14 B. & S	17 B.W.G.	15 B. & S.	18 B.W.G.	17 B. & S.
4		13 B. & S.	16 B.W.G	17 B.W.G.	15 B. & S.	16 B. & S.	17 B. & S	19 B.W.G.
	6	16 B.W.G.	14 B. & S.	17 B.W.G	15 B. & S.	16 B. & S	17 B. & S.	19 B.W.G.
5		14 B & S.	17 B.W.G.	15 B. & S.	18 B.W.G.	17 B. & S.	17 B. & S.	19 B. & S.
	7	17 B.W.G.	15 B. & S.	16 B. & S.	18 B.W.G.	17 B. & S.	19 B.W.G.	19 B. & S.
	8	15 B. & S	16 B. & S.	18 B.W.G.	17 B. & S.	19 B.W.G.	19 B. & S.	20 B.W.G.
6		15 B. & S	16 B. & S	18 B.W.G.	17 B. & S.	18 B. & S.	19 B. & S	20 B. & S.
	9	18 B.W.G	17 B. & S	19 B.W.G.	18 B. & S.	18 B. & S.	20 B.W.G.	20 B. & S.
7		18 B.W.G.	17 B. & S.	19 B.W G.	18 B. & S.	19 B. & S.	20 B. & S.	21 B. & S.
	10	17 B & S.	19 B.W.G.	18 B. & S.	19 B. & S.	20 B.W.G.	20 B. & S.	21 B. & S.
8		19 B.W.G.	18 B. & S.	19 B. & S	20 B.W.G.	20 B. & S.	21 B. & S.	22 B. & S.
	11	18 B. & S.	18 B. & S.	19 B. & S.	20 B. & S.	20 B. & S.	22 B.W.G.	23 B.W.G.
9		18 B. & S.	19 B & S	20 B.W.G.	20 B. & S.	21 B. & S.	22 B. & S.	23 B.W.G.
	12	19 B. & S.	20 B.W.G.	20 B. & S.	21 B. & S.	22 B.W.G.	23 B.W.G.	23 B. & S.
10		20 B.W.G.	20 B. & S.	21 B. & S.	21 B. & S.	22 B & S	23 B. & S.	24 B.W.G.
	13	20 B. & S.	21 B. & S.	22 B.W.G.	22 B. & S.	23 B.W.G	24 B.W.G.	24 B. & S.
11		20 B. & S	21 B. & S.	22 B. & S.	23 B.W.G.	23 B. & S	24 B. & S.	25 B. & S.
	14	22 B.W.G	22 B. & S.	23 B.W.G.	24 B.W.G.	24 B.W.G	25 B. & S.	25 B. & S.
12		22 B.W.G.	22 B. & S.	23 B. & S.	24 B.W.G.	24 B. & S.	25 B. & S.	26 B. & S.
13	15	23 B.W.G.	23 B. & S	24 B W.G.	24 B. & S.	25 B. & S.	26 B. & S.	27 B. & S.
	16	24 B.W.G	24 B. & S.	24 B & S.	25 B. & S	26 B. & S.	27 B. & S	29 B.W.G.
14		24 B.W.G.	24 B. & S.	25 B. & S.	25 B. & S.	26 B. & S	27 B. & S.	29 B.W.G.
	17	24 B. & S.	25 B. & S	25 B. & S.	26 B. & S.	27 B. & S.	29 B.W.G	30 B.W.G.
15		24 B. & S.	25 B. & S.	26 B. & S.	26 B. & S.	27 B. & S.	28 B. & S.	29 B. & S.
16		25 B. & S	26 B. & S	27 B. & S.	27 B. & S.	28 B. & S.	29 B. & S.	30 B. & S.
	18	26 B. & S.	26 B. & S.	27 B. & S.	29 B.W.G.	30 B.W.G.	29 B. & S.	30 B. & S.
17		26 B. & S.	27 B & S.	29 B.W.G.	30 B.W.G	29 B. & S.	30 B. & S.	—
	19	27 B. & S.	29 B.W.G.	30 B W.G.	29 B. & S	30 B. & S.	—	—
18		28 B. & S.	29 B. & S.	30 B.W.G.	29 B. & S	30 B. & S.	—	—
19		30 B.W.G.	29 B. & S.	30 B. & S.	—	—	—	—
	20	30 B.W.G.	29 B. & S	30 B. & S.	—	—	—	—
20	21	29 B. & S.	30 B. & S.	—	—	—	—	—

ture specified for dynamo magnets, and the currents given in the latter column being those which will injure the insulation.

Table CXIX., page 681, is self-explanatory. It gives the carrying capacities of wires from No. 0000 to No. 30 B. & S., and from No. 0000 to No. 31 B. W. G. for current densities of 1000, 2000, 3000, 4000, 5000, and 6000 amperes per square inch, respectively.

TABLE CXXII.—STRANDING OF STANDARD CABLES.

Number of Wires in Cable.	Diameter of Cable as Compared with Diameter of Single Wire.	Stranding of Cable.		
		Arrangement of Strands.	Number of Concentric Layers in Each Strand.	Number of Wires in Concentric Layers. (Commencing from Inner Layer.)
3	$2\frac{1}{4}$	1× 3	1	3
7	3	1× 7	2	1+6
12	$4\frac{1}{4}$	1× 12	2	3+9
19	5	1× 19	3	1+6+12
27	$6\frac{1}{4}$	1× 27	3	3+9+15
37	7	1× 37	4	1+6+12+18
48	$8\frac{1}{4}$	1× 48	4	3+9+15+21
49	9	7× 7	2	1+6
61	9	1× 61	5	1+6+12+18+24
75	$10\frac{1}{4}$	1× 75	5	3+9+15+21+27
91	11	1× 91	6	1+6+12+18+24+30
108	$12\frac{1}{4}$	1×108	6	3+9+15+21+27+33
127	13	1×127	7	1+6+12+18+24+30+36
133	15	7× 19	3	1+6+12
147	$14\frac{1}{4}$	1×147	7	3+9+15+21+27+33+39
169	15	1×169	8	1+6+12+18+24+30+36+42
192	$16\frac{1}{4}$	1×192	8	3+9+15+21+27+33+39+45
217	17	1×217	9	1+6+12+18+24+30+36+42+48
259	21	7× 37	4	1+6+12+18
343	37	7×7×7	2	1+6
427	27	7× 61	5	1+6+12+18+24

In Table CXX., page 682, the current capacities of copper rods from ½ inch to 4 inches in diameter, for temperature increases of 10°, 20°, 30°, and 40° C., respectively, are compiled. The values contained in this table have been obtained from actual tests on bare rods of commercial copper of about 98 per cent. conductivity, horizontally suspended.

Table CXXI., pages 684 and 685, is designed as a short-cut

TABLE CXXIII.—Number and Size of Wires in Cables of Given Cross-Section.

Cross-Section of Cable, in Circular Mils.	7	12	19	27	37	48	61	75	91	108	127	133	147	169	192	217	343	427
	Diameter of each Wire, in Inches, if Number of Wires in Cable is:																	
4,000,000	.756	.577	.458	.385	.329	.289	.256	.231	.210	.192	.177	.173	.165	.154	.145	.136	.107	.097
3,500,000	.707	.540	.430	.360	.308	.270	.240	.216	.196	.180	.166	.162	.154	.144	.135	.127	.101	.091
3,000,000	.655	.500	.397	.333	.285	.250	.222	.200	.182	.167	.154	.150	.143	.133	.125	.118	.094	.084
2,500,000	.597	.456	.363	.304	.260	.228	.203	.183	.166	.152	.140	.137	.130	.122	.114	.107	.085	.077
2,000,000	.534	.408	.324	.272	.233	.204	.181	.163	.148	.136	.125	.123	.117	.109	.102	.096	.076	.068
1,750,000	.500	.381	.303	.255	.217	.191	.169	.153	.139	.127	.117	.115	.109	.102	.095	.090	.071	.064
1,500,000	.463	.354	.281	.235	.201	.177	.157	.141	.128	.118	.109	.106	.101	.094	.088	.083	.066	.059
1,250,000	.422	.323	.257	.215	.184	.161	.143	.129	.117	.108	.099	.097	.092	.086	.081	.076	.060	.054
1,000,000	.377	.289	.229	.193	.164	.144	.128	.116	.105	.096	.089	.087	.083	.077	.072	.068	.054	.048
950,000	.368	.281	.224	.188	.160	.141	.125	.113	.102	.094	.086	.085	.080	.075	.070	.066	.053	.047
900,000	.359	.274	.218	.183	.156	.137	.121	.110	.100	.091	.084	.082	.078	.073	.069	.064	.051	.046
850,000	.348	.266	.212	.177	.152	.133	.118	.107	.097	.089	.082	.080	.076	.071	.067	.062	.050	.045
800,000	.338	.258	.205	.172	.147	.129	.115	.103	.094	.086	.079	.078	.074	.069	.065	.061	.048	.043
750 000	.327	.250	.199	.167	.142	.125	.111	.100	.091	.083	.077	.075	.071	.067	.063	.059	.047	.042
700,000	.316	.242	.192	.161	.138	.121	.107	.097	.088	.080	.074	.073	.069	.064	.060	.057	.045	.041
650,000	.305	.233	.185	.155	.133	.116	.103	.093	.085	.078	.072	.070	.067	.062	.058	.055	.044	.039
600,000	.293	.224	.178	.149	.127	.112	.099	.089	.081	.075	.069	.067	.064	.060	.056	.053	.042	.038
550.000	.280	.214	.170	.143	.122	.107	.095	.086	.078	.071	.066	.064	.061	.057	.054	.050	.040	.036
500,000	.267	.204	.162	.136	.116	.102	.091	.082	.074	.068	.063	.061	.058	.054	.051	.048	.038	.034
450,000	.254	.194	.154	.129	.110	.097	.086	.076	.070	.065	.060	.058	.055	.052	.048	.046	.036	.033
400,000	.239	.183	.145	.122	.104	.091	.081	.073	.066	.061	.056	.055	.052	.049	.046	.043	.034	.031
350,000	.224	.171	.136	.114	.097	.085	.076	.068	.062	.057	.053	.051	.049	.046	.043	.040	.032	.029
300,000	.207	.158	.126	.105	.090	.079	.070	.063	.057	.053	.049	.047	.045	.042	.040	.037	.030	.027
250,000	.189	.144	.115	.096	.082	.072	.064	.058	.052	.048	.044	.043	.041	.038	.036	.034	.027	.024
200,000	.169	.129	.103	.086	.074	.065	.057	.052	.047	.043	.040	.039	.037	.034	.032	.030	.024	.022
150,000	.146	.112	.089	.075	.064	.056	.050	.045	.041	.037	.034	.033	.032	.030	.028	.026	.021	.019
100,000	.120	.091	.073	.061	.052	.046	.040	.037	.033	.030	.028	.027	.026	.024	.023	.022	.017	.015

in armature winding calculations. The size of the single wire required to carry the given current having been found, this table gives directly, without further calculation, the size of wire for a subdivided armature conductor. The scope of this table, including all sizes from No. 0000 to No. 20 B. & S., and from No. 0000 to No. 21 B. W. G., and giving subdivisions from 2 to 24, is such that it will answer for all the usual cases occurring in practice.

TABLE CXXIV.—SIZE AND WEIGHT OF CABLES.

SIZE OF CABLE, B. & S. GAUGE.	AREA OF CABLE, CIRCULAR MILS.	MAXIMUM DIAMETER OF COPPER STRAND, INCHES.	OUTSIDE DIAMETER OF CABLE, INCHES.	WEIGHT PER FOOT OF CABLE, POUNDS.
....	1,000,000	$1\frac{1}{2}$	$1\frac{7}{8}$	3.7
....	900,000	$1\frac{3}{8}$	$1\frac{3}{4}$	3.0
....	800,000	$1\frac{11}{32}$	$1\frac{23}{32}$	2.9
....	750,000	$1\frac{5}{16}$	$1\frac{11}{16}$	2.8
....	700,000	$1\frac{1}{4}$	$1\frac{5}{8}$	2.5
....	650,000	$1\frac{7}{32}$	$1\frac{19}{32}$	2.4
....	600,000	$1\frac{3}{16}$	$1\frac{9}{16}$	2.1
....	550,000	$1\frac{1}{8}$	$1\frac{1}{2}$	2.0
....	500,000	$1\frac{1}{16}$	$1\frac{7}{16}$	1.9
....	450,000	1	$1\frac{3}{8}$	1.6
....	400,000	$\frac{15}{16}$	$1\frac{5}{16}$	1.5
....	350,000	$\frac{7}{8}$	$1\frac{1}{4}$	1.3
....	300,000	$\frac{13}{16}$	$1\frac{3}{16}$	1.2
....	250,000	$\frac{3}{4}$	$1\frac{1}{8}$	1.0
0000	211,600	$\frac{11}{16}$	$1\frac{1}{16}$	.82
000	168,100	$\frac{5}{8}$	1	.65
00	133,225	$\frac{9}{16}$	$\frac{15}{16}$	.55
0	105,625	$\frac{1}{2}$	$\frac{7}{8}$	.46
1	83,521	$\frac{7}{16}$	$\frac{13}{16}$	.38
2	66,564	$\frac{3}{8}$	$\frac{3}{4}$	.30
3	52,441	$\frac{11}{32}$	$\frac{23}{32}$	.27
4	41,616	$\frac{5}{16}$	$\frac{11}{16}$	.22
5	33,124	$\frac{9}{32}$	$\frac{21}{32}$	.20
6	26,244	$\frac{1}{4}$	$\frac{5}{8}$	.18

Tables CXXII., CXXIII., and CXXIV. give the data of flexible copper wire cables, such as are used for dynamo connections. In Table CXXII., page 686, the diameter of the copper strand of cables containing from 3 to 427 wires is given, and the arrangement of the various stranded cables is shown. The unit of the diameter in each case is the diameter of the wire employed in making up the cable, so that the diameter in inches is found by multiplying the given figure by the diameter of the wire. For instance, the diameter of the

copper strand of a cable consisting of 133 No. 13 B. & S. wires is 15 × .072 = 1.08 inches. From the last column it will be seen that cables are made up in either of two ways: (1) one single wire forms the center layer, 6 wires the second layer, 12 wires the third layer, etc.; or (2) the center layer consists of 3 wires, the second layer of 9 wires, the third of 15 wires, etc. Up to 9 concentric layers are used in this manner; cables requiring a larger number of wires are made

TABLE CXXV.—IRON WIRE FOR RHEOSTATS AND STARTING BOXES.

Size of Wire.		For Rheostats.				For Starting Boxes.		Resistance, Ohms per Foot.	Length, Feet per Ohm.	Weight, Pounds per Foot.
		Wood Frame.		Iron Frame.						
B. & S. Gauge.	Diameter in Inches.	Safe Capacity. Amps.	Resistance of Circuit Required on 110 Volts. Ohms.	Safe Capacity. Amps.	Resistance of Circuit Required on 110 Volts. Ohms.	Safe Capacity. Amps.	Resistance of Circuit Required on 110 Volts. Ohms.			
8	.1285	17.4	6.32	20.3	5.42	43.6	2.52	.00398	250.	.040
9	.1144	14.6	7.53	17.1	6.43	36.6	3.01	.00578	173.	.033
10	.1019	12.3	8.94	14.3	7.69	30.8	3 57	.00728	137.	.0275
11	.0907	10.3	10.68	12.0	9.17	25.8	4.26	.00918	108.	.0218
12	.0808	8.7	12.64	10.1	10.89	21.7	5.07	.01157	86.4	.0173
13	.072	7.3	15.07	8 5	12.94	18.3	6.01	.01459	68.5	.0137
14	.0641	6.1	18.03	7.1	15.49	15.3	7.19	.01840	54.3	.0109
15	.0571	5.1	21.57	6.0	18.33	12.9	8.53	.02320	43.1	.0063
16	.0508	4.3	25.58	5.0	22.00	10.8	10.19	.02925	34.1	.00685
17	.0453	3.6	30.56	4.2	26.19	9.1	12.09	.03688	27.1	.00543
18	.0403	3.0	36.67	3.5	31.43	7.6	14.47	.04652	21.4	.00430
19	.0359	2.52	43.65	2.9	37.93	6.3	17.46	.06032	16.5	.00341
20	.032	2.17	50 69	2.5	44.00	5.4	20.37	.07396	13.5	.00271
21	.0285	1.82	60.44	2.1	52.38	4.5	24 44	.09332	10.7	.00231
22	.0253	1.53	71.90	1.77	62.15	3.8	28 95	.11769	8.49	.001838
23	.0226	1.28	85.94	1.5	73.33	3.2	34 38	.14843	6.73	.001457
24	.020	1.08	101.85	1.2	91.67	2 3	47.83	.18717	5.34	.001155

up by stranding together 7 individual cables, each composed of the proper number of concentric layers. In the 343-wire cable, each of the 7 cables so stranded is again subdivided into 7 cables, each of the small cables consisting of 7 wires in two concentric layers. The figures given in Table CXXIII., page 687, are the diameters of the wire required to produce a given sectional area when used for making up a cable of a given number of wires. Thus, the size of wire for a

147-wire cable of 400,000 circular mils sectional area is given as .052 inch. The nearest gauge sizes are No. 15 and No. 16 B. & S.; the former is to be taken if it is not desirable to go below the specified area, and the latter may be used if a

TABLE CXXVI.—CARRYING CAPACITY OF GERMAN SILVER RHEOSTAT COILS. (18% Commercial German Silver.)

Size of Wire, B. & S.	Permissible Current in a ⅝-Inch Spiral, 3½ Inches Long, Stretched Horizontally to 7 Inches, for a Rise in Temperature of:					Decrease in Current Capacity for Every ¼ Inch Increase in Length of Spiral. Amps.
	50° C. (90° F.)	75° C. (135° F.)	100° C. (180° F.)	125° C. (225° F.)	150° C. (270° F.)	
10	9 23	12.03	14.04	16.15	17.03	1.100
11	7.75	10 07	11.79	13.46	14.29	.964
12	6.52	8.47	9.88	11.31	12.03	.812
13	5.48	7.15	8.32	9.50	10.11	.703
14	4.50	6.00	7.00	8.00	8.50	.600
15	3.89	5.03	5.86	6.71	7.15	.481
16	3.22	4.21	4.90	5.62	5.99	.406
17	2.73	3.59	4.13	4.71	5.04	.311
18	2.28	2.98	3.47	3.95	4.22	.287
19	1.89	2.46	2.90	3.30	3.48	.236
20	1.62	2.12	2.46	2.76	2.98	.203
21	1.36	1.78	2.10	2.35	2.50	.170
22	1.14	1.50	1.76	2.00	2.11	.149
23	.96	1.25	1.47	1.61	1.78	.126
24	.81	1.04	1.23	1.39	1.48	.105
25	.68	.88	1.03	1.19	1.24	.089
26	.57	.74	.87	.99	1.05	.074
27	.48	.63	.73	.83	.885	.063
28	.40	.52	.61	.70	.74	.052
29	.34	.44	.51	.59	.625	.039
30	.29	.37	.43	.50	.52	.037
31	.24	.31	.36	.41	.44	.031
32	.20	.26	.31	.35	.37	.027
33	.17	.22	.25	.29	.31	.022
34	.14	.19	.21	.245	.256	.019
35	.12	.16	.18	.205	.215	.016
36	.10	.13	.15	.174	.185	.013
37	.084	.11	.13	.144	.156	.011
38	.071	.092	.11	.121	.130	.009
39	.060	.077	.090	.100	.110	.008
40	.050	.065	.077	.088	.092	.007

slight shortcoming of the cross-section is immaterial. Table CXXIV., page 688, gives the maximum bare and outside diameters and the weights of flexible cables from 1,000,000 circular mils down to 26,244 circular mils area, the latter being

equivalent to a 6 No. B. & S. wire. The figures in the third column are based on the data given in Table CXXIII. for 343-wire cables; to obtain the diameter given in the fourth column, $\frac{3}{8}$ of an inch is added to the former dimensions, the thickness of insulation on flexible dynamo and power cables being usually about $\frac{3}{16}$ of an inch. Table CXXIV. will be found useful for designing the cable-lugs of brush holders and the cable-receiving parts of switches, etc.

Tables CXXV. and CXXVI., finally, contain all the data necessary for selecting the proper size of wire for resistance coils. Table CXXV., page 689, gives the safe current-carrying capacities, the resistance required, the specific resistance, the specific length, and the specific weight of *tinned iron* wires from No. 8 to No. 24 B. & S. gauge for wooden and iron rheostats as well as for starting boxes. The resistance of the shunt circuit corresponding to each current strength is calculated for the case of 110 volts; this resistance must be multiplied by 2 for 220 volts, by 5 for 550 volts, and by $\frac{E}{110}$, where E is the E. M. F. at shunt terminals, for any other voltage E. The resistance so obtained is the minimum resistance that should be provided for the voltage in question. Table CXXVI., page 690, gives the current capacities for temperature increases of 50°, 75°, 100°, 125°, and 150° C., respectively, of *German Silver* rheostat coils from No. 10 to No. 40 B. & S. wire. The figures are the results of tests made with spirals wound to a solid length of $3\frac{1}{2}$ inches on a $\frac{3}{8}$-inch mandrel, and afterwards stretched to a length of 7 inches; thus making the space between the turns equal to the diameter of the wire. The last column of this table serves to correct the values given for the 7-inch spiral in the case that its length, that is to say, its number of turns, is increased; twice the current strengths given in this column are to be deducted from the respective table-value for every additional inch length of spiral above 7 inches. Thus, for instance, the current capacity of an $8\frac{1}{2}$-inch spiral of No. 12 German Silver wire for a temperature increase of 125° C. is

$$11.31 - 2 \times (8\tfrac{1}{2} - 7) \times .812 = 11.31 - 2.44 = 8.87 \text{ amperes.}$$

Table CXXVI. refers to the so-called 18 per cent. German Silver, containing 18 per cent. of nickel, this alloy being usu-

ally employed for resistance coils. From a great number of tests it has been found that the average resistance of this material is a trifle over eighteen times the resistance of copper at 75° F. German Silver is also made with 30 per cent. of nickel, but in this composition it is now but rarely used for electrical purposes. The resistance of the 30 per cent. alloy is about twenty-eight times that of copper.

APPENDIX III.

LOCALIZATION AND REMEDY OF TROUBLES IN DYNAMOS AND MOTORS IN OPERATION.

LOCALIZATION AND REMEDY OF TROUBLES IN DYNAMOS AND MOTORS IN OPERATION.*

Classification of Dynamo Troubles.—The constructive expedients to be employed when designing a dynamo having been treated in the text, the following pages are intended for the consideration of the attendant in whose care the machine is placed, and upon whose competency depends, in no small measure, its efficiency and even its life. While general precautions and preventive measures do not always insure freedom from trouble, neglect and carelessness with any machinery are usually followed by accidents of some sort. The success of an electric plant depends to a very great extent upon the promptness with which the attendant is able to remedy such difficulties.

The usual troubles in dynamo-electric machinery may be classified as follows:

1. Sparking at commutator.
2. Heating of armature and field magnets.
3. Heating of commutator and brushes.
4. Heating of bearings.
5. Noises.
6. Abnormal speed.
7. Dynamo fails to generate.
8. Motor stops or fails to start.

1. Sparking at Commutator.—The most common trouble met with in running continuous-current dynamos and motors is the sparking at the commutator. Since excessive sparking wears, and eventually even destroys, the commu-

* Compiled from "Practical Management of Dynamos and Motors," by Crocker and Wheeler.

tator and brushes, and produces heat which may become injurious to the armature and bearings, it is of the greatest importance that sparking, upon its discovery, should be promptly checked.

CAUSES OF SPARKING.—Sparking may be due to any of the following causes: (1) Brushes not set at neutral points. (2) Brushes make poor contact with commutator. (3) Vibration, or chattering, of brushes. (4) Short-circuited or reversed coil in armature. (5) Weak field. (6) Unequal distribution of magnetism. (7) High resistance brush. (8) Vibration of machine. (9) Commutator rough, eccentric, or having "high" or "flat" bars. (10) Broken circuit in armature. (11) Ground in armature. (12) Armature overloaded.

PREVENTION OF SPARKING.—It is seen from the preceding that sparking at the commutator may be due to one, or more, of many causes. The first step in the speedy prevention of sparking, therefore, is the detection of the trouble that causes it. If due to any one cause, the finding of the trouble is comparatively easy, since in each case the symptoms are different and quite well pronounced; but when two or more causes combine in effecting excessive sparking, it requires more care to locate the faulty conditions.

FAULTY ADJUSTMENT.—Causes (1) to (3) are due to faulty adjustment of the brushes, and sparking in consequence of these causes can be easily prevented by resetting the brushes. In the first case, when the brushes are *not at neutral points*, they must be set exactly *opposite* each other, if the machine is *bipolar; 90°* apart, if it is *fourpolar; 60°*, if it has *six poles*, etc. The proper points of contact for so setting the brushes are best determined by counting the commutator bars.

To find the *opposite* bar to No. 1, add 1 to *half* the number of bars; to find the *90°* bar, add 1 to *one-quarter* the number of bars; to find the *60°* bar, add 1 to *one-sixth* the number of bars, etc. For instance, if the commutator has *48* bars, one-half the number is 24, and $24 + 1 = 25$, hence bar No. 25 is opposite bar No. 1; since one-quarter of 48 is 12, bar No. 13 is the one 90° from No. 1; and 1 added to one-sixth of 48,

or 8, gives No. 9 as the bar 60° from No. 1. Mark bars Nos. 1 and 25 in case of a bipolar machine, bars 1 and 13 in case of a four-pole machine, or 1 and 9 in case of a six-pole machine, and adjust the brushes so that each set rests on one of the marked bars, taking care that all the brushes of one set are exactly in line with each other, so that none is ahead or behind the others. This being done, the *rocker-arm* or *brush-yoke*, as the movable bracket is called to which the brush-holders are attached, is shifted backward or forward until sparking is reduced to a minimum, or disappears.

If the sparking is due to the second cause, *poor contact*, an examination of the brushes will show that they touch only at one corner, or only at one edge, or that there is dirt or oil between them and the commutator. In case of *metallic brushes*, they should then be filed or bent until they rest evenly on the commutator. *Carbon brushes* are fitted by pasting a band of sandpaper around the commutator and revolving the armature while the brushes are firmly pressed in their proper positions. In this manner, the tips of the brushes are hollowed out to the exact shape of the commutator, and after removal of the sandpaper, the brush-contact is found to be perfect.

The third cause, *vibration of brushes*, is frequently met with carbon brushes which are set *radially* to the commutator. Such brushes, when the commutator has become sticky, are thrown into rapid vibration by the running of the machine, and create a chattering noise. The ensuing spark is easily stopped by cleaning and slightly lubricating the commutator. In applying the lubricant, which may consist of ordinary machinery-oil, vaseline, or specially prepared *commutator-compound*, care should be exercised in using it very sparingly, preferably by rubbing it over the commutator with the finger. For, since all oils are non-conductors of electricity, too much lubrication will *insulate* the brushes from the commutator, and will thus prevent the machine from operating.

FAULTY CONSTRUCTION AND WRONG CONSTRUCTION.—Causes (4) to (8) are a consequence either of *faulty construction* of, or *wrong connections* in, the machine, and can be properly eliminated only by repairing the machine.

If a coil in the armature has been accidentally *short-circuited* or *reversed* while connecting up the armature, it will, when rotated in the magnetic field, generate local currents which give rise to sparking. Short-circuits in the coils are frequently caused by solder dropping between two commutator bars when fastening the wires to the commutator. Defects of this kind can be readily detected by careful inspection, and are easily remedied by removal of the obstruction. Reversals are due to mistaking the terminals of the armature coils, so that the end of a coil is connected to a bar where the beginning of the coil should be, or *vice versa.* In this case the faulty coils must be disconnected and properly reattached to the commutator.

A *weak field* may be due to a *break*, or to a *short-circuit*, or to a *ground* in the field coils. These faults can be easily repaired if they are external or accessible. When internal, the only remedy is to partially, or wholly, rewind and replace the faulty coil.

Unequal distribution of magnetism, which is usually traced to weakness of one of the pole-tips, causes one brush to spark more than the other. In this case the polepieces must be re-shaped so as to weaken the strong tip or to strengthen the weak tip.

The brushes of a machine must be of sufficient conductivity to carry the current generated by the armature without undue heating. Their material as well as their cross-section must therefore be suitably selected. Brushes of *abnormally high resistance* become very hot, cause sparking, and reduce the output of the machine. By using new brushes of proper material and thickness, the sparking due to this cause is immediately stopped.

Vibration of machine is usually due to an imperfectly balanced armature or pulley, to a bad belt, or to an unsteady foundation, or sometimes to poor design of the field frame. In the two former cases, re-balancing of the rotating parts, or relacing the belt, respectively, will remove the vibration; in the two latter cases, the weak parts must be strengthened or rebuilt.

WEAR AND TEAR.—Causes (9) to (11) are consequent upon wear and tear, and are therefore the most usual.

A commutator, after some time, shows *grooves* and *ridges*, wears more or less *eccentric*, and develops *projections* and *flats*. To avoid sparking from these causes, the commutator should from time to time be smoothed by means of a fine file or sandpaper, but *never* by means of emery-paper. Emery, containing the metal particles taken from the commutator, would lodge in the insulation between the bars, and would thus cause short-circuits, which would give rise to worse sparking than the unevenness of the commutator did before smoothing.

A *broken circuit* in the armature is either located in the commutator connection or in the coil itself. If the commutator connection has worked loose, the break can easily be repaired; but if the wire inside the coil has broken, the coil must be removed and the armature rewound.

Grounds in the armature are due to breakdown of the insulation between two wires or between a wire and the armature core. To repair a ground, the defective insulation must be replaced.

EXCESSIVE CURRENT.—If a machine is *overloaded*, cause (12), the winding is forced to carry too much current, which causes excessive heating of the armature. Sparking due to excessive current can be reduced by decreasing the load upon the machine, or by decreasing the strength of the magnetic field in the case of a dynamo, or increasing it in the case of a motor.

2. Heating of Armature and Field Magnets.—Injurious heating in a dynamo or motor can be detected by placing the hand on the various parts. If any part so examined feels so hot that the hand cannot remain in touch without discomfort, the safe limit of temperature has been passed, and the trouble should be remedied. If the heat has become so great as to produce *odor* or *smoke*, the current should be shut off and the machine stopped immediately. The above simple method of testing for heat should be applied shortly after the machine has been started up, because after the machine has run for some time, any abnormal heating effect will spread until all parts are nearly equal in temperature, and it will be almost impossible to locate the trouble.

Abnormal heating of the armature or field magnets may be due to *excessive current*, to *short-circuits*, to *moisture* in the coils, or to excessive generation of *eddy currents* in the armature core or polepieces.

Excessive current in the armature is remedied by reducing the load, or eliminating whatever other cause is responsible for it, which may be either a *short-circuit*, or a *leak*, or a *ground* on the *line*. To decrease the current in the field circuit, increase the field resistance by *inserting rheostat*, in case of a *shunt* machine, or by *shunting*, in case of a *series* machine, or by *both* of these methods in a *compound* machine.

If heating is caused by a *short-circuit in the winding*, the faulty coil must be located, and repaired or replaced by a new one.

The presence of *moisture* is revealed by the formation of *steam* when the machine becomes hot. A moist machine should be stopped and the moisture expelled by *baking* the affected part in a moderately warm oven for 4 or 5 hours, or by passing a current through it which should be regulated to maintain a temperature of about 75° Centrigrade.

Heating due to *eddy currents* in the iron is indicated by the fact that, after a short trial run, the iron parts will feel warmer than the windings; it is always the result of some constructional fault of the machine, such as insufficient lamination of the armature core, misproportioning of the armature teeth, improper arrangement of the polepieces, etc.

3. Heating of Commutator and Brushes.—Heating of the commutator and brushes may be due to *sparking* at the brushes, *arcing* in the commutator, or to *excessive resistance*.

Sparking always heats both the commutator and the brushes. Heating from this cause is checked by shifting the rocker arm, or by applying the proper remedies for the sparking. Frequently heating of the commutator is produced by the causes which induce sparking without being accompanied by the visible manifestation of the latter.

Arcing within the commutator, either between one bar and the next, or between the bars and the commutator shell, is due to *defective* or *punctured insulation;* if the heating is traced to this cause, the commutator must be taken apart and repaired.

Heating of commutator and brushes is often due to *insufficient contact area*, the cross-section of the brushes being too small to carry the current without overheating. In this case, either thicker brushes of the same material as the old ones, or new brushes of a higher-conductivity material, should be substituted.

Sometimes connections or joints in the brush-holder or cable terminal become loose by the vibration of the machine, and cause heating of the joints due to *increased contact resistance.* By tightening all contacts before every run, however, this trouble can always be prevented.

4. Heating of Bearings.—Heating of the bearings may arise from *lack of lubrication*, presence of *grit*, or from *friction* due to roughness or tight fit of shaft, faulty alignment of bearings, excessive belt pull, or to end-thrust or side-thrust of armature.

The first two causes, and also excessive belt tension, can be easily detected and remedied, while a rough, sprung, bent, or tight shaft has to be turned or filed down true in the lathe. When the bearings are out of line, they must be unscrewed and properly adjusted, by filing out the bolt holes if necessary, so that they will stay in line, with the armature central to the polepieces, when the screws are tightened.

Friction due to *end-thrust* may be relieved by lining up the belt, shifting armature collar or pulley, turning off shoulder on shaft, or filing off bearing until a sufficient clearance between the two is obtained. In case of *side-thrust*, the armature must be re-centered by adjusting the bearings or the polepieces.

Water or *ice cooling* of the bearings should only be used in cases of extreme necessity, and should never be attempted if there is the slightest danger of wetting the commutator or the armature.

5. Causes and Prevention of Noises in Dynamos.—The humming noise often issued by an electric machine is produced by *vibration* due to armature or pulley being *out of balance.* The armature and pulley should always be balanced separately by slowly rolling the armature, first without

and then with pulley, upon a knife-edge track and attaching weights to the places which show a tendency to remain on top. An excess of weight on one side of the armature and an equal excess on the opposite side of the pulley will not produce a balance when running, though it does when standing still; on the contrary, it will give the shaft a strong tendency to *wobble.* A perfect balance is only obtained when the weights are directly opposite in the same line perpendicular to the shaft.

Rattling noises are sometimes caused by *striking* of the armature against one or more of the polepieces, by *scraping* of the shaft collar or pulley hub against the bearing, or by *looseness of screws* or other parts. In these cases, the machine should be immediately stopped and the parts properly adjusted.

Singing or *hissing* of the brushes is usually occasioned by *rough* or *sticky commutator* or by *unevenness of the brushes*, especially when the commutator or the brushes are new, and have not yet worn smooth. A sparing application of oil or vaseline to the commutator, with a rag or the finger, will in most cases stop the noise. If not, shortening or lengthening of the brushes may be resorted to, and, if hissing still continues, it will be necessary to sandpaper, file, or turn down the commutator.

A *humming* sound is often heard in toothed-armature machines, due to the *sudden changes of magnetic conditions* as each tooth passes the edge of the polepieces. This sound is reduced or stopped by filing away the ends of the polepieces, so that the armature teeth pass each pole-edge gradually.

Slipping of the belt causes an intermittent *squeaking* noise, which can be stopped by tightening the belt. If the belt is *poorly laced*, so that the joint is rough, a *pounding* is emitted every time the joint passes over the pulleys. A properly made joint will immediately obviate this difficulty.

6. Adjustment of Speed.—Too high or too low speed is generally a serious matter in either dynamo or motor, and it is always desirable and often imperative to shut off immediately, and make a careful investigation of the trouble.

Low speed in a *motor* may be caused by *overloading*, or *short-circuits*, or *grounds* in armature, or by *excessive friction* in the

bearings. Accidental *weakness of the field* due to a break, short-circuit, or ground in the field coils, or to weakness of the field current, has the effect, on a *constant-potential* circuit, of making a motor run too fast if lightly loaded, or too slow if heavily loaded, or even to stop or to run backward if the overload is excessive. A *series motor* on a *constant-potential* circuit, or *any* motor on a *constant-current* circuit, is liable to run too fast if the load is very much reduced. Constant-current motors, except when directly coupled, must, therefore, be provided with *automatic governors* or *cut-outs*, which act to reduce the power if the speed becomes too great.

7. Failure of Self-Excitation.—The inability of a generator to *excite* or *build up* its field magnetism is in most cases due to weakness or absence of the *residual magnetism*, caused either by vibration or jar, or by counter-magnetization effected by the proximity of another dynamo or by the earth's magnetism, or by accidentally reversed current through the fields.

Residual magnetism can be restored by sending a current from any dynamo or battery through the field coil; if, upon starting the machine, it fails to generate, apply the battery current in the opposite direction, since the magnets may have enough polarity to prevent the battery from building them up in the direction first tried. By shifting the brushes backward from the neutral point, the armature magnetization can be made to assist the field.

Other causes for dynamo failing to generate are *reversed connections*, *reversed direction of rotation*, *short-circuit* in machine or external circuit, *connection of field coils in opposition*, *open circuit*, and *faulty position of brushes.* Any of these troubles can be detected by carefully inspecting and testing the machine, and, when found, can readily be eliminated.

A *break*, *poor contact*, or *excessive resistance* in the field circuit or regulator of a *shunt dynamo* will make the magnetization weak, and prevent its building up. This may be detected and overcome by cutting out the rheostat for a moment, taking care not to make a short-circuit.

An abnormally *high resistance* anywhere in the circuit of a *series machine* will prevent it from generating, since the field coil is in the main circuit. The magnetization in this case

can be started up by short-circuiting the machine for an instant. Either of the above two expedients should be applied very carefully, and not until the polepieces have been tested with a piece of iron to make sure that the magnetization is weak.

8. Failure of Motor.—The stopping of a motor, or its refusal to start, may be due to *excessive overload*, to *open circuit*, or to a *wrong connection*.

While a moderate overload causes the motor to run below its normal speed, an *excessive overload* will stop it entirely. The abnormal load is not always exterior to the motor, but may be due to unusual friction in the motor itself, occasioned by jamming of the shaft, bearings, or other parts, or by armature touching polepieces. In either case, the current should be turned off instantly, the load or the friction reduced, and the current supplied again for a moment, just long enough to see if trouble still exists. If the motor comes promptly up to speed, it may then be left in circuit.

If the stopping of the motor is caused by *open circuit*, a melted fuse or broken wire will be found upon examination of the motor, provided its brushes are in contact and the external circuit is in proper order. If there is no visible break, the armature and field coils must be tested for continuity by means of a battery (or magneto) and electric bell. On a constant-potential circuit, if current is excessive, it indicates a *short-circuit*. If the field is at fault, the polepieces will be found non-magnetic, or but very weakly magnetized.

The possible complications of *wrong connections* are so great that a very careful and systematic examination and comparison of the connections with the diagram furnished by the manufacturer is necessary to locate the trouble.

INDEX.

(*Numbers indicate pages.*)

E

F

L

M

N

Q

R

S